14.13 Economics and Psychology (Lecture 4)

Xavier Gabaix

February 12, 2004

1 Prospect Theory value of the game

Consider gambles with two outcomes: x with probability p, and y with probability $1 - p$ where $x \geq 0 \geq y$.

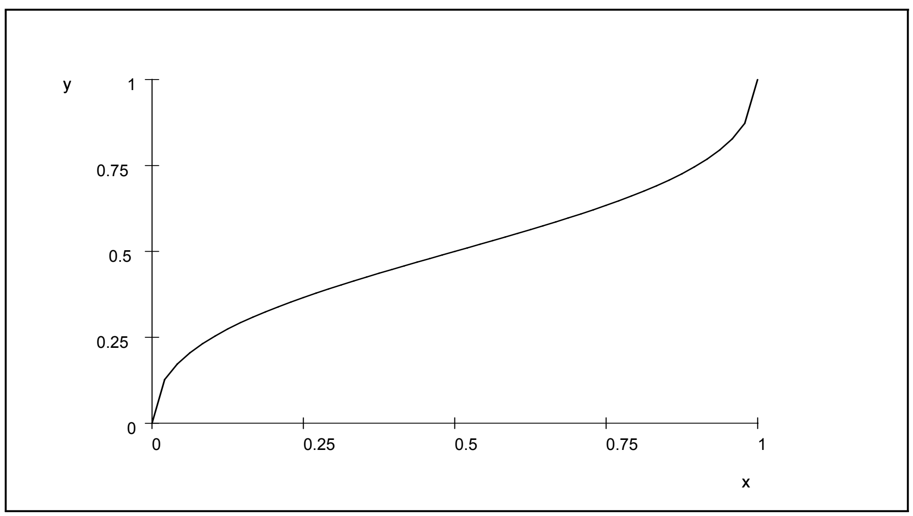
The PT value of the game is

$$V = \pi(p)\, u(x) + \pi(1 - p)\, u(y)$$

- In prospect theory the probability weighting π is concave first and then convex, e.g.

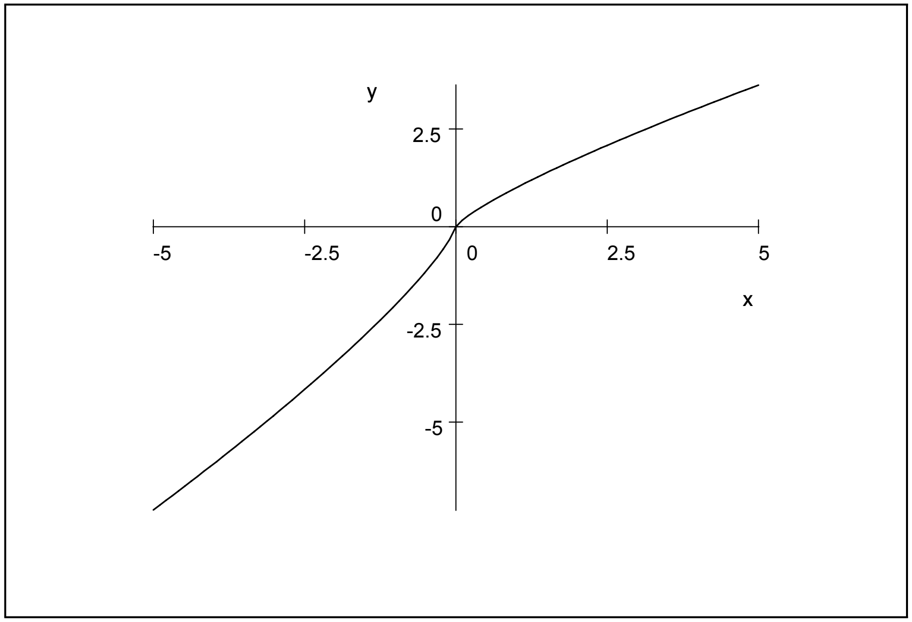
$$\pi(p) = \frac{p^{\beta}}{p^{\beta} + (1-p)^{\beta}}$$

for some $\beta \in (0,1)$. In the figure below p is on the horizontal axis and $\pi(p)$ on the vertical one.



