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Problem Set #4

Due October 19, 2005

1. Tirole exercise 6.4, p. 250.

2. Tirole exercise 6.6, p. 251.

3. Consider a repeated duopoly model. Firms 1 and 2 choose quantities q_{it} at $t = 0, 1, 2, \ldots$. As in the Cournot model, the firms' products are undifferentiated and a market clearing condition determines the market price. Assume that demand fluctuates over time, so that this market price is $P_t(q_{1t}, q_{2t}) = A_t - (q_{1t} + q_{2t})$. Suppose that the A_t are independent random variables with $A_t \sim U[0, 12]$. The firms do not know A_t when they choose q_{it}. Firm i cannot see q_{-it} and therefore cannot be sure what A_t was even after seeing P_t. Assume that firms have no costs of production.

(a) Find the fully collusive output q^m and the Cournot equilibrium of the one period game. What are the per firm profits in each? What would the distribution of market prices be in each?

(b) Suppose that A_t is observed after the firms choose q_{it} but before they choose q_{it+1}. For what discount factors could the firms sustain collusion by choosing $q_{it} = q^m/2$ as long as no deviation has been observed and permanently reverting to the Cournot equilibrium if any firm has ever deviated.

(c) Now go back to the original assumption that A_t is never observable. Suppose the firms try to sustain collusion via strategies that are initally fully collusive and permanently revert to the Cournot equilibrium if the firms ever observe $P_t < -3$. For what discount factors will this punishment make it unprofitable for the firms to deviate to $q^m/2 + dq$? Show that this condition is in fact sufficient to give a collusive equilibrium.

(d) How is the equilibrium described above similar to and different from the equilibrium that motivates Porter's empirical work and the equilibrium of the two-state version of the Green-Porter model described in Tirole's text (and in class)? Do you think the equilibrium would be a good one to use to motivate tests for collusion?

4. In Porter's (1983) *Bell Journal* he assumes a log-log specification of demand. Suppose that he had instead decided to use a linear specification:

$$Q_t = \alpha_0 + \alpha_1 P_t + \alpha_2 Lakes_t + u_t.$$

(a) Show that for this demand curve the optimal price for a monopolist with a constant marginal cost of c to set is

$$P_t = c - \frac{1}{\alpha_1} Q_t.$$

Given this result, what functional form would Porter have choosen for the supply curve in the model? How would he have measured the degree of collusion during collusive phases?

(b) What pricing rule would result with this demand curve if the industry instead consisted of perfectly competitive firms with total costs of the form $c(Q_t) = c_0 Q_t + c_1 Q_t^2$ setting price equal to marginal cost? (For extra credit comment on whether this is the equilibrium of a Bertrand-like pricing game). Could one use an approach like Porter's to distinguish between these two models of behavior? Talk about why this is an important question.

(c) Go back to assuming that marginal costs are constant. Suppose that demand is linear, but that the opening of the Great Lakes also affects the slope of demand and that there are additive seasonal shifts in demand so that the correct specification of demand is

$$Q_t = \alpha_0 + \alpha_1 P_t + \alpha_2 Lakes_t + \alpha_{3-14} Seasxx_t + \alpha_{15} Lakes_t P_t + U_{1t}.$$

How would a monopolist set prices in such an environment? What supply curve would you estimate if you were redoing Porter's estimation with this model of demand? How would one use the parameter estimates to derive a measure of the degree of collusion comparable to Porter's θ? Are there multiple ways to do this?