14.30 Introduction to Statistical Methods in Economics

Spring 2009

14.30 Exam I
Spring 2008

Instructions: This exam is closed-book and closed-notes. You may use a calculator. Please read through the exam first in order to ask clarifying questions and to allocate your time appropriately. In order to receive partial credit in the case of computational errors, please show all work. You have approximately 85 minutes to complete the exam. Good luck!

1.(30 points) Short Questions
Don't spend much time on these questions, a short answer to each suffices.

(a) For a sequence of n independent trials, each of which can result in a "success" (with probability p) or a "failure" (probability $1-p$), what is the p.d.f. $f_X(x)$ of the total number x of successes? Be careful about specifying the p.d.f. for *all* real numbers.

(b) You are given the joint p.d.f.

$$f_{XY}(x,y) = \begin{cases} c\exp\{-(x+y)(x-y)\} & x \in [-2,2], y \in [-2,2] \\ 0 & \text{otherwise} \end{cases}$$

where c is a positive constant such that the density integrates to 1. Are X and Y independent?

(c) A pregnant woman goes to her obstetrician complaining of visual disturbances. "Don't worry", her OB tells her, "only a small fraction of miscarriages or other adverse events are preceded by visual disturbances." Explain briefly, preferably using a formula, why the patient should be annoyed, not reassured.

True/false/uncertain: Always give a brief explanation if the statement is true, or examples if the statement is false or uncertain.

(d) Two events based on the same realization of a physical randomization device (e.g. a single card drawn from a deck of cards) can't be independent.

(e) In principle, one can always recover the joint distribution of random variables given their marginal distributions, although calculations can at times be difficult.

2. (20 points)

(a) Verify whether the the following function a valid CDF. If yes, draw a graph of the corresponding PDF.

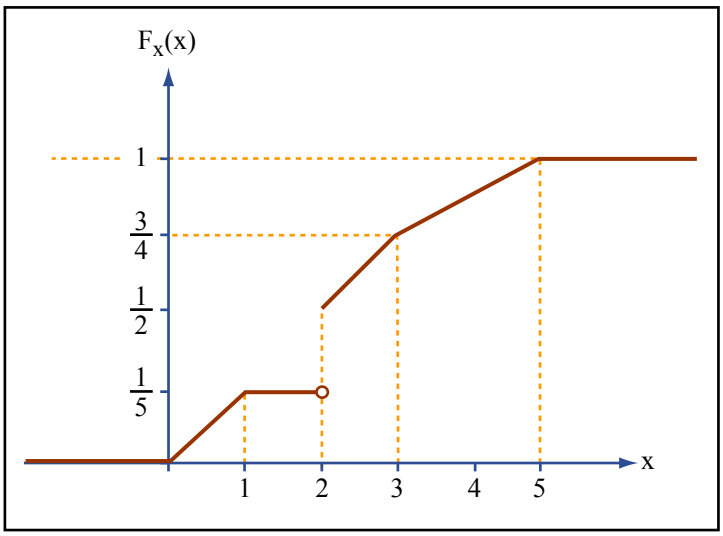

Image by MIT OpenCourseWare.

(b) Verify that the following function is a valid PDF and draw the corresponding CDF.

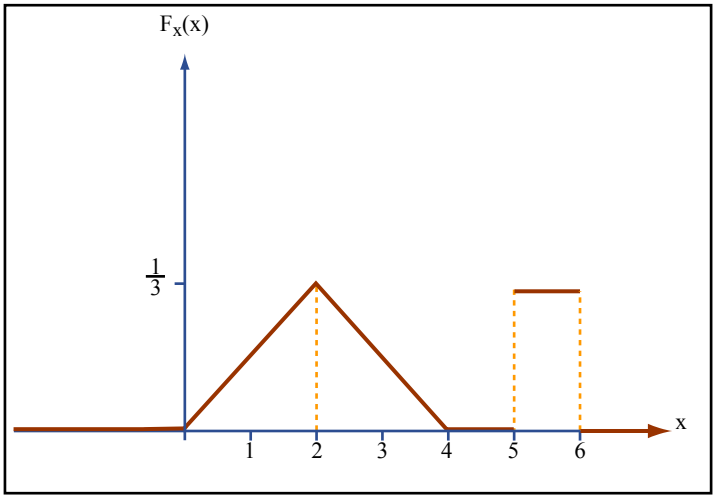

Image by MIT OpenCourseWare.

3. (30 points) Lucy is a chimp living in a cognitive scientist's laboratory. She spends her days playing games with letters, but doesn't attach any specific meaning to, nor does she have any preference for any given letter.

(a) The scientist gives Lucy a box with 16 cubes, each of which has a letter printed on it. Some of the letters appear multiple times - more specifically, the letters are

C IIII M NN O PP SSSS W

Lucy is then asked to arrange the cubes in a row. Reading the letters of the cubes, is the row more likely to start with "MISSISSIPPI" or "WISCONSIN"? Hint: it may be more practical to check whether the ratio of the probabilities is greater than 1 instead of calculating both probabilities, so you may be able to cancel many terms.

(b) It turned out that Lucy preferred to juggle around with the cubes or throw them at the experimenter, so the scientist decides to replace them with a keyboard with 16 keys with the following letters: C (1 key), I (4 keys), M (1 key), N (2 keys), O (1 key), P (2 keys), S (4 keys), and W (1 key) - just to be clear, the number of keys for each letter is the same as the number of cubes in the previous part. Lucy hits each key with equal probability. If she types a chain of 16 characters, is it more likely to start with "MISSISSIPPI" or "WISCONSIN"?

(c) In the experiment in (b), is the chain of 16 characters more likely to contain "MISSISSIPPI" or "WISCONSIN", starting at an arbitrary position of the sequence?

4. (20 points) In many dry regions, crops are irrigated with *circle irrigation*: the water for irrigation is pumped up from a well at the center of the field and spread over the field. This is done by sprinklers which are mounted on a long straight pipe which moves like the arm of a clock over a circular area around the center. This means in particular that crops are planted only on a circle of a given radius R (the standard length for these systems is about $\frac{1}{4}$ mile) around the well. The total quantity of water used for irrigation is normalized to 1.

(a) Suppose, there is a continuum of sprinklers along the pipe which dispense the same amount of water at any given point in time, and the arm moves around the circle at a constant speed. Putting a system of straight coordinates (x, y) with the origin at the center of the circle (so that the circle is given by the coordinates (x, y) such that $x^2 + y^2 \leq R^2$), what is, up to a proportionality factor (i.e. no need to calculate complicated integrals), the joint p.d.f. of the coordinate at which a random drop of water falls? Intuitively, what does this mean in terms of how much water a given area of the field receives? Do any parts of the field receive more water than others?

(b) If you want to plant the same crop across the entire circle, you would want all plants to receive about the same amount of water. You can control the distribution of water over the area through the amount q of water discharged by the sprinkler at a distance r from the center given the position of the pipe, as given by the angle ω formed between the pipe and a line from the center to a given point on the edge of the circle. How would you have to choose $q(r, \omega)$ if the system rotates at a constant speed?