Networks

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Networks in Economics

- Network: collection of nodes and links between them. . . local interaction structure
- Social and economic networks: nodes are individuals or firms, links relationships
- Many activities shaped by networks
  - exchange of information and opinions
  - trade of goods and services, intermediation
  - job opportunities
  - friendships, business partnerships, political/trade alliances
  - risk-sharing, favors, cooperation
  - innovation, technology adoption
  - credit and financial flows
  - peer effects: education, crime, voting

Outcomes in Networks

Questions for economics

- How do networks form?
- Which network structures are likely to emerge?
- What is the impact of network architecture on economic outcomes?
- How does an individual's position in the network affect his actions and welfare?
- Which networks are socially optimal?

Networks also constitute an important area of research in sociology, computer science, mathematics, and statistical physics.

Notation

- *players* (or nodes, vertices): $N = \{1, \ldots, n\}$
- *network* (or graph) with nodes N: $(g_{ij})_{i,j \in N}$ with $g_{ij} \in \{0, 1\}, \forall i, j \in N$
- i has a *link* (or connection, relationship) to j iff $g_{ij} = 1$; alternative notation, $ij \in g \iff g_{ij} = 1$
- a *network* is *undirected* if $g_{ij} = g_{ji}, \forall i, j \in N$; *directed* otherwise

American Politics

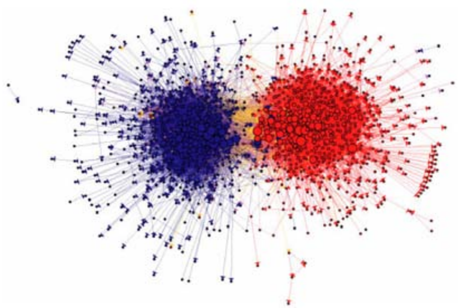

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Figure: Political blogs prior to the 2004 U.S. Presidential election: two well-separated clusters (Adamic and Glance 2005). The web does not automatically guarantee that each individual obtains a diversity of opinions. . . networks can increase “herding” instead of “wisdom of the crowds.”

Social Contagion

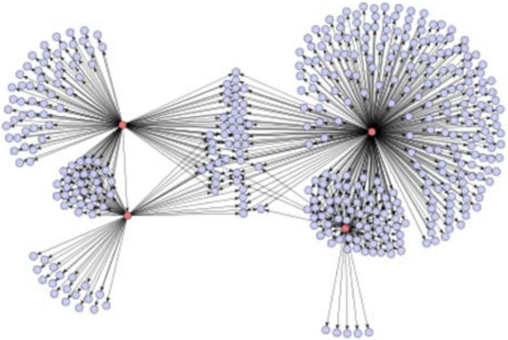

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Figure: E-mail recommendations for a manga book: information cascade in the adoption of new products/technologies (Leskovec et al. 2007)

Florentine Elite

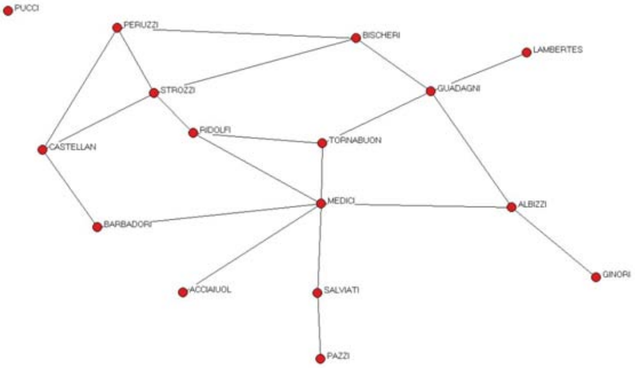

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Figure: Structure of marriages between key families in 15th century Florence. The Medicis emerged as the most influential family, dominating Mediterranean trade, even though they were not as powerful or wealthy as other families. Their central position in the intermarriage network explains why (Padgett and Ansell 1993).

