Treatment Effects I

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Treatment effects about how outcome of interest (earnings) is affected treatment (job training) program.

Like structural model, outcome of interest is the left hand side variable, treatment is a right-hand side variable.

Binary (endogenous) right-hand side variable with heterogenous coefficients.

Have terminology all their own.

i is individual,

$D_i \in \{0, 1\}$ is treatment indicator ($D_i = 1$ is enrollment in training)

Y_{i0} potential outcome occurs when not treated ($D_i = 0$),

Y_{i1} potential outcome when treated ($D_i = 1$).

Observed outcome will be

$$Y_i = D_i Y_{i1} + (1 - D_i) Y_{i0}.$$

Y_{i0} and Y_{i1} are counterfacutals; both not observed.

$$Y_i = D_i Y_{i1} + (1 - D_i) Y_{i0}.$$

Treatment effect:

$$\beta_i = Y_{i1} - Y_{i0}.$$

Not identifiable; only one of Y_{i1} and Y_{i0} are observed.

Individual heterogenity in effect of treatment.

Some objects may be identified. Average treatment effect:

$$ATE \stackrel{def}{=} E[\beta_i].$$

Average treatment effect on treated:

$$TT \stackrel{def}{=} E[\beta_i | D_i = 1].$$

Local average treatment effect and other effects described below.

$$Y_i = D_i Y_{i1} + (1 - D_i) Y_{i0}.$$

Random coefficient interpretation.

$$\begin{aligned} Y_i &= Y_{i0} + (Y_{i1} - Y_{i0}) D_i = \alpha_i + \beta_i D_i, \\ \alpha_i &= Y_{i0}, \beta_i = Y_{i1} - Y_{i0}. \end{aligned}$$

Treatment effect β_i is the coefficient of D_i and the constant α_i and slope β_i may vary over individuals.

ATE is average of slope over entire population.

TT is average of slope over treated subset where $D_i = 1$.

Intellectual history: $\beta_i = Y_{i1} - Y_{i0}$ is "counterfactual," Rubin (70's).

Econometricians know as "movement along a curve," Wright (1928).

Here consider identification and estimation of various effects.

Constant Treatment Effects

Constant treatment effects:

$$\beta_i = \bar{\beta}$$

$ATE = TT = \bar{\beta}$.

For $\bar{\alpha} = E[\alpha_i]$ and $\varepsilon_i = \alpha_i - \bar{\alpha}$,

$$Y_i = \bar{\alpha} + \bar{\beta} D_i + \varepsilon_i.$$

Simple linear model with additive disturbance and constant coefficients.

General model is linear model with additive disturbance but random slope coefficient.

Note equivalence between random α_i and a constant plus disturbance $\alpha_i = \bar{\alpha} + \varepsilon_i$.

Instrument Z_i will identify $\bar{\beta}$ and $\bar{\alpha}$ in usual way.

Z_i uncorrelated with ε_i and correlated with D_i, that is

$$\begin{aligned} 0 &= Cov(Z_i, \varepsilon_i) = Cov(Z_i, \alpha_i) = Cov(Z_i, Y_{i0}), \\ Cov(Z_i, D_i) &\neq 0. \end{aligned}$$

Then

$$\bar{\beta} = Cov(Z_i, Y_i)/Cov(Z_i, D_i).$$

Estimate in the usual way.

Nothing new here but terminology.

Constant treatment effect too strong.

Will not hold for effect of training, schooling, etc.

β_i will vary over individuals.

Random Assignment

Random assignment means D_i not related to individual characteristics.

Assume

$$E[Y_{i0}|D_i] = E[Y_{i0}],$$

The mean of outcome without treatment does not depend on treatment..

Equivalently $E[\alpha_i|D_i] = E[\alpha_i]$.

Slightly more general than independence.

To see what happens under this assumption note first that

$$E[\beta_i|D_i]D_i = \begin{cases} 0, & D_i = 0, \\ E[\beta_i|D_i = 1], & D_i = 1 \end{cases} = E[\beta_i|D_i = 1]D_i.$$

Then

$$\begin{aligned} E[Y_i|D_i] &= E[\alpha_i + \beta_i D_i|D_i] = E[\alpha_i] + E[\beta_i|D_i]D_i \\ &= E[\alpha_i] + E[\beta_i|D_i = 1]D_i. \end{aligned}$$

$$E[Y_i|D_i] = E[\alpha_i] + E[\beta_i|D_i = 1]D_i.$$

Simple linear regression, constant is $E[\alpha_i]$ slope is $TT = E[\beta_i|D_i = 1]$.

Assume also that

$$E[Y_{i1}|D_i] = E[Y_{i1}],$$

Then we find that $ATE = TT$, since

$$\begin{aligned} E[\beta_i|D_i &= 1] = E[Y_{i1}|D_i = 1] - E[Y_{i0}|D_i = 1] \\ &= E[Y_{i1}] - E[Y_{i0}] = E[\beta_i]. \end{aligned}$$

Summarizing, if $E[Y_{i0}|D_i] = E[Y_{i0}]$ then by usual formula for OLS regression with constant and dummy variable,

$$TT = \frac{Cov(D_i, Y_i)}{Var(D_i)} = E[Y_i|D_i = 1] - E[Y_i|D_i = 0].$$

If in addition $E[Y_{i1}|D_i] = E[Y_{i1}]$ then

$$ATE = E[Y_i|D_i = 1] - E[Y_i|D_i = 0].$$

Discussion

Random assignment too strong for many applications.

Individuals may choose whether to accept the treatment or not.

May drop out of training programs

May opt out of medical treatment.

If decisions related to (α_i, β_i) then (α_i, β_i) and D_i not independent.

