14.381: Statistics

Problem Set 4

1. Let $X_1, \dots, X_n$ be iid Poisson (λ) and let λ have a Gamma (α, β) distribution (the conjugate family for Poisson)

$$\pi(\lambda) = \lambda^{\alpha-1} \frac{\exp\{-\lambda/\beta\}}{\Gamma(\alpha)\beta^{\alpha}}$$

   (a) Find the posterior distribution for λ.

   (b) Calculate posterior mean and variance. *Hint:* mean of Gamma (α, β) is $\alpha\beta$; the variance is $\alpha\beta^2$.

   (c) Discuss whether the prior vanishes asymptotically.

   (d) Assume that α is an integer. Show that the posterior for $\frac{2(n\beta+1)}{\beta}\lambda$ given X is $\chi^2(2(\alpha + \Sigma X_i))$.

   (e) Using result of (d), suggest a 95%-credible interval for λ.

2. Suppose that conditional on τ a random variable X has normal distribution with mean zero and variance $\frac{1}{\tau}$. The prior for τ is Gamma (α, β).

   (a) Find the posterior for τ.

   (b) Compare the prior mean for τ and the posterior mean.

3. Let X be a random variable with exponential distribution

$$f(x \mid \beta) = \frac{1}{\beta} e^{-y/\beta} \quad ; x > 0,\ \beta > 0.$$

   One wants to test $H_0 : \beta = \beta_0$ against $H_a : \beta \neq \beta_0$.

   (a) Suggest a 5% level test.

   (b) Draw the power function.

   (c) Provide the formula for the p-value.

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