Betweenness Centrality

A measure of power

- $P(i,j)$: # shortest paths connecting i to j
- $P_k(i,j)$: # shortest paths connecting i and j that include k
- Betweenness centrality of node k

$$B_k = \frac{1}{(n-1)(n-2)} \sum_{i,j \neq k} \frac{P_k(i,j)}{P(i,j)}$$

- B_k: fraction of paths between all pairs of nodes going through k
- Medicis have the highest betweenness centrality, 0.52. All other families have centralities < 0.26.

Why only *shortest paths*?

International Trade and R&D

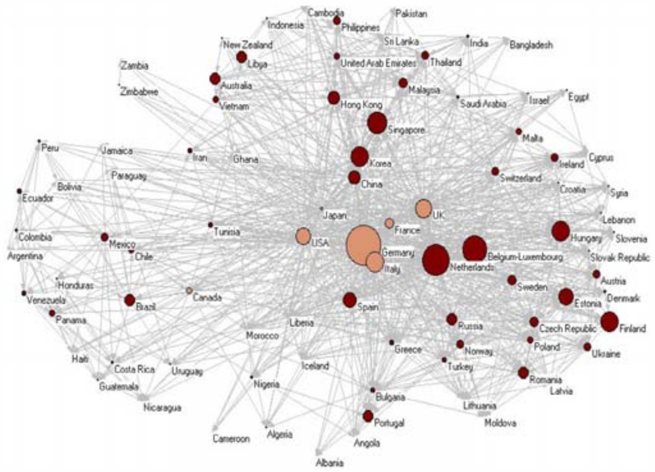

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Figure: Betweenness centrality in international trade is correlated with indirect research and development spillovers (Franco, Montresor, and Marzetti 2009).

Financial Networks

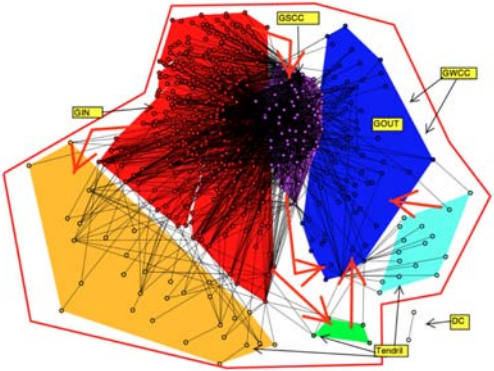

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Figure: Network of loans among financial institutions: how do financial interactions affect the health of each bank and the whole system? Centrality is correlated with interest rates of loans (Bech and Atalay 2008).

Bonacich (1972) Centrality

A node's centrality depends on the centrality of its neighbors...

Fix $\alpha > 0$. Solve the system

$$x_i = 1 + \alpha \sum_{j \neq i} g_{ij} x_j, \forall i \in N$$

to obtain $x = (I - \alpha G)^{-1}\mathbf{1}$ (inverse exists for small α), where $G = (g_{ij})_{i,j \in N}$. Define

$$M(G, \alpha) = (I - \alpha G)^{-1} = \sum_{k=0}^{+\infty} \alpha^k G^k.$$

G^k keeps track of paths of length k, and $m_{ij}(G, \alpha)$ counts all weighted paths from i to j (length k paths weighted by α^k).

Definition 1

The *Bonacich centrality* $b_i(g, \alpha)$ of node i in g for parameter α is the total weight of paths starting at i,

$$b_i(g, \alpha) = m_{i1}(G, \alpha) + \cdots + m_{in}(G, \alpha).$$

Peer Effects

Player $i \in N$ selects effort $x_i \geq 0$. Player i's payoff for $x = (x_i)_{i \in N}$ is

$$u_i(x) = x_i - \frac{1}{2}x_i^2 + \alpha \sum_{j \neq i} g_{ij} x_i x_j.$$

$\alpha > 0, g_{ij} = 1$: strategic complementarity between i and j,

$$BR_i(x_{-i}) = 1 + \alpha \sum_{j \neq i} g_{ij} x_j.$$

In a Nash equilibrium,

$$x_i = 1 + \alpha \sum_{j \neq i} g_{ij} x_j, \forall i \in N \iff x_i = b_i(g, \alpha).$$