- A useful parametrization of the PT value function is a power law function

$$u(x) = |x|^{\alpha} \text{ for } x \geq 0$$
$$u(x) = -\lambda |x|^{\alpha} \text{ for } x \leq 0$$



Meaning - Fourfold pattern of risk aversion u

- Risk aversion in the domain of likely gains
- Risk seeking in the domain of unlikely gains
- Risk seeking in the domain of likely losses
- Risk aversion in the domain of unlikely losses

See tables on next page.

Preferences between Positive and Negative Prospects

Positive Prospects			Negative Prospects		
Problem 3: (4,000, .80)	<	(3000).	Problem 3': (-4,000, .80)	>	(-3000).
$N = 95$	[20]	[80]*	$N = 95$	[92]*	[8]
Problem 4: (4,000, .20)	>	(3,000, .25).	Problem 4': (-4,000, .20)	<	(-3,000, .25).
$N = 95$	[65]*	[35]	$N = 95$	[42]	[58]
Problem 7: (3,000, .90)	>	(6,000, .45).	Problem 7': (-3,000, .90)	<	(-6,000, .45).
$N = 66$	[86]*	[14]	$N = 66$	[8]	[92]*
Problem 8: (3,000, .002)	<	(6,000, .001).	Problem 8': (-3,000, .002)	>	(-6,000, .001).
$N = 66$	[27]	[73]*	$N = 66$	[70]*	[30]

Percentage of Risk-Seeking Choices

	Gain		Loss	
Subject	$p \leq .1$	$p \geq .5$	$p \leq .1$	$p \geq .5$
1	100	38	30	100
2	85	33	20	75
3	100	10	0	93
4	71	0	30	58
5	83	0	20	100
6	100	5	0	100
7	100	10	30	86
8	87	0	10	100
9	16	0	80	100
10	83	0	0	93
11	100	26	0	100
12	100	16	10	100
13	87	0	10	94
14	100	21	30	100
15	66	0	30	100
16	60	5	10	100
17	100	15	20	100
18	100	22	10	93
19	60	10	60	63
20	100	5	0	81
21	100	0	0	100
22	100	0	0	92
23	100	31	0	100
24	71	0	80	100
25	100	0	10	87
Risk seeking	78[a]	10	20	87[a]
Risk neutral	12	2	0	7
Risk averse	10	88[a]	80[a]	6

[a] Values that correspond to the fourfold pattern.

Note: The percentage of risk-seeking choices is given for low ($p \leq .1$) and high ($p \geq .5$) probabilities of gain and loss for each subject (risk-neutral choices were excluded). The overall percentage of risk-seeking, risk-neutral, and risk-averse choices for each type of prospect appears at the bottom of the table.

Properties of power law PT value functions

- they are scale invariant, i.e. for any $k > 0$

Consider a gamble and the same gamble scaled up by k:

$$g = \begin{cases} x & \text{with prob } p \\ y & \text{with prob } 1 - p \end{cases}$$

$$kg = \begin{cases} kx & \text{with prob } p \\ ky & \text{with prob } 1 - p \end{cases}$$

then

$$V^{PT}(kg) = k^{\alpha} V^{PT}(g)$$

- if someone prefers g to g' then he will prefer kg to kg' for $k > 0$

- if $x, y \geq 0$, $V(-g) = -\lambda V(g)$

- if $x', y' \geq 0$ and someone prefers g to g' then he will prefer $-g'$ to $-g$

2 How robust are the results?

- Very robust: loss aversion at the reference point, $\lambda > 1$

- Medium robust: convexity of u for $x < 0$

- Slightly robust: underweighting and overweighting of probabilities $\pi(p) \gtrless p$

3 In applications we often use a simplified PT (prospect theory):

$$\pi(p) = p$$

and

$$u(x) = x \text{ for } x \geq 0$$
$$u(x) = \lambda x \text{ for } x \leq 0$$

4 Second order risk aversion of EU

- Consider a gamble σ and $-\sigma$ with $50:50$ chances.

