14.16 STRATEGY AND INFORMATION

FINAL ASSIGNMENT

Question 1

A seller uses the *winner and loser pay* auction to sell a good to one of $n \geq 2$ buyers. In this auction, buyers simultaneously submit bids and the highest bidder (ties are broken randomly) obtains the good. However, both the highest and the lowest bidder(s) *pay* their own bids. Assume that each buyer i has a private value v_i, which is independently and *uniformly* distributed in $[0, 1]$.

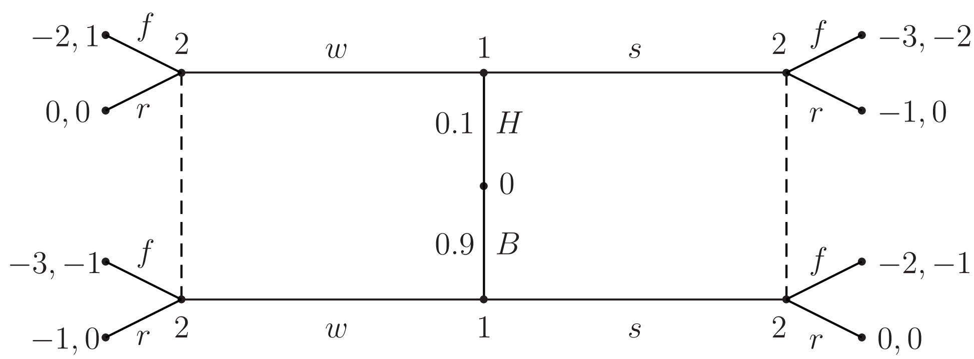
(a) Find a symmetric Bayesian Nash equilibrium described by a strictly increasing bidding function (you do not have to argue that the identified strategies constitute an equilibrium).

(b) Prove that the bid of every buyer type v_i in the *winner and loser pay* auction does not exceed the bid of type v_i in the standard *first-price* auction. What buyer types bid exactly the same amount in the two auctions?

(c) Which of the two auction formats discussed in (b) generates a higher revenue for the seller?

Question 2

Consider the following signaling game.



(a) Find a Nash equilibrium that is not a sequential equilibrium.

(b) Find all sequential equilibria.

Question 3

Two players repeatedly play the following simultaneous move game for T periods. Assume that both players observe all past actions and discount payoffs by $\delta \in (0, 1)$.

	C	D
A	$5, 5$	$-1, -2$
B	$6, -1$	$0, -3$

(a) Find all subgame perfect equilibria of the game repeated for $T = 2016$ periods.

(b) Assume now that $T = \infty$, so that the stage game is played an infinite number of times. For what values of δ does the following strategy profile, in which (A, C) is played along the equilibrium path at every date, constitute a subgame perfect equilibrium? Play starts in Phase I and evolves as follows.

- In Phase I, player 1 is supposed to choose action A and player 2 chooses C. If either player deviates from Phase I, play switches to Phase II.
- In Phase II, player 1 is supposed to play B and player 2 should play D for *a single period*. If both players comply with these strategies, play reverts to Phase I in the next period. Otherwise, play continues in Phase II. (Players are stuck in Phase II until (B, D) is observed.)

Question 4

Consider the following cooperative game between a capitalist c and a set of $n \geq 1$ workers W. The capitalist owns a factory and workers can contribute with labor. Workers cannot produce anything by themselves; together with the capitalist, any group of w workers can generate an amount of output that is worth $f(w)$, where $f : [0, \infty) \to [0, \infty)$ is a concave increasing function with $f(0) = 0$. Formally, the set of players is $N = \{c\} \cup W$ and the value of a coalition $S \subseteq N$ is

$$v(S) = \begin{cases} 0 & if\ c \notin S \\ f(|S \cap W|) & if\ c \in S \end{cases}.$$

(a) Show that the core of this game is given by

$$\mathcal{C} = \left\{ x \in \mathbb{R}^N \mid 0 \leq x_i \leq f(n) - f(n-1), \forall i \in W \ \& \ \sum_{i \in N} x_i = f(n) \right\}.$$

Interpret the payoffs in this set. (Note: showing that the core coincides with the set $\mathcal{C}$ proceeds in two steps. One needs to argue that (1) any core point satisfies the properties defining $\mathcal{C}$; and (2) any payoff vector in $\mathcal{C}$ belongs to the core.)

(b) Find the Shapley value of this game.

(c) Are there any n and f satisfying the hypotheses such that the Shapley value does not belong to the core?

MIT OpenCourseWare
https://ocw.mit.edu

14.16 Strategy and Information
Spring 2016

For information about citing these materials or our Terms of Use, visit: https://ocw.mit.edu/terms.