Bonacich centrality measures influence. Connection exploited by

- Ballester, Calvo-Armengol, and Zenou (2006)
- Acemoglu, Carvalho, Ozdaglar, Tahbaz-Salehi (2012)
- Bramoulle, Kranton, and D'Amours (2014)

Bargaining Power

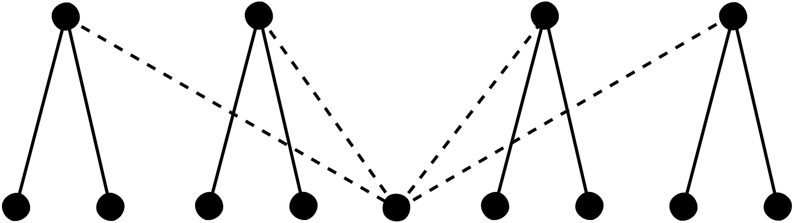

Figure: The position with the greatest number of connections is not necessarily the strongest (Manea 2012).

Intermediation Power and Long Chains

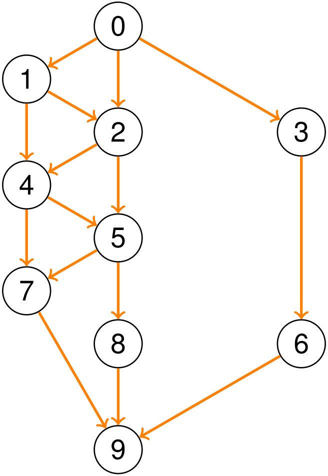

Figure: Trade does not take place along the shortest intermediation chain due to seller's incentives to exploit competition (Manea 2016).

A Large Financial Network

Fedwire enables financial institutions to transfer funds electronically

- primary US network for large-value and time-critical payments
- operated by the Fed
- 10,000 banks
- resilient and redundant

In 2007, average daily value of transfers over Fedwire was $2.7 trillion, and daily number of payments was 537,000.

Structure of Observed Networks

- Degree distribution: number of connections is uneven and small on average
  Fedwire: 10,000 banks; average of 3 links per bank, a few banks with >1,000 links
- Clustering: a node's connections tend to be linked with one another

$$Cl_i(g) = \frac{|\{jk \in g | ij, ik \in g\}|}{|\{jk | ij, ik \in g\}|}$$

  What fraction of pairs of *i*'s neighbors are linked to each other?
  Fedwire: high clustering, average $> .5$
- Diameter: small average distance between nodes
  Fedwire: average distance 2.6

Do We Live in a Small World?

In a 1929 play, Frigyes Karinthy suggested we live in a small world: any two of the $1.5B$ people living then linked through at most 5 acquaintances.

Stanley Milgram (1967) “The Small World Problem”

- Participants asked to route letters to persons whom they did not know directly. Letters distributed to subjects in KS and NE. . . told the name, profession, and residential details of target persons from MA.
- Subjects need to pass the letter on to someone whom they knew and would be likely to know the target or pass it on to someone else, etc., with the objective of reaching the target.
- 42 of the 160 letters made it to their target, with a median number of intermediaries equal to 5.5. . . six degrees of separation

Similar studies for other networks

- Albert, Jeong, and Barabasi (1999): diameter of the Internet in 1998 (800M websites) was 11
- Watts and Strogatz (1998): average distance of 3.7 in actor network

Why Small Distances?

- Most people have thousands of acquaintances.
- Taking the conservative estimate that each individual has 100 relatives, friends, colleagues, and acquaintances with whom they are in regular contact, we end up with roughly 10,000 friends of friends and 1M friends of friends of friends.
- Moving out 4 links, we cover a nontrivial part of most countries.
- While this overestimates the reach of a network since it treats the network like a tree and ignores clustering, it provides an idea of orders of magnitude.

