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Problem Set #6

Due November 16, 2005

1. Consider a standard model of horizontal differentiation involving two firms and consumers who are uniformly distributed on $[0, 1]$ with the consumer at location x receiving utility $v_0 - tx - p_0$ if he purchases from the firm at location 0, $v_1 - t(1-x) - p_1$ if he purchases from the firm at location 1 and zero if he purchases from neither firm.

(a) Suppose we model advertising by firm 1 as raising v_1. Does advertising make firm 1 tough or soft?

(b) Suppose instead we model advertising as increasing the degree of differentiation in the market without affecting the consumers' rankings of the goods, *i.e.* suppose it increases t. Does advertising make firm 1 tough or soft?

(c) Suppose instead that customers initially do not necessarily know of the existence of both products (say each potential customer is informed about each product only with probability p and that these probabilities are independent so that a consumer knows about both products with probability p^2) and can not purchase a product they do not know about. In this model suppose the effect of advertising by firm 1 is to increase the probability with which customers know of the existence of firm 1. Does advertising now make firm 1 tough or soft?

2. Consider an incumbent monopolist facing a threat of entry by a potential entrant. In the first period, the incumbent can lobby the government to require extensive testing of the output, which boosts marginal cost to both the incumbent and the entrant. When the incumbent spends L on lobbying the regulations which get passed result in profit functions of the form $\pi_i(x_I, x_E) = (x_i - cL)(1 - 2x_i + \text{Min}\{x_{-i}, 1\})$, where $i = I, E$ refers to the incumbent or entrant and x_i is the price firm i chooses in the second stage. Suppose also that after observing L the entrant decides whether or not to enter the market and pay a fixed cost of E.

For what values of E is entry accomodated/deterred? What level of L is chosen in each case?

3. Consider the following model of brand proliferation. A continuum of consumers (of mass 1) are located around a circle of circumference one. In the first period, firm 1 has the opportunity to introduce any number N of brands and position them anywhere it likes around the circle. The cost of doing this is NE_1. Firm 2 then chooses whether to enter, in which case it introduces and positions a single brand at a cost of E_2. If firm 2 enters, assume that there is differentiated product price competitions with consumers having value $v - td$ for a product located at a distance d from them.

(a) If firm 1 introduces two brands at points which are opposite each other on the circle, and firm 2 introduces a single brand half way between two of these show that the equilibrium prices and profits are $p_1 = 7t/12$, $p_2 = 5t/12$, $\pi_1 = 49t/144 - 2E_1$, $\pi_2 = 25t/144 - E_2$. Explain intuitively why firm 1 chooses a higher price than firm 2.

(b) Find values of v, t, E_1, and E_2 for which firm 1 would choose $N = 1$ if entry were not possible, but "overinvests" in brand proliferation and chooses $N = 2$ in this model to deter entry.

(c) Suppose we added a third stage to this game where firm 1 could withdraw any of its brands if it desired before price competition occurs (but not get back the sunk costs of introducing the brands). Given the parameter values from part (b) show that if firm 2 were to introduce a brand located in exactly the same place as one of firm 1's brands, then firm 1 would in equilibrium withdraw that brand. What does this imply about the feasibility of entry deterrence through brand proliferation?

4. (a) Consider a game in which two firms simultaneously choose actions a_1 and a_2 to maximize their profit functions $\pi_1(a_1, a_2)$ and $\pi_2(a_1, a_2)$. Suppose that $\pi_i(a_i, a_{-i})$ is concave in a_i and that the game has an unique interior Nash equilibrium. Show that the game has strategic complements if $\frac{\partial^2 \pi_i}{\partial a_i \partial a_j} > 0$ and strategic substitutes if $\frac{\partial^2 \pi_i}{\partial a_i \partial a_j} < 0$.

(b) Use the result above to show that Cournot competition with linear demand has strategic substitutes. (If you're curious, try using the result above to find a demand curve for which this isn't true.)

(c) Consider a model of differentiated product price competition where two firms with a constant marginal cost of c compete by simultaneously setting prices p_1 and p_2 and firm i's demand is $D_i(p_i, p_j) = A - bp_i^2 + dp_j$. When is this a game with strategic complements and when is it a game with strategic substitutes?

5. Consider the following two period model of learning-by-doing. In each of two periods, the demand for a nondurable good is given by $P(Q) = 4 - Q$ where Q is the total quantity of the good produced.