D_i may be correlated with both α_i and β_i.

Two approaches: a) Instrumental variables (IV). b) Selection on observables.

IV Identification of Treatment Effects

Two cases: Dummy instrument and continuous instrument.

Dummy Instruments

$Z_i \in \{0,1\}$, $0 < \Pr(Z_i = 1) = P < 1$. (Why?)

Assume throughout that that mean independence holds, as in

$$E[\alpha_i|Z_i] = E[Y_{i0}|Z_i] = E[Y_{i0}] = E[\alpha_i].$$

Wald formula for IV limit: By usual least squares formula for slope when right-hand side variable is a dummy,

$$\frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)} = \frac{Cov(Z_i, Y_i)/Var(Z_i)}{Cov(Z_i, D_i)/Var(Z_i)} = \frac{E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]}{E[D_i|Z_i = 1] - E[D_i|Z_i = 0]}$$

Plugging in $Y_i = \alpha_i + \beta_i D_i$, and using mean independence of α_i we find

$$\begin{aligned}\frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)} &= \frac{E[\alpha_i|Z_i = 1] - E[\alpha_i|Z_i = 0] + E[\beta_i D_i|Z_i = 1] - E[\beta_i D_i|Z_i = 0]}{E[D_i|Z_i = 1] - E[D_i|Z_i = 0]} \\ &= \frac{E[\beta_i D_i|Z_i = 1] - E[\beta_i D_i|Z_i = 0]}{E[D_i|Z_i = 1] - E[D_i|Z_i = 0]}.\end{aligned}$$

In general, β_i and D_i correlated.

Not possible separate them out in general.

Random Intention to Treat

Medical trials have random assignment to treatment, where no one not assigned receive treatment.

Z_i is treatment assignment, $Z_i = 0$ means not assigned.

When $Z_i = 0$ we have $D_i = 1$.

Here

$$TT = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)}$$

To show, note first that $\{D_i = 1\} \subseteq \{Z_i = 1\}$, so that

$$\{D_i = 1\} \cap \{Z_i = 1\} = \{D_i = 1\}.$$

It follows that

$$TT = E[\beta_i | D_i = 1] = E[\beta_i | D_i = 1, Z_i = 1]$$

$$TT = E[\beta_i | D_i = 1, Z_i = 1]$$

Then by iterated expectations and $D_i = 0$ when $Z_i = 0$,

$$\begin{aligned}
\frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)} &= \frac{E[\beta_i D_i | Z_i = 1] - 0}{E[D_i | Z_i = 1] - 0} = \frac{E[E[\beta_i | D_i = 1, Z_i = 1] D_i | Z_i = 1]}{E[D_i | Z_i = 1]} \\
&= \frac{E[TT \cdot D_i | Z_i = 1]}{E[D_i | Z_i = 1]} = TT.
\end{aligned}$$

Has led to widespread use of instrumental variables in biostatistics.

Medical trials have Z_i randomly assigned.

People need not take treatment though.

IV gives effect of treatment on treated.

The Local Average Treatment Effect

Average treatment effect for a particular group that is sometimes interesting.

In addition to $E[Y_{i0}|Z_i] = E[Y_{i0}]$ assume

Independence: $D_i = \Pi(Z_i, V_i)$ *and* (β_i, V_i) *is independent of* Z_i;

Monotonicity: $\Pi(1, V_i) \geq \Pi(0, V_i)$ *and* $\Pr(\Pi(1, V_i) > \Pi(0, V_i)) > 0$.

$\Pi(z, v)$ is reduced form.

Ex: Threshold crossing or index model

$$D_i = 1(D_i^* \geq 0), D_i^* = Z_i - V_i$$

Reduced form is "selection equation"

$$LATE = E[\beta_i|\Pi(1, V_i) > \Pi(0, V_i)].$$

Average treatment effect for those whose behavior changes with instrument

Ex: Y_i is the log of earnings, D_i completing high school, and Z_i is a quarter of birth dummy,

LATE is average over dropouts who would have remained in school had their quarter of birth been different and those who remained in school but would have dropped out if their quarter of birth were different.

Average returns to completing high school for potential dropouts.

Interesting parameter; not returns to schooling over the whole population.

Show

$$LATE = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)}.$$

Let $T_i = \Pi(1, V_i) - \Pi(0, V_i)$. By monotonicity, $T_i \in \{0, 1\}$. Also,

$$\begin{aligned} E[\beta_i D_i | Z_i &= 1] - E[\beta_i D_i | Z_i = 0] \\ &= E[\beta_i \Pi(1, V_i) | Z_i = 1] - E[\beta_i \Pi(0, V_i) | Z_i = 0] \\ &= E[\beta_i \Pi(1, V_i)] - E[\beta_i \Pi(0, V_i)] = E[\beta_i T_i]. \end{aligned}$$

and similarly

$$E[D_i | Z_i = 1] - E[D_i | Z_i = 0] = E[\Pi(1, V_i)] - E[\Pi(0, V_i)] = E[T_i].$$

Then

$$\frac{\text{cov}(Z_i, Y_i)}{\text{cov}(Z_i, D_i)} = \frac{E[\beta_i T_i]}{E[T_i]} = E[\beta_i | T_i = 1] = E[\beta_i | \Pi(1, V_i) > \Pi(0, V_i)].$$

LATE Empirical Example

Empirical example: Angrist and Krueger (1991).

1980 U. S. Census for males born in 1930-1939.

2SLS estimator with 3 instruments is .1077 (.0195)

FULL estimator with 180 instruments is .1063 (.0143; with many instruments correction).

Returns to schooling of "potential dropouts" is about 11 percent.