Small World Networks

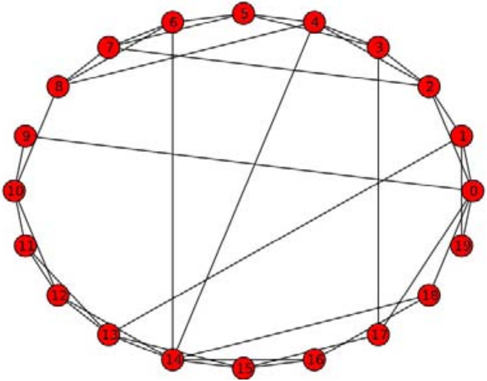

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Figure: It only takes a small number of links randomly placed in a network to generate small diameter. . . combine with a regular and clustered base network to generate networks that simultaneously exhibit high clustering and low diameter (Watts and Strogatz 1998). Research inspired by synchronization of cricket chirps.

Economic Models of Small Worlds

- Jackson and Rogers (2007): individuals form links in order with randomly selected predecesors and then with randomly selected neighbors of their neighbors (e.g., meeting friends of friends)
- Iijima and Kamada (2015): individuals have multidimensional characteristics (e.g, ethnicity, job, location, opinions, hobbies) and form links with others who are close on k dimensions. . . homophily

Homophily

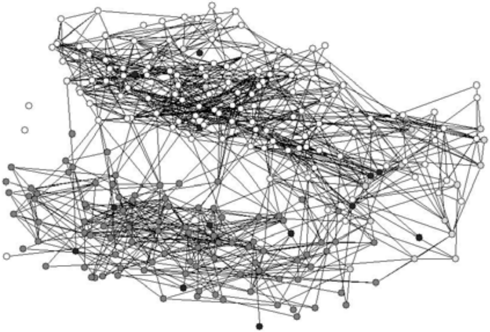

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Figure: The National Longitudinal Adolescent Health Data Set—friendships among high school students coded by race (Currarini, Jackson and Pin 2008). Homophily: 52% of the students are white and yet 86% of whites' friendships are with other whites. Similarly, 38% of the students are black but 85% of blacks' friendships are with other blacks.

How Do People Find Jobs?

Granovetter (1973) “The Strength of Weak Ties”

- Most people find jobs through acquaintances, not close friends.
- People have more acquaintances than friends.
- Clustering of strong ties: we become friends with close friends of our friends.
- Workers are more likely to get referrals through an acquaintance than a close friend to managers whom they do not know→ importance of weak ties.

Strategic Network Formation

Jackson and Wolinsky (1996): model of network formation where links require mutual consent

- explicitly model the costs and benefits from every network
- how do networks form?
- predict how individual incentives translate into network outcomes
- which networks are best for society?
- tension between individual incentives and social welfare due to network externalities

Payoffs

- $N = \{1, \ldots, n\}$: players
- $\mathcal{G}$: set of undirected networks with nodes in any $N' \subseteq N$
- *value function* $v : \mathcal{G} \to [0, \infty)$ assigns a total worth to any $g \in \mathcal{G}$
- v is component additive if $v(g) = \sum_{\text{connected components } h \text{ of } g} v(h)$
- V: set of all component additive value functions
- an *allocation rule* $u : \mathcal{G} \times V \to [0, \infty)^n$ describes how the value of every network is distributed among players
- $u_i(g, v)$: payoff of player i in network g for value function v
- u is component balanced if for all connected components h of g,

$$\sum_{i \text{ is a node in } h} u_i(g, v) = v(h)$$

Definition 2 (Efficiency)

A network g is *efficient* with respect to v if $v(g) \geq v(g'), \forall g' \in \mathcal{G}$.

Pairwise Stability

Definition 3 (Pairwise stability)

A network g is *pairwise stable* with respect to u if

(i) for all $ij \in g$, $u_i(g, v) \geq u_i(g - ij, v)$ (and $u_j(g, v) \geq u_j(g - ij, v)$);

(ii) for all $ij \notin g$, if $u_i(g, v) < u_i(g + ij, v)$ then $u_j(g, v) > u_j(g + ij, v)$.

Part (i): no player wants to delete a link unilaterally

Part (ii): no pair of unlinked players finds it mutually beneficial to form a new link. This accounts for the communication and coordination that are important in the formation of relationships in networks. . . not captured by standard non-cooperative solution concepts.

How “Stable” Are Pairwise Stable Networks?

Pairwise stability is a necessary, but not sufficient, condition for a network to be stable over time.

