

Problem Set #2 Solutions

Problem 1:

a) A monopolist solves the following problem:

$$\max_Q P(Q)Q - C(Q) = 100Q - \frac{1}{2}Q^2 - 10Q$$

with first-order condition (FOC)

$$90 - Q = 0$$

$$Q^m = 90$$

$$P^m = 55$$

$$\pi^m = 4050$$

b) Given the results from part a,

$$q^* = 45$$

$$P^* = P^m = 55$$

$$\pi^* = 2025$$

On the other hand, a Cournot competitor, as in the punishment phases, solves the following problem:

$$\max_{q_i} \left[100 - \frac{1}{2}(q_i + q_j) \right] q_i - 10q_i$$

with FOC

$$90q_i - \frac{1}{2}q_j - q_i = 0$$

which implies the following best response function:

$$q_i = 90 - \frac{1}{2}q_j$$

Due to symmetry, we know that $q_i^C = q_j^C$; hence,

$$q_i^C = q_j^C = 60$$

$$P^C = 40$$

$$\pi^C = 1800$$

Last, suppose I decide to cheat; what quantity do I produce? I should produce the best response to what the other firm (who is not cheating) is producing. Remember, as long as I am cheating, I might as well cheat “as well as possible.” Hence, if I cheat, or “deviate” from the proposed equilibrium, I should

$$q^d = 90 - \frac{1}{2}(45) = 67.5$$

$$P^d = 43.75$$

$$\pi^d = 2278.125$$

If I “play along” with the cooperative strategy, the PDV of my profit stream is

$$2025(1 + \delta + \delta^2 + \dots) = \frac{2025}{1 - \delta}$$

But, if I deviate, the PDV is

$$2278.125 + 1800(\delta + \delta^2 + \dots) = 2278.125 + \frac{1800\delta}{1 - \delta}$$

The cooperative strategy is an equilibrium if

$$\begin{aligned} \frac{2025}{1 - \delta} &\geq 2278.125 + \frac{1800\delta}{1 - \delta} \\ 2025 &\geq 2278.125(1 - \delta) + 1800\delta \\ &= 2278.125 - 478.125\delta \\ 478.125\delta &\geq 253.125 \\ \delta &\geq 0.5294 \end{aligned}$$

- c) With a 2 period detection lag, the deviator can cheat for two periods. If I deviate, the PDV is

$$2278.125 + 2278.125\delta + 1800(\delta^2 + \delta^3 + \dots) = 2278.125(1 + \delta) + \frac{1800\delta^2}{1 - \delta}$$

The cooperative strategy is an equilibrium if

$$\begin{aligned} \frac{2025}{1 - \delta} &\geq 2278.125(1 + \delta) + \frac{1800\delta^2}{1 - \delta} \\ 2025 &\geq 2278.125(1 - \delta^2) + 1800\delta^2 \\ &= 2278.125 - 478.125\delta^2 \\ \delta^2 &\geq 0.5294 \\ \delta &\geq 0.7276 \end{aligned}$$

- d) Firms competing in price in a homogeneous goods market – this is a Bertrand game. Firms bid the price down to marginal cost. Hence, the only equilibrium is for both firms to charge a price equal to the marginal cost of 10. The total market quantity is 180.
- e) When both firms cooperate by setting price to be the monopoly price $P^m = 55$, each firm makes a profit of 2025 in each period (See Part a). In the punishment phases, firms are Bertrand competitors and make zero profit. What about when one firm cheats? A firm's optimal cheating strategy is to set price equal to $P^m - \varepsilon$, i.e., just undercut the other firm and capture the whole market. In this way, the cheating firm essentially acts as the sole monopolist and makes the entire monopoly profit of 4050. The cooperative strategy is an equilibrium if

$$\frac{2025}{1-\delta} \geq 4050$$

$$\frac{1}{2} \geq 1-\delta$$

$$\delta \geq \frac{1}{2}$$

Price competition is more likely to sustain a monopoly outcome because, due to the fierce competition inspired by price competition, the punishment is much harsher.

- f) Firms are capacity constrained – no one can produce the full market demand at a price equal to the marginal cost. Firms keep undercutting the rival's price down to marginal cost, at which point no firm makes profit and can benefit from raising its price. Since neither firm can meet the full market demand, some consumers are forced to either not buy or buy from the other firm. Firms then begin again to undercut each other back down in price. In this way, prices cycle up and down. This phenomenon, known as Edgeworth cycles, actually happens.

Problem 2:

- a) The joint profit-maximization outcome is the same as in part a of Problem 1. In the cartel, the total production quota, allotted somehow among the five firms, must be

$$Q = Q^m = 90$$

The cartel can actually allocate the 90 units any way it wants; evenly dividing it among the five firms seems likely. The price is the monopoly price of 55; profits are distributed among the five firms in the same way that the quantities are.

- b) In the five firm cartel, each firm makes profit 810 (=4050/5). Suppose one firm decides not to enter into the cartel. Let Q_4 be the total quantity produced by the four firms in the cartel. Then the non-cartel firm solves the following problem:

$$\max_q \left[100 - \frac{1}{2}(Q_4 + q) \right] q - 10q$$

with FOC

$$90 - \frac{1}{2}Q_4 - q = 0$$

$$q = 90 - \frac{1}{2}Q_4$$

Knowing the non-cartel firm will behave in this way, the cartel maximizes its joint profit as follows:

$$\max_{Q_4} \left[100 - \frac{1}{2} \left(Q_4 + 90 - \frac{1}{2}Q_4 \right) \right] Q_4 - 10Q_4 = \frac{1}{2} \left(90 - \frac{1}{2}Q_4 \right) Q_4$$

The FOC is

$$90 - Q_4 = 0$$

$$Q_4 = 90$$

Hence, the equilibrium is

$$Q_4 = 90$$

$$q = 45$$

$$P = 32.5$$

$$\pi^{non-cartel} = (P - 10)q = 1012.5$$

It thus pays not to enter the cartel.

