Problem 1.1 *(Phase Transitions in the Erdös-Renyi Model)*
Consider an Erdös-Renyi random graph $G(n, p)$.

(a) Let A_l denote the event that node 1 has at least $l \in \mathbb{Z}^+$ neighbors. Do we observe a phase transition for this event? If so, find the threshold function and justify your reasoning.

(b) Let B denote the event that a cycle with k edges (for a fixed k) emerges in the graph. Do we observe a phase transition for this event? If so, find the threshold function and justify your reasoning.

Problem 1.2 *(Problem 1.2 from Jackson)*

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Problem 1.3 *(Clustering in the Configuration Model)*

(a) Consider a graph g with n nodes generated according to the configuration model with a particular degree distribution $P(d)$. Show that the overall clustering coefficient is given by

$$Cl(g) = \frac{\langle d \rangle}{n}\left[\frac{\langle d^2 \rangle - \langle d \rangle}{\langle d \rangle^2}\right]^2,$$

where $\langle d \rangle$ is the expected degree under distribution $P(d)$, i.e., $\langle d \rangle = \sum_d dP(d)$ and similarly $\langle d^2 \rangle = \sum_d d^2 P(d)$.

(b) *(Optional for Bonus)*: Consider a power-law degree distribution $P(d)$ given by

$$P(d) = cd^{-\alpha} \qquad \text{for } \alpha < 3.$$

Show that the overall clustering coefficient satisfies

$$Cl(g) \sim n^{-\beta}, \qquad \beta = \frac{3\alpha - 7}{\alpha - 1}.$$

Discuss the monotonicity properties of the overall clustering coefficient as a function of n for different values of α.

Problem 1.4 *(Clustering in the Small World Model)*

(a) Consider the small-world model of Watts and Strogatz with rewiring probability p. Show that when $p = 0$, the overall clustering coefficient of this graph is given by

$$Cl(g) = \frac{3k - 3}{4k - 2}.$$

(b) *(Optional for Bonus)*: Show that when $p > 0$, the overall clustering coefficient is given by

$$Cl(g) = \frac{3k - 3}{4k - 2}(1 - p)^3.$$

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