- deviations by only one or two players
- changing a single link at a time
- myopic incentives; no foresight, no dynamics

First Application

A Distance Based Utility Model (Bloch and Jackson 2007)

Players obtain utility from direct connections but also from indirect ones, and utility deteriorates with distance.

The payoff of player i in network g is

$$u_i(g) = \sum_{j \neq i} b\left(d_{ij}(g)\right) - n_i(g)c$$

- $d_{ij}(g)$ is the distance between players i and j in g ($d_{ij}(g) = \infty$ if i and j are not connected by any path)
- $b(k)$: benefit a player receives from a node k links away; $b(\infty) = 0$; b s. decreasing
- $n_i(g)$: player i's number of neighbors in g
- c: cost of a link

Efficiency vs. Stability in the Connections Model

Proposition 1

Suppose that $b(1) - b(2) < c < b(1) + (n/2 - 1)b(2)$. Then the only efficient networks are stars encompassing all nodes. However, if $b(1) < c$ then stars are not pairwise stable.

Consider a connected component of an efficient network. Suppose it has k players and $m \geq k - 1$ links.

The value of direct connections due to m links is $2m\,(b(1) - c)$. This leaves $k(k-1)/2 - m$ pairs of players with a distance ≥ 2. The value of each indirect connection $\leq b(2)$. Total welfare at most

$$2m\,(b(1) - c) + (k(k-1) - 2m)\,b(2) = 2m\,(b(1) - c - b(2)) + k(k-1)b(2)$$

Since $b(1) - c - b(2) < 0$, if $m \geq k - 1$ the expression above is maximized for $m = k - 1$. All bounds are binding iff the component is a star.

Continuation of the Proof

If two stars, with $k_1 \geq 1$ and $k_2 \geq 2$ nodes, respectively, each lead to non-negative welfare, then a single star with $k_1 + k_2$ nodes leads to higher welfare.

Then stars among all n players are the only efficient networks iff they generate positive payoffs, i.e.,

$$(n-1)\,(2(b(1)-c)+(n-2)b(2)) > 0,$$

which follows from our hypothesis.

If $b(1) < c$, stars are not stable because the center player would gain $c - b(1)$ from deleting a link.

Discussion

- Efficient network formation is difficult to achieve in general.
- Forming a link creates a positive externality on others because it reduces the distance between other players.
- The externality is not internalized in a model with self-interested players. Stable networks have too few links.

Second Application

A Coauthor Model

- Each player is a researcher who spends time writing papers.
- Two players linked in g work together on a project.
- The amount of time researcher i spends on a given project is inversely related to the number of projects $n_i(g)$ he is involved in.
- Player i's payoff is

$$u_i(g) = \sum_{\{j|ij \in g\}} \left(\frac{1}{n_i(g)} + \frac{1}{n_j(g)} + \frac{1}{n_i(g)n_j(g)} \right),$$

for $n_i(g) > 0$ and $u_i(g) = 1$ if $n_i(g) = 0$.

The output of a project undertaken by i and j depends on the total time $1/n_i(g) + 1/n_j(g)$ invested by the two collaborators and some synergy in the production process, $1/(n_i(g)n_j(g))$.

Links have negative externalities—each researcher prefers that his coauthors have fewer links. Costs of forming links are implicit in diluted synergies when efforts are spread among more projects.

Efficiency vs. Stability in Coauthor Networks

Proposition 2

1. *If n is even, then the efficient network consists of $n/2$ separate pairs.*
2. *Any pairwise stable network can be partitioned into cliques. If $m \geq m'$ are the cardinalities of two distinct cliques, then $m > m'^2$.*

It is sufficient to consider the case $n_i(g) > 0, \forall i \in N$. Total welfare satisfies

$$\begin{aligned}
\sum_{i \in N} \sum_{\{j | ij \in g\}} \left(\frac{1}{n_i(g)} + \frac{1}{n_j(g)} + \frac{1}{n_i(g) n_j(g)} \right) &= 2n + \sum_{i \in N} \frac{1}{n_i(g)} \sum_{\{j | ij \in g\}} \frac{1}{n_j(g)} \\
&\leq 2n + \sum_{i \in N} \frac{1}{n_i(g)} n_i(g) \\
&= 3n,
\end{aligned}$$

with equalities iff g consists of $n/2$ disjoint links.

