14.13 Behavioral Economics (Lecture 15)

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1 Problems with happiness research

- Measurement problem. Even if the true relation between happiness was linear ($u = ky$), you will get a concave pattern if the scale of happiness is bounded and the scale of wealth is unbounded.
  - the meaning of 7 on the scale 1,2,...,10 changes.
    * Similarly, if you ask people how high they are on the scale 1,2,...,10 then the average answer may not differ across years even if height goes up

- Problem with relative consumption theory
  - people don't take steps to change locations to be ahead of others

2 Self-control problems and hyperbolic discounting

Reading: Thaler, chapter on Intertemporal Choice, *Winner's Curse.*

2.1 Orthodox intertemporal choice

2.1.1 A three-period consumption problem

- Assume that at date 0 the individual picks consumption $c_0.c_1, c_2$ for dates 0, 1, and 2. He has wealth W_0 and can put savings in the bank with interest rate r
  - at time 1 he has wealth $W_1 = (1 + r)(W_0 - c_0)$
  - at time 2 he has wealth $W_2 = (1 + r)(W_1 - c_1)$
  - at time 2 he consumes remaining wealth $c_2 = W_2$.
  - The agent maximizes $u(c_0, c_1, c_2)$ over $c_0.c_1, c_2, W_1, W_2$ subject to the above three constraints.

- We can substitute out for W_1, W_2 and reduce the consumption problem to:

$$\max_{\{c_0, c_1, c_2\}} u\left(c_0, c_1, c_2\right)$$

subject to the constraint:

$$W_0 = c_0 + \frac{c_1}{(1+r)} + \frac{c_2}{(1+r)^2}.$$

- In other words, the NPV of consumption is equal to the value of wealth, W_0, at period 0.

- For a $T+1$ period problem the constraint would take the form

$$W_0 = c_0 + \frac{c_1}{(1+r)} + \frac{c_2}{(1+r)^2} + ... + \frac{c_T}{(1+r)^T}.$$

- Postulate

$$u(c_0, c_1, c_2) = \Delta_0 u(c_0) + \Delta_1 u(c_1) + \Delta_2 u(c_2)$$

- To solve this problem, set up a Lagrangian:

$$\max_{\{c_0,c_1,c_2\}} \Delta_0 u(c_0) + \Delta_1 u(c_1) + \Delta_2 u(c_2) \\ + \lambda \left[W_0 - \left(c_0 + \frac{c_1}{(1+r)} + \frac{c_2}{(1+r)^2} \right) \right]$$

- In general when the Lagrangian maximization is $\max L = F - \lambda G$ the first order conditions are

$$\frac{\partial}{\partial c_i} L = 0.$$

- So, our first order condition's are:

$$\Delta_0 u'(c_0) = \lambda$$
$$\Delta_1 R u'(c_1) = \lambda$$
$$\Delta_2 R^2 u'(c_2) = \lambda$$

and the budget constraint is

$$W_0 = c_0 + \frac{c_1}{(1+r)} + \frac{c_2}{(1+r)^2}.$$

- To solve specify $u(c) = \ln c$.

- Then

$$c_t = \Delta_t (1+r)^t \frac{1}{\lambda}$$

- This leads to

$$W_0 = \sum_{t=0}^{2} \frac{\Delta_t}{\lambda}$$

or

$$\frac{1}{\lambda} = \frac{W_0}{\sum_{t=0}^{2} \Delta_t}$$

- Thus

$$c_t = (1+r)^t \frac{\Delta_t}{\sum_{t=0}^{2} \Delta_t} W_0$$

- Assume that $r = 0$. Then

$$c_t = \frac{\Delta_t}{\sum_{t=0}^{2} \Delta_t} W_0$$

2.1.2 Time Consistency

- Imagine that the agent can reoptimize at times 1 and 2. Will he stick to time 0 determind consumption path?

- Assume $W_0 = 1$.

- At time 1 the wealth is $W_1 = (1 + r)(W_0 - c_0) = 1 - c_0 = \frac{\Delta_1 + \Delta_2}{\sum_{t=0}^{2} \Delta_t}$.

- The two period problem is analogous to the three period one we just considered with consumer consuming c_1', c_2' at times 1 and 2, respectively.

- Postulate that the weights he employes Δ_1',Δ_2' are the same as Δ_0,Δ_1, respectively (i.e. the weights depend only on distance in time from present).

- The agent consumes

$$c_1' = \frac{\Delta_0}{\sum_{t=0}^{1} \Delta_t} \frac{\Delta_1 + \Delta_2}{\sum_{t=0}^{2} \Delta_t}$$

$$c_2' = \frac{\Delta_1}{\sum_{t=0}^{1} \Delta_t} \frac{\Delta_1 + \Delta_2}{\sum_{t=0}^{2} \Delta_t}$$

- Hence

$$\frac{c_2'}{c_1'} = \frac{\Delta_1}{\Delta_0}.$$

- If those consumptions c_0', c_1' are the same as c_1, c_2 planned at original time 0, then

$$\frac{c_1'}{c_0'} = \frac{c_2}{c_1} = \frac{\Delta_2}{\Delta_1}.$$

- Hence, the condition of consistency is

$$\frac{\Delta_1}{\Delta_0} = \frac{\Delta_2}{\Delta_1}.$$

- In other words, there is α such that

$$\begin{aligned} \Delta_1 &= \alpha \Delta_0 \\ \Delta_2 &= \alpha \Delta_1 = \alpha^2 \Delta_0. \end{aligned}$$

- Proposition. There is no time inconsistency iff there exist Δ_0 and α such that

$$\Delta_t = \alpha^t \Delta_0,$$

i.e.

$$u = \Delta_0 \left(\ln c_0 + \alpha \ln c_1 + \alpha^2 \ln c_2 \right).$$

- Proposition. If the agent, at $t = 0$, maximizes

$$V_{t=0} = \sum_{t=0}^{T} \Delta_s u\left(c_s\right)$$

and at $t = t_1$ he maximizes

$$V_{t=t_1} = \sum_{t=0}^{T-t_1} \Delta_s u\left(c_{t_1+s}\right)$$

the the decision problem is time consistent iff there exist Δ_0 and α such that

$$\Delta_t = \Delta_0 \alpha^t.$$

Time consistent means that the optimal consumption $\left(c_1^*, c_2^*, ..., c_t^*\right)$ decided at time 0 is always optimal at any $t_1 > 0$.