14.13 Lecture 16

Xavier Gabaix

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1 Is exponential discounting (and hence dynamic consistency) a good assumption?

The property of *dynamic consistency* is appealing.

- Early selves and late selves agree!
  - self$_{t=0}$ decides C_0 and plans for C_1, C_2...
  - self$_{t=1}$ decides C_1 and plans for C_2, C_3...
- Can simply maximize at beginning of problem without worrying about later selves overturning the decisions of early selves. But, sometimes there does appear to be a conflict between early selves and late selves:

- I'll quit smoking next week...
- I'll start the problem set early, so I won't need to work all night...
- I'll go to sleep now, but get up early so I can finish the problem set...
- I'll exercise this weekend...
- I'll eat better food...
- I'll call my grandparents next week...
- I'll start studying for my finals at the beginning of reading period....
- I'll stop procrastinating on my term paper...

Early selves say “be good” (get up at 7 to finish problem set)

Late selves want “instant gratification” (keep hitting snooze button)

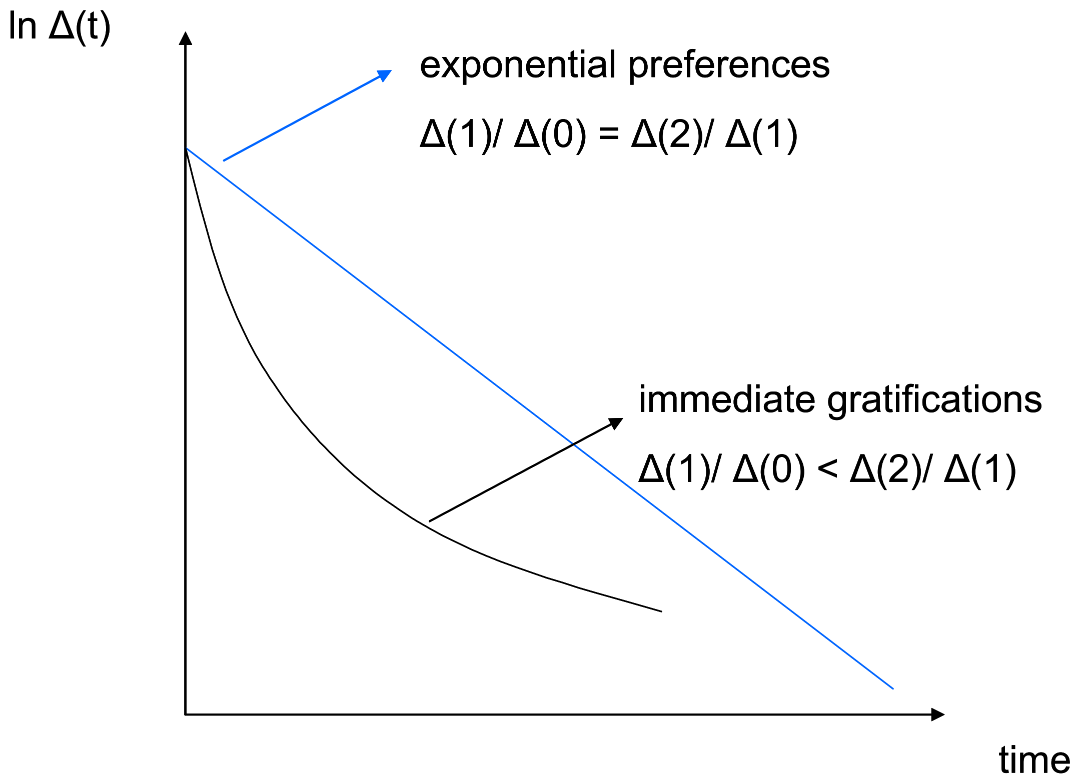
When discount functions are not exponential, the intertemporal choice model generates a conflict between early selves and late selves: dynamic inconsistency.

Dynamically inconsistent model predicts “self-control problems” like procrastination, laziness, addiction, etc...

Motivation for dynamically inconsistent preferences: Measured discount functions don't appear to be exponential.

Instead, short-run discount rates are measured to be higher than long-run discount rates.

Early selves want later selves to be patient. Later selves don't want to be patient.



ln Δ(t)
exponential preferences
Δ(1)/ Δ(0) = Δ(2)/ Δ(1)
immediate gratifications
Δ(1)/ Δ(0) < Δ(2)/ Δ(1)
time

2 Discounting evidence

Thaler (1981)

- What amount makes you indifferent between \$15 today and $\$X$ in 1 month? ($X = 20$)

If your preferences were exponential, the initial utility is

$$V_0 = \sum_t \delta^t u(c_t)$$

where t is expressed in years. Call V' the utility from accepting \$15 today and V'' the utility form accepting $\$X$ in 1 month

$$V' - V_0 = u(c_0 + 15) - u(c_0)$$
$$V'' - V_0 = \delta^t \left(u(c_t + X) - u(c_t)\right)$$

You are indifferent iff

$$V' - V_0 = V'' - V_0$$

$$\iff u(c_0 + 15) - u(c_0) = \delta^t \left(u(c_t + X) - u(c_t)\right)$$

$$\iff 15u'(c_0) = \delta^t X \ u'(c_t)$$

by Taylor expansion as $15 \ll c_0$ and $X \ll c_t$.