In the first period, firm 1 (a monopolist) produces quantity q_1^1 of a good at a constant marginal cost of 2. After the first period, firm 2 has the opportunity to pay a sunk cost of E and enter the market. If firm 2 enters, then in the second period firm 1 and firm 2 compete as Cournot duopolists (otherwise firm 1 is again a monopolist). Firm 2 has a constant marginal cost of 2. Because of the experience it gained in the first period, however, firm 1 can produce the good at at a lower marginal cost. Write $MC(q_1^1)$ for the second period *marginal cost* of firm 1 when its first period output was q_1^1 and assume that $MC(q_1^1) \in [1, 2]$.

(a) What are the firms' outputs and profits in the second period as a function of $MC(q_1^1)$.

(b) Assume that the function relating first period output and second period marginal cost is

$$MC(q_1^1) = \begin{cases} 2 & \text{if } q_1^1 \leq 1 \\ 2\frac{1}{2} - \frac{1}{2}q_1 & \text{if } q_1^1 \in [1, 3] \\ 1 & \text{if } q_1^1 > 3. \end{cases}$$

Assume also that firm 2 observes q_1^1 before making its entry decisions and choosing its second period output. Show that if $E = \frac{1}{4}$ it is not optimal for firm 1 to choose a q_1^1 which is sufficiently large so as to deter entry.

(c) Again suppose $E = \frac{16}{81}$ (or any other value which is such that firm 1 wants to "accomodate" entry) and that firm 2 observes q_1^1 before choosing its second period output. What output level does firm 1 choose?

(d) Suppose now that firm 2 is unable to observe q_1^1. Without doing the calculations, how would you expect firm 1's first period output to differ from the answer to part (c)? Would you expect it to be greater than or less than one? Go ahead and solve for q_1^1 to see if you're right. How would the answers to the qualitative parts of this question change if the firms engaged in price competition instead of Cournot competition?

6. Consider a market with two competing standards, e.g. VHS and Beta VCRs.

(a) In the standard model of (one-sided) network externalities, consumers get utility $v + u(x)$ if they make the same choice as x other consumers, with $u(x)$ a strictly increasing function. Suppose there are a continuum of consumers of unit mass and both goods are produced competitively and therefore sold at price $c < v$. Show that this model has three equilibria: one in which all consumers buy a good compatible with standard one, one in which all consumers buy a good compatible with standard two, and one in which the two standards have identical market shares. In what sense is the split-market equililbrium unstable?

(b) Consider now a two-sided analysis of the above model. Assume that in the first stage S producers simultaneously choose between standards at the same time as the consumers are deciding which standard to buy, i.e. consumers decide between standards before they know how many firms are producing each standard. In the second stage, the two standards become two independent markets. In each, the firms play some kind of price competition game against other firms producing to the same standard and consumers who have decided on this standard choose between them. Let $\pi(S_i, B_i)$ be the equilibrium payoff of a producer that chooses standard i if S_i producers and a mass B_i of buyers are in the standard i market, and let $u(S_i, B_i)$ be the corresponding consumer payoff (i.e. the gross surplus minus the equibrium price). Assume that the market has positive cross-population externalities and negative within population externalities in the sense that $\pi(S_i, \gamma S_i)$ is strictly increasing in γ and weakly decreasing in S_i and $u(S_i, \gamma S_i)$ is strictly increasing in S_i and weakly decreasing in γ.

Treating the set of firms as a continuum, show that the two-sided model also has exactly three equilibria.

(c) Consider a particular example. Suppose that after the firms choose which standard to use they are spread evenly around a Salop-circle of circumference one. After consumers choose which standard to use they learn their location on this Salop circle. (Assume this is chosen uniformly and not observable to the firm.) Suppose that the transporation costs parameter in market i, $t(B_i)$ is an increasing function of the number of consumers in the market. (This would happen with road congestion in a physical transporation cost model. Why it should happen with tastes is less clear.) Derive the payoff for the firms and consumers conditional on the number of firms and consumers in a market and show that they satisfy the assumptions in part (b) if $t(B)$ is constant or does not increase too quickly in B.

(d) Taking integer constraints into account, show that a model like this can easily have a *strict* Nash equilibrium with two firms adopting standard one and one firm adopting standard two. One might have thought that the argument you made in part (b) would have ruled out such an equilibrium. Why doesn't it? Could the same thing happen in a discrete-agent version of the model described in part (a)?

(e) Write down a model of men and women choosing between two dating sites that would also fit the two-sided model of part (b). What does part (d) suggest about differences between competing homosexual dating sites and competing heterosexual dating sites?