Problem 1. *(Dying Links in Preferential Attachment) - 30 Points*
Consider a preferential attachment process where at each step a single node with m links is born. Suppose that each newborn agent attaches a fraction α (with $1 > \alpha > 0$) of its links by choosing their other end uniform at random, and a fraction $1 - \alpha$ of its links by preferential attachment.

1. (15 Points) Using the continuous time mean field model, obtain a differential equation for the expected degree of a node that was born at time i. Write the boundary condition for the differential equation.

2. (5 Points) Derive the degree distribution for this network.

   Now additionally assume that at each step qm links are destroyed, where $\frac{1-\alpha}{2} \geq q \geq 0$ and the links are selected uniformly at random out of all links that exist at the beginning of the period.

3. (15 Points) Using the continuous time mean field model, obtain a differential equation for the expected degree of a node that was born at time i. Write the boundary condition for the differential equation.

4. (5 Points) Derive the degree distribution for the new network.

Problem 2. *(Repeated Bertrand Competition with Different Marginal Costs of Production) - 30 Points*
Consider an infinitely-repeated Bertrand competition game between two firms, firm 1 and firm 2. There is a mass 1 of consumers, that will buy only if the minimum price is $\leq R$. Marginal cost of production is c_1 for firm 1 and c_2 for firm 2, such that $c_1 < c_2 < R$. Both firms have the same discount factor $\delta \in [0, 1)$. Characterize a threshold value for this discount factor such that above this threshold, the two firms can support a collusive equilibrium in which they both charge a price equal to R.

Problem 3. *(Public Goods Game) - 30 Points*
Consider n individuals and a graph G that represents their social network. Each individual i can choose her effort level $x_i \geq 0, x_i \in \Re^+$ (that may represent her share in a public good that she shares with her neighbors). Individual i's utility function is given by:

$$u_i = x_i - \frac{1}{2}x_i^2 - \sum_{j \in N_i} x_i x_j,$$

where N_i denotes the neighborhood of individual i.

- (20 Points) Show that the game defined above is a potential game. Provide the potential function.

- (10 Points) Let x_{-i} represent the vector of effort levels of all individuals except i. What is the best response of individual i to x_{-i}?

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