Problem 1. [Sudoku as a game of cooperation]
In the popular game of *Sudoku*, the number placement puzzle, the objective is to fill a 9×9 grid so that each column, each row, and each of the nine 3×3 boxes (also called blocks or regions) contains the digits from 1 to 9 only one time each (see figure). The puzzle setter provides a partially completed grid and the puzzle solver is asked to fill in the missing numbers. Consider now a multi-agent extension of the Sudoku game, in which each of the 81 boxes is controlled by one agent. The agent's action space is choosing a number from 1 to 9 to put in her box, i.e., $\mathcal{A}_i = \{1, \cdots, 9\}$, where $\mathcal{A}_i$ denotes the action space of agent i. Let α denote the vector of choices of the agents. Then, agent i's utility is given by:

$$u_i(\alpha) = -(n_i^R(\alpha) + n_i^C(\alpha) + n_i^B(\alpha)),$$

where $n_i^R(\alpha)$, $n_i^C(\alpha)$, $n_i^B(\alpha)$ is the number of repetitions of numbers in i's row, column and block respectively. Show that the multi agent Sudoku game is a potential game and write down explicitly the associated potential function.

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Figure 1: Sudoku

Problem 2. Consider the network formation game discussed in Lecture 12.

(i) Find conditions for which the ring network, in which the agents form a circle (each one has two neighbors), is a pairwise stable equilibrium of the game.

(ii) Find conditions for which a network that consists of two disjoint rings in a pairwise stable equilibrium of the game.

Problem 3. [The Stag Hunt Game - A Game of Social Cooperation]
The stag hunt is a game which describes a conflict between safety and social cooperation. Other names for it or its variants include "assurance game", "coordination game", and "trust dilemma". Inspired by the philosopher Jean-Jacques Rousseau, the game involves two individuals that go out on a hunt. Each can individually choose to hunt a stag or hunt a hare. If an individual hunts a stag, he must have the cooperation of his partner in order to succeed. An individual can get a hare by himself, but a hare is worth less than a stag. The game is succinctly described by the payoff matrix below:

	stag	hare
stag	a, a	$0, b$
hare	$b, 0$	$b/2, b/2$

In particular, if they both cooperate and hunt a stag, they succeed and get a. Alternatively, one goes for hare, succeeds and get a lower payoff b, whereas the other that went for stag gets 0, since stag hunting needs cooperation. Finally, if both go for hare, then they both obtain $b/2$. The main assumption is that $a > b > 0$.

(i) Compute all Nash Equilibria of the stag hare game, both in pure and mixed strategies.

(ii) Show that the pure strategy Nash Equilibria are evolutionary stable. How about the mixed strategy equilibrium?

(iii) Consider the continuous time replicator dynamics for the stag hare game. Write down their expression and show that the pure strategy Nash equilibria are asymptotically stable.

Problem 4. [The Chain Store Paradox]
Consider a monopolist with a store in each one of n towns. There is a separate entrant considering entry into each one of the towns. Entry is sequential, and this is a perfect information game. In particular, at $n = 1$, the first entrant decides whether to enter, and then the monopolist decides whether to fight or not; at $n = 2$, the second entrant decides after observing past actions etc. The monopolist makes a profit equal to 2 if there is no entry into the relevant market, a profit equal to 1 if there is entry and no fighting, and -1 if there is fighting. The entrant gets 0 if it does not enter,1 if it enters and there is no fighting, and -1 if there is fighting. The monopolist's total payoff is the discounted sum of the profits from the n towns, where the discount factor is $0 < \delta < 1$.

(i) Show that there exists a unique subgame perfect equilibrium of the game.

(ii) Consider now the case when $n = \infty$ (infinite horizon game). Show that for $\delta > 2/3$, there exists a subgame perfect equilibrium, in which no entrant ever chooses to enter. Describe in detail the strategy profile that supports this subgame perfect equilibrium. Finally, draw a parallel between this equilibrium and the cooperation equilibrium in the infinitely repeated prisoner's dilemma.

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