Assume now that $c_0 \simeq c_t$

$$\iff 15 = \delta^t X$$

$$\iff -\ln \delta = \frac{1}{t} \ln \frac{X}{15}$$

- What is a "reasonable" δ?
  - Economists will say that at a yearly horizon, $\delta \simeq 0.95$.
  - Why? People solve

$$\max_{c_0 + \frac{c_1}{1+r} = W} u(c_0) + \delta u(c_1)$$

$$L = u(c_0) + \delta u(c_1) - \lambda \left(c_0 + \frac{c_1}{1+r} \right)$$

$$\begin{cases} \frac{\partial L}{\partial c_0} = 0 \\ \frac{\partial L}{\partial c_1} = 0 \end{cases}$$
$$\Leftrightarrow \begin{cases} u'(c_0) - \lambda = 0 \\ \delta u'(c_1) - \frac{\lambda}{1+r} = 0 \end{cases}$$
$$\Rightarrow u'(c_0) = \delta(1+r)u'(c_1)$$

which is the Euler's equation

One can observe that at the **macroeconomic** level $c_0 \simeq c_1$ which implies

$$\delta(1+r) = 1 \Rightarrow \delta = \frac{1}{1+r} \simeq 1 - r = 0.95$$

where the equilibrium level of the interest rate is $r = 5\%$ per year.

- At the **microeconomic** level

$$
\begin{aligned}
-\ln \delta &= \frac{1}{t} \ln X/15 \\
&= \frac{1}{1/12} \ln 20/15 \\
&= 345\% \text{ per year}
\end{aligned}
$$

- Why?
  - different attitudes towards small amounts and large amounts
  - borrowing constraints

- What makes you indifferent between \$15 today and $\$X$ in ten years? ($X = 100$)

$$
\begin{aligned}
-\ln \delta &= \frac{1}{\tau} \ln X/15 \\
&= \frac{1}{10} \ln X/15 \\
&= 19\% \text{ per year}
\end{aligned}
$$

Benzion, Rapoport and Yagil (1989)

- What amount makes you indifferent between \$40 today and $\$X$ in half a year? ($X = 50$)

$$40 = X\delta^{\tau}$$

so

$$\begin{aligned} -\ln\delta &= \frac{1}{\tau}\ln X/40 \\ &= \frac{1}{.5}\ln X/40 \\ &= 45\% \text{ per year} \end{aligned}$$

- What makes you indifferent between \$40 today and $\$X$ in four years? $(X = 90)$

$$\begin{aligned} -\ln \delta &= \frac{1}{\tau} \ln X/40 \\ &= \frac{1}{4} \ln X/40 \\ &= 20\% \text{ per year} \end{aligned}$$

- In most experiments, shifting out both rewards by the same amount of time lowers the implied discount rate (e.g., Kirby and Herrnstein, *Psychological Science*, 1996).

- For example, \$45 right now is preferred to \$52 in 27 days.

$$-\ln \delta > \frac{1}{27/365} \ln 52/45$$
$$= 195\% \text{ per year}$$

- But, \$45 in six days is inferior to \$52 in 33 days (now $-\ln \delta < 195\%$ per year).

- With exponential discounting, no preference reversal i.e. if X now $> Y$ in Δt, then X at $t > Y$ at $t + \Delta t$. Indeed

- X now $> Y$ in $\Delta t \Leftrightarrow X \geq \delta^{\Delta t} Y$
- X at $t > Y$ at $t + \Delta t \Leftrightarrow \delta^t X \geq \delta^{t+\Delta t} Y$
- here $X = \$45$, $Y = \$52$, $\Delta t = 27$days and $t = 6$days.

Vast body of experimental evidence, demonstrates that discount rates are higher in the short-run than in the long-run.

Consider a final thought experiment:

- Choose a ten minute break today or a fifteen minute break tomorrow.
- Choose a ten minute break in 100 days or a fifteen minute break in 101 days.

- If $V = \sum \Delta(t) u(c_t)$, what is $\Delta(t)$
  - big reward: U_B
  - small reward: U_S
  - $t_1 = 1$ day, $t = 100$ days

$$\left.\begin{array}{c} U_S\ \Delta(0) > U_B\ \Delta(t_1) \\ U_S\ \Delta(t) < U_B\ \Delta(t+t_1) \end{array}\right\} \Rightarrow \frac{\Delta(t_1)}{\Delta(0)} < \frac{U_S}{U_B} < \frac{\Delta(t+t_1)}{\Delta(t)}$$

$$\text{minus 1 and divide by } t_1 \text{ both sides } \Rightarrow \frac{\frac{\Delta(t_1)-\Delta(0)}{t_1}}{\Delta(0)} < \frac{\frac{\Delta(t+t_1)-\Delta(t)}{t_1}}{\Delta(t)}$$

$$\text{when } t_1 \longrightarrow 0,\ \frac{\Delta'(0)}{\Delta(0)} < \frac{\Delta'(t)}{\Delta(t)} \tag{1}$$

Rewrite the discounting function as

$$\Delta(t) = e^{-\int_0^t \rho(s)ds}$$

where ρ is the discount rate or the rate of time discounting (it measures the impatience), the higher ρ the more impatient.
Note that with exponential preferences $\rho(s) = -\ln \delta$ as $\Delta(t) = e^{t \ln \delta}$.

Generalize (1) for $t > \tau > 0$

$$\frac{\Delta'(0)}{\Delta(0)} < \frac{\Delta'(\tau)}{\Delta(\tau)} < \frac{\Delta'(t)}{\Delta(t)}$$

and note that $\frac{\Delta'(t)}{\Delta(t)} = -\rho(t)$

$$\Rightarrow \rho(0) > \rho(\tau) > \rho(t)$$

i.e. ρ is decreasing