14.13 Economics and Psychology
MIT Spring 04

Problem Set #2

1 Optimism and Pessimism of PT Maximizers

Tim owns a house in the Boston area. His company offered him a job in Europe that he accepted. In consequence, he decided to sell the house. He does not have much time, thus he just plan to post a take-it-or-leave-it offer at price x. For any price $x \in [1; 2]$ in millions of dollars, Tim assesses the probability q of closing the deal to equal

$$q = 2 - x$$

If he doesn't find a buyer, he can always sell it to a friend for $1million.

Tim is a Prospect Theory maximizer and he integrates over different accounts (house and money). In particular, he values any two-outcome distribution of changes to his reference point, say s with probability p and t with probability $1 - p$ at

$$V = v(t) + (v(s) - v(t))\, p$$

whenever $s > t \geq 0$ or $s < t \leq 0$. Here

$$v(z) = |z|^{\frac{1}{2}} \text{ if } z \geq 0 \text{ and } v(z) = -2\,|z|^{\frac{1}{2}} \text{ if } z < 0$$

Tim's reference point already includes all the changes required by the move to Europe other than the sale of the house.

1. [6 points] Assume that Tim is a pessimist and his reference point is based on presumption that he sells the house for $1million. Thus, he will see it as a gain of $x - 1$ if he obtains x higher than that. What price x would Tim ask for?

2. [6 points] Now, assume that Tim is an optimist and his reference point is based on presumption that he sells the house for $2million. Thus, he will see any price x below this as a loss of $2 - x$. What price x would Tim ask for?

3. [5 points] Is there a difference between prices in questions 1 and 2? If not, try to explain why not. If yes, tell which one is higher and explain intuitively why the prices are different.

2 Lucas Calculation

Remind from class the Lucas discussion of the loss of welfare due to the business cycle. The welfare is

$$V = Eu\left(c + \varepsilon_t\right)$$

where c is an average consumption, and ε_t is the random cyclic element equal to $-\sigma c$ with probability 1/2 and $+\sigma c$ with probability 1/2. We measure the welfare loss associated with the business cycle $\varepsilon_t$by the fraction Λ of consumption that people would accept to give up in order to avoid consumption variability. This means that Lucas' Λ solves

$$V = Eu\left(c + \varepsilon_t\right) = u\left(\left(1 - \Lambda\right) c\right)$$

1. [6 points] Assume that the agent is an EU maximizer with

$$u\left(c_t\right) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

for some positive $\gamma \neq 1$. Show that for small σ,

$$\Lambda \simeq \frac{\gamma}{2}\sigma^2$$

2. Now, assume that the agent is a Prospect Theory maximizer with reference point at c, and

$$v\left(c_t\right) = \frac{\left(c_t - c\right)^{1-\gamma}}{1-\gamma} \text{ if } c_t \geq c \text{ and } v\left(c_t\right) = -\lambda \frac{\left|c_t - c\right|^{1-\gamma}}{1-\gamma} \text{ if } c_t < c$$

for $\lambda > 1$ and some positive $\gamma \in (0, 1)$. Let us modify the definition of Lucas' welfare loss Λ^{PT} so that agents are indifferent between Prospect A: $c_t = c + \varepsilon_t$, $\varepsilon_t = \pm\sigma c$ as above, and Prospect B, a constant $c_t = \left(1 - \Lambda\right) c$.

   (a) [3 points] Calculate the PT value $V^{PT}\left(A\right)$ of prospect A

   (b) [3 points] Calculate the PT value $V^{PT}\left(B\right)$ of prospect B

   (c) [3 points] Calculate the value of Λ^{PT} that makes PT agents indifferent between A and B.Show it follows "first order risk aversion" for small σ.

   (d) [3 points] For small σ, which is bigger, Λ^{EU} or Λ^{PT}?

3. [extra points – harder] . Suppose that the agent consumes over many periods $c_t = c + \varepsilon_t$, ε_t independent and identically distributed, distribution as above. So his EU utility is:

$$V^{EU} = E\left[\sum_{t=1}^{T} \frac{c_t^{1-\gamma}}{1-\gamma}\right]$$

Units for t are months.

   (a) [2 points] Write down the equivalent PT value, for a 1 month frame.

(b) [2 points] Write down the equivalent PT value, for a "2 month frame" in which consumptions are put in lumps of 2 months. This is, the agents consider prospects of $(c_t + c_{t+1})$, $(c_{t+2} + c_{t+3})$, ... We assume that T is even. The reference point for those 2 month frames is $2c$.

(c) [4 points] Which frame makes PT agents better off, the 1 month frame or the 2 month frame? You can just give an intuition for the answer, or (better) a derivation.

3 Heuristics and biases

[17 points] Choose one among the heuristics and biases discussed in class and provide a real life example when it is *important*. Describe the situation, the bias and analyze the consequences of the bias in the situation you present.