Proof of Part (2)

Suppose g is pairwise stable and $ij \notin g$. Then i benefits form a link with j if

$$\frac{1}{n_j(g)+1}\left(1+\frac{1}{n_i(g)+1}\right) > \left(\frac{1}{n_i(g)} - \frac{1}{n_i(g)+1}\right)\sum_{h, ih \in g}\frac{1}{n_h(g)} \iff$$

$$\frac{n_i(g)+2}{n_j(g)+1} > \frac{1}{n_i(g)}\sum_{\{h|ih \in g\}}\frac{1}{n_h(g)}.$$

Similarly, if $ik \in g$, i has incentives to maintain the link to k if

$$\frac{n_i(g)+1}{n_k(g)} \geq \frac{1}{n_i(g)-1}\sum_{\{h|h \neq k, ih \in g\}}\frac{1}{n_h(g)}.$$

- If k maximizes $n_k(g)$ and $n_j(g) \leq n_k(g)$, then i would benefit from a link to j.
- Since each of k's $n_k(g)$ neighbors are willing to coauthor with k, they also want to coauthor with one another (their degrees do not exceed $n_k(g)$). Then each of k's neighbors has $n_k(g)$ coauthors... clique.

Comparisons between the Two Models

Connections and coauthorship models

- different strategic settings; simple structures of efficient networks
- pairwise stable networks tend to be inefficient due to externalities
- different sources of externalities
  - connections model: when stars are efficient, the center may not have incentives to maintain links with solitary players; however, such links bring valuable indirect connections to other players; the indirect value is not reflected in the payoffs of the center
  - coauthor model: by forming additional connections, a player dilutes the value of his existing partnerships; the inefficiency stems from the fact each player sees more benefit from adding a link than the loss in value for other links, while existing partners see only harm
- in either model, social and private incentives are not aligned

General Tension between Efficiency and Stability

Definition 4

The allocation rule u satisfies *equal treatment of equals* if $u_i(g, v) = u_j(g, v)$ for all i and j that share the same set of neighbors outside $\{i, j\}$.

Theorem 1

If $n \geq 3$, there exists no component balanced allocation rule u that satisfies equal treatment of equals s.t. for every value function v under which singleton components generate the same value, at least one efficient network is pairwise stable.

[Example of component balanced u satisfying equal treatment of equals

$$u_i(g, v) = \frac{v(h)}{\text{number of nodes in } h}$$

if i belongs to the connected component h of g.]

Proof

It is sufficient to consider the case $n = 3$. Suppose $v(g)$ takes the values 0, 1, $1 + \varepsilon$ and 1 depending on whether g has $0, 1, 2$ or 3 links, resp.

Efficient networks have two links. Suppose the network $\{12, 23\}$ is stable.

Since u is component balanced, $u_3(\{12\}, v) = u_3(\emptyset, v) = v(\emptyset) = 0$. By equal treatment of equals,

$$u_3(\emptyset, v) = u_1(\emptyset, v) = u_2(\emptyset, v) = 0$$
$$u_1(\{12\}, v) = u_2(\{12\}, v) = 1/2$$
$$u_1(\{12, 23, 13\}, v) = u_2(\{12, 23, 13\}, v) = u_3(\{12, 23, 13\}, v) = 1/3.$$

Pairwise stability of $\{12, 23\}$ implies that player 2 does not want to delete a link, $u_2(\{12, 23\}, v) \geq u_2(\{12\}, v) = 1/2$. Equal treatment of equals leads to

$$u_1(\{12, 23\}, v) = u_3(\{12, 23\}, v) \leq 1/4 + \varepsilon/2.$$

Contradiction with the stability of $\{12, 23\}$ when $\varepsilon < 1/6$, since 1 and 3 have incentives to form a link,

$$u_1(\{12, 23, 13\}, v) = u_3(\{12, 23, 13\}, v) = 1/3 > 1/4 + \varepsilon/2.$$

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