- Question: what risk premium Π would people pay to avoid the small risk σ?

- We will show that as $\sigma \to 0$ this premium is $O\left(\sigma^2\right)$. This is called *second order risk aversion*.

- In fact we will show that for twice continuously differentiable utilities:

$$\Pi(\sigma) \cong \frac{\rho}{2}\sigma^2,$$

  where ρ is the curvature of u at 0 that is $\rho = -\frac{u''}{u'}$.

- Let's generalize and consider an agent starting with wealth x. The agent takes the gamble iff:

$$B(\Pi) = \frac{1}{2}u(x + \Pi + \sigma) + \frac{1}{2}u(x + \Pi - \sigma) \geq u(x)$$

i.e. $\Pi \geq \Pi^*$ where:

$$B(\Pi^*) = u(x)$$

- Assume that u is twice differentiable and take the Taylor expansion of $B(\Pi)$ for small σ and Π:

$$u(x+\Pi+\sigma) = u(x) + u'(x)(\Pi+\sigma) + \frac{1}{2}u''(x)(\Pi+\sigma)^2 + o(\Pi+\sigma)^2$$

$$u(x+\Pi-\sigma) = u(x) + u'(x)(\Pi-\sigma) + \frac{1}{2}u''(x)(\Pi-\sigma)^2 + o(\Pi-\sigma)^2$$

hence

$$B(\Pi) = u(x) + u'(x)\Pi + \frac{1}{2}u''(x)\left[\sigma^2+\Pi^2\right] + o\left(\sigma^2+\Pi^2\right)$$

Then use the definition $B(\Pi^*) = u(x)$ to get

$$\Pi^* = \frac{\rho}{2}\left[\sigma^2+\Pi^{*2}\right] + o\left(\sigma^2+\Pi^{*2}\right)$$

- to solve : $\Pi^* = \frac{\rho}{2}\left[\sigma^2+\Pi^{*2}\right]$ for small σ, call $\rho' = \rho/2$.

- *Barbaric way*:

  - find the roots of $\Pi^{*2} - \frac{1}{\rho'}\Pi^* + \sigma^2 = 0$.

    * compute the discriminant

    $$\Delta = \frac{1}{\rho'^2} - 4\sigma^2$$

    * the roots are $\Pi^* = \frac{1}{2\rho'} \pm \frac{1}{2}\left(\frac{1}{\rho'^2} - 4\sigma^2\right)^{\frac{1}{2}}$

    $$\Pi^* = \frac{1}{2\rho'} \pm \frac{1}{2}\left(\frac{1}{\rho'^2} - 4\sigma^2\right)^{\frac{1}{2}}$$

    * as when there is no risk, the risk premium should be 0, then the

relevant root is:

$$\Pi^* = \frac{1}{2\rho'} - \frac{1}{2}\left(\frac{1}{\rho'^2} - 4\sigma^2\right)^{\frac{1}{2}}$$

– take the Taylor expansion for small σ

$$\begin{aligned}\Pi^* &= \frac{1}{2\rho'} - \frac{1}{2\rho'}\left(1 - 4\rho'^2\sigma^2\right)^{\frac{1}{2}} \\ &= \frac{1}{2\rho'} - \frac{1}{2\rho'}\left(1 - \frac{1}{2}4\rho'^2\sigma^2 + o(\sigma^2)\right) \\ &= \rho'\sigma^2\end{aligned}$$

– then remember that $\rho' = \rho/2$:

$$\boxed{\Pi^* = \frac{\rho}{2}\sigma^2}$$