Problem Set #7

Due December 4, 2013

1. Consider the following four stage game involving two firms. Initially firm 1 is a monopolist. In the first stage, it sets price p_1 and receives profits $\pi_1^1 = (p_1 - c)D(p_1)$. The demand function is initially unknown to the firms. They share a common prior, believing that demand is $\bar{\theta}d(p)$ with with probability q and $\underline{\theta}d(p)$ with probability $1 - q$. Assume $\bar{\theta} > \underline{\theta}$. Demand is the same in both periods of the game. Firm 1 learns the true value of θ after the first stage. Firm 2, however, does not observe firm 1's demand.

In the second stage firm 1 has the option of bulding a fire with $100 bills so that he may reduce his profits to any level he likes. At the end of the second stage firm 1 is required by law to disclose its remaining profits (but not the amount of money it burned).

In the third stage firm 2 may enter the market at a cost of $E > 0$.

Finally in the fourth stage the firms compete earning profits $\pi_1^m(\theta)$ and 0 if firm 2 didn't enter and $\pi_1^D(\theta)$ and $\pi_2^D(\theta)$ if firm 2 did enter.

(a) If $q\pi_2^D(\bar{\theta}) + (1 - q)\pi_2^D(\underline{\theta}) > E$, show that there is no perfect Bayesian equilibrium where firm 1 burns $(\bar{\theta} - \underline{\theta})d(p_1)$ in the high demand state to pretend that it is low demand state.

(b) Find sufficient conditions for the existence of a separating PBE where firm 1 burns money only when demand is low. How does the welfare analysis of such an equilibrium differ from that of Fudenberg and Tirole's signal-jamming model.

2. Spiegler's "Market for Quacks" paper considers a market in which N firms produce products that yield random payoffs: a consumer who pays p_i for firm i's product gets utility $1 - p_i$ with probability α and $-p_i$ with probability $1 - \alpha$. Firms have no costs of production.

His base model is a simple simultaneous move game with a unit mass boundedly rational consumers. Firms simultaneously choose prices $p_1, \ldots, p_N$. Each consumer then finds exactly one user of each brand; asks them what their utility was from their most recent purchase of that brand; and they buys the product that gave the highest utility.

(a) Compute the static Nash equilibrium of the game where the firms simultaneously choose prices $p_1, \ldots, p_N$.

(b) How do equilibrium prices behave in the limit as $\alpha \to 0$? Can you provide some intuition for why this occurs?

3. Justin Johnson's recent working paper "Unplanned Purchases and Retail Competition" develops a model in which boundedly rational consumers are made better off by the fact that their behavior departs from rationality. Try to think about how this works in his paper and whether there are other situations in which a similar mechanism seems plausible. No need to hand in anything on this question if you don't want to, but if you come up with any idea you like you could.

4. Consider a Butters-style model of advertising by a monopolist. A continuum of consumers of unit mass have values $v_1 \sim U[0,1]$ for firm 1's product. Firm 1 has no production costs. It does, however, have advertising costs. An advertising campaign that will inform a fraction $x \in [0,1]$ of consumers about the existence of firm 1's product costs cx^2. Assume that uninformed consumers never buy from firm 1 and informed consumers buy if and only if $p_1 \leq v$.

(a) Holding x fixed what price p^m will the monopolist choose and what will its profits be? What advertising level x^m does the monopolist choose? Does Butters' result about the advertising level being socially optimal carry over to this model? Can you provide some intuition for why it does or does not carry over?

(b) Consider now an asymmetric duopoly model. Assume that firm 1's advertising technology is as above. Assume that the consumers' utilities for purchasing from firm 1 are also as above *if* they don't buy from firm 2. If a consumer buys from firm 2, however, then he or she has no need for firm 1's product.

Assume that firm 2 has a superior product and a superior advertising technology. All consumers get utility $v_2 = 1$ from firm 2's product and all consumers are aware of firm 2's product.

Consider a two-stage game in which firm 1 first chooses x (which is observed by both firms) and firms 1 and 2 then compete by simultaneously choosing prices p_1 and p_2. What values of p_1 and p_2 must be chosen in the second stage after firm 1 chooses the equilibrium level of x?

(c) What are the firms' profits in this equilibrium (as a function of x)? What advertising level would firm 1 choose?

5. (Consider a variant on the Edelman-Ostrovsky-Schwarz model of Google's advertising auction. Suppose that there are three firms with per-click values v_1, v_2, and v_3, drawn indepedently from some distribution F. Suppose there are two slots for ads. Assume that the number of clicks that firm j will receive while in position i is $\alpha_i w_j$, where $\alpha_1 > \alpha_2$ and w_1, w_2, and w_3 are potentially different.

(a) What is the socially efficient allocation of advertisers to slots?

(b) Suppose Google wanted to allocate slots using a VCG mechanism. What would this mechanism look like in this environment, e.g. what would firms announce, and what payments would firms make?

(c) Suppose that w_j is known only to firm j and that Google allocates slots via a simple unweighted GSP auction. Give an example to show that the equilibrium outcome can be different from the social optimum.

(d) Suppose that Google uses a weighted GSP auction using the w_j as weights, i.e. the firm in position i makes a per-click payment of $b_{i+i} w_{i+1}/w_i$. Derive the formula for the optimal dropout strategy of firms in an ascending bid auction. Do these strategies imply that firms will drop out in the efficient order?

NOTE: I may not cover Edelman-Ostrovsky-Schwarz until after the problem set is due. If so, treat this problem as something extra to help review for the final exam and don't hand it in.

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