- c) Suppose there are n firms. Let us check if an n -firm cartel is stable; does a firm have incentive not to enter. Firms in the n -firm cartel make profit $4050/n$. Using reasoning analogous to part b, a non-cartel firm solves the following problem:

$$\max_q \left[100 - \frac{1}{2}(Q_{n-1} + q) \right] q - 10q$$

with FOC

$$90 - \frac{1}{2}Q_{n-1} - q = 0$$

$$q = 90 - \frac{1}{2}Q_{n-1}$$

Knowing the non-cartel firm will behave in this way, the cartel maximizes its joint profit as follows:

$$\max_{Q_{n-1}} \frac{1}{2} \left(90 - \frac{1}{2}Q_{n-1} \right) Q_{n-1}$$

The FOC is

$$90 - Q_{n-1} = 0$$

$$Q_{n-1} = 90$$

Hence, the equilibrium is

$$Q_{n-1} = 90$$

$$q = 45$$

$$P = 32.5$$

$$\pi^{non-cartel} = 1012.5$$

Therefore, comparing the profit to firms inside the n -firm cartel to the profit of the non-cartel firm suggests

$$n < 4 \Rightarrow \text{Stable cartel}$$

$$n = 4 \Rightarrow \text{Indifferent}$$

$$n > 4 \Rightarrow \text{Incentive to not join}$$

Due to the symmetry of the firms, there is no difference between the $(n-1)$ -firm cartel and a single Stackelberg leader; look at the FOC to confirm this. With a large number of firms, collusive profits are heavily diluted; thus, despite the fact that a Stackelberg follower does comparatively worse than the leader, it still pays to hold out and gain the undissipated Stackelberg follower profits.

Problem 3:

- a) Each firm solves the following problem:

$$\max_{P_i} P_i Q_i - C(Q_i) = (P_i - 10)(80 - 2P_i + P_j)$$

with FOC

$$80 - 4P_i + P_j + 20 = 0$$

$$P_i = \frac{100 + P_j}{4}$$

Due to symmetry, we know that $P_R = P_W$; hence,

$$P_R = P_W = 100/3$$

$$\pi = 2 \left(\frac{70}{3} \right)^2 = 1088.\bar{8}$$

b) Wave knows Rah's best response function is

$$P_R = \frac{100 + P_W}{4}$$

because Rah's profit-maximization problem has not changed; therefore, Wave solves the following problem:

$$\max_{P_W} (P_W - 10) \left(80 - 2P_W + \frac{100 + P_W}{4} \right)$$

with FOC

$$80 - 4P_W + 25 + \frac{1}{2} P_W + 20 - 2.5 = 0$$

$$122.5 - 3.5P_W = 0$$

$$P_W = 35$$

Therefore,

$$P_W = 35$$

$$P_R = 33.75$$

$$\pi_W = 1093.75$$

$$\pi_R = 1128.125$$

Wave should take advantage of the commitment strategy because it (slightly) increases his profits.

Problem 4:

For simplicity, label the points at which the players move as nodes one through seven, numbering them up and down, left to right; player one moves at nodes 1, 4, 5, and 6, while player two moves at nodes 2 and 3. The two nodes connected by the dotted line are in fact a single node. The dotted line signifies the fact that player one does not know at which node he was. To find an SPNE, we employ backwards induction.

At node 6, player one will choose u_2 , whether player two had chosen U or D at node three. At node five, player one will choose d_2 ; at node four, he will choose u_2 . Stepping back to nodes two and three, we can decide what is optimal for player two knowing what player one will do at the ensuing nodes. At node three, player two will choose U; at node two, he will also choose U. Last,

player one will begin the game by choosing d_1 . We can represent the equilibrium strategies for the two players as follows:

$$\begin{pmatrix} d_1, u_2, d_2, u_2 \\ U, U \end{pmatrix}$$

where the first vector is the strategies for player one and the second vector is the strategies for player two, strategies are ordered by node.

Problem 5:

- a) This is analogous to the GE/Westinghouse case when GE began posting all its transaction prices. This is likely to facilitate collusion because it monitors the actions of the firms; colluding firms leave monitoring and detecting cheating to the Commission.
- b) The environmental policies are likely to inhibit collusion. Collusion is enforced by the threat of future punishment, i.e., the attractiveness of colluding is the collusive future revenue stream. The fact that the product will be banned in five years truncates the possible benefit to colluding; less incentive to collude is more incentive to not collude.
- c) Uncertainty tends to hinder collusion primarily by making detection more difficult. For example, a low price may indicate low demand or cheating by a firm, producing less output.
- d) Multimarket contact is likely to facilitate collusion. It makes punishment easier; as referred to in the lecture notes, “May relax incentive constraint; transfer ‘slack’ in non-cheating condition from one market to another to expand scope of cooperative behavior.”

Problem 6:

- a) The major reason the two prices are different is probably price discrimination. Suppose some consumers have much lower search costs than others; they enjoy surfing the web bargain-hunting or are more techno-savvy. Similarly, it may be discrimination between different types of consumers; college students and yuppies pay less than older adults. Business travelers may not search around for low prices in the same way that pleasure travelers do. Possibly, the two rooms are not really identical; there may be small differences between the products that we do not observe.
- b) The hotels may want to prevent prices from being posted because it helps attract more consumers to the lower prices. Fewer consumers pay the high prices relished by the hotels. Priceline, on the other hand, may not want prices posted because it makes their service less attractive in the future. With prices made public, people may be able to bargain directly with the hotels, or prices may become standardized at some lower price, making Priceline obsolete.