14.13 Economics and Psychology
MIT Spring 04

Problem Set #4

1 Learning about Experiential Utility

Consider a family of utilities from consuming quantity x of good X and quantity y of good Y:

$$U(x, y; \theta) = x + \theta\sqrt{y} + \varepsilon$$

where ε is distributed according to the normal distribution $N(0, 1)$ and independent of x and y, and θ is a parameter. The prices of both goods are $p_X = p_Y = 1$.

A consumer experiential utility has the above form with $\theta = 1$. Her time 1 decision utility however has $\theta = 2$. She is a bayesian updater. THE CONSUMER OBSERVES HER FIRST PERIOD EXPERIENTIAL UTILITY u BEFORE SECOND PERIOD CHOICE BUT SHE DOES NOT OBSERVE θ NOR ε.

1. [8 points] At time 1 the consumer spends total of 2 on the portfolio of both goods. How much of good Y is she buying?

2. Assume, that the consumer beliefs that experientially $\theta = 2$ for sure. She observes her experiential utility in period 1, updates her belief on experiential θ, and then buys goods X and Y for period 2 consumption, spending the total of 4.

(a) [4 points] What is her time 2 belief about θ?

(b) [4 points] How much does she spend on good Y if she wants to maximize her experiential utility at time 2?

3. [9 points] Now, assume that the consumer initially believes that experiential θ is distributed according to the normal distribution $N(2, 1)$. She observes her time 1 utility and updates her belief about experiential θ. Compute the expected value of consumer's updated belief about θ. Hint: the expected value is a function of time 1 realization of experiential utility u.

2 Intertemporal choice

Consider a consumer with temporaneous utility of consumption

$$u(c) = \frac{c^{1-\rho} - 1}{1 - \rho}$$

for some parameter $\rho \in (0, 1)$ who discounts future temporanous utilities that are one period ahead by δ_1 and utilities that are two period ahead by δ_2. The consumer has positive wealth W_0 and is going to consume it over three periods $0, 1, 2$. He has access to bank deposits which pays the interest rate $r > 0$. Thus we can formulate his problem as:

$$\max_{\{c_0, c_1, c_2\}} u(c_0) + \delta_1 u(c_1) + \delta_2 u(c_2)$$

subject to the constraint:

$$W_0 = c_0 + \frac{c_1}{1 + r} + \frac{c_2}{(1 + r)^2}.$$

1. [15 points] Compute consumptions c_0, c_1, c_2.

2. [10 points] What condition on δ_1 and δ_2 ensures time consistency of the consumer?