14.381: Statistics

Problem Set 2

1. Let $X_1, X_2, \ldots, X_n$ be iid observations. Find minimal sufficient statistics

   (a) $f(x \mid \theta) = \frac{2x}{\theta^2},\ 0 < x < \theta,\ \theta > 0;$

   (b) $f(x \mid \theta) = e^{-(x-\theta)} \cdot \exp\left\{-e^{-(x-\theta)}\right\},\ -\infty < x < \infty,\ -\infty < \theta < \infty;$

   (c) $f(x \mid \theta) = \frac{2!}{x!(2-x)!}\theta^x(1-\theta)^{2-x},\ x \in \{0, 1, 2\},\ 0 \le \theta \le 1.$

2. Let $X_1, \ldots, X_n$ be independent random variables with pdfs

$$f_{X_i}(x \mid \theta) = \frac{1}{2i\theta} \quad \text{for} - i(\theta - 1) < x < i(\theta + 1)$$

   and zero otherwise. Find a two-dimensional sufficient statistic for θ.

3. Suppose that a random variable X has a Poisson distribution with unknown parameter λ. Assume taht we want to estimate $\theta = e^{-2\lambda}$. Show that the only unbiased estimator of θ is $\delta(X) = 1$ if X is an even integer, and $\delta(X) = -1$ if X is an odd integer. *Note:* It is another example of an inappropriate unbiased estimator.

4. Assume $X_1, \ldots, X_n$ are iid with mean μ and variance σ^2 (both unknown). Let us estimate mean by

$$\hat{\mu} = \sum_{i=1}^{n} \omega_i X_i$$

   (i) Under what condition is $\hat{\mu}$ unbiased?

   (ii) Among all unbiased $\hat{\mu}$ find the one with the smallest variance.

   (iii) What $\{\omega_i\}$ whould lead to the smallest MSE?

5. Let $X_1, \ldots, X_n$ be iid with pdf

$$f(x \mid \theta) = \theta x^{\theta - 1} \quad 0 \leq x \leq 1, \quad \theta > 0.$$

(a) Find the MLE of θ, and show its consistency.

(b) Find the method of moments estimator of θ.

(c) Find limit distributions of for both estimators.

- *Hint 1:* Find the distribution of $Y_i = -\log X_i$.
- *Hint 2:* Use delta-method.

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