# Demand Estimation with Imperfect Competition

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Fall 2007

Methods widely applied.

Also illustrates GMM and estimation by simulation.

Berry, Levinsohn, Pakes (1996),

Motivated by an empirical application to estimating demand for different types of cars.

Data is prices, market shares, attributes of car models over several years.

Some microeconomic data on consumers.

Approach: Use individual maximizing model and aggregate (by simulation) to get share equations, as a function of observed attributes and one disturbance per model, set equal to empirical shares, invert to get disturbances, and , solve for disturbances, do instrumental variables (to account for endogenous prices).

Model:

Let $i$ index individuals and $j$ index car types.

Individual utility function:

$$U(\zeta_i, p_j, x_j, \xi_j, \theta), j = 1, ..., J; j = 0 \text{ is no car}$$

$\zeta_i$ : individual heterogeneity

$p_j$ : price of type $j$

$x_j$ : observed attributes of cars

$\xi_j$ : unobserved attribute, scalar, one per car; this is structural disturbance.

$\theta$ : parameters of utility function.

Consumer chooses $j^{th}$ car if

$$\zeta_i \in A_j = \left\{ \zeta : U(\zeta, p_j, x_j, \xi_j; \theta) \geq U(\zeta, p_r, x_r, \theta)(r = 0, 1, \right\}$$

Let $p = (p_1, \ldots, p_J),\ x = (x_1, \ldots, x_J),\ \xi = (\xi_1, \ldots, \xi_J)$.

Let $F_0$ be the CDF of $\zeta$. The $i^{th}$ share is

$$s_j(p, x, \xi; \theta) = \int_{\zeta \in A_j} F_0(d\zeta)$$

Few closed forms.

Functional forms: Starting point is linear utility and multinomial logit.

Utility function

$$\begin{aligned} U(\zeta_i, p_j, x_j, \xi_j; \theta) &= x_j\beta - \alpha p_j + \xi_j + \varepsilon_{ij} = \delta_j + \varepsilon_{ij} \\ \delta_j &= x_j'\beta - \alpha p_j + \xi_j \\ \zeta_i &= (\varepsilon_{i0}, ..., \varepsilon_{iJ}). \end{aligned}$$

Distribution of $\zeta_i$: $\varepsilon_{i0}, ..., \varepsilon_{iJ}$; i.i.d. Type I extreme value.

Share equation is

$$s_j(\delta_0, ..., \delta_J) = \frac{e^{\delta_j}}{\sum_{k=0} e^{\delta_k}}$$

Gives linear log-share equations

$$\ln(s_j/s_k) = \delta_j - \delta_k = (x_j - x_k)'\beta - \alpha(p_j - p_k) + \xi_j - \xi_k.$$

Logit demand:

$$\ln(s_j/s_0) = \delta_j - \delta_0 = (x_j - x_0)'\beta - \alpha(p_j - p_0) + \xi_j - \xi_0.$$

Could do linear IV on this, plugging in estimated shares.

Instruments: BLP use $x's$ for other products; this assumes that unobserved attribute $\xi_j$ and and observed attributes $x_j$ are uncorrelated.

Do GMM assuming $\xi_j - \xi_0$ uncorrelated across $j$ but possibly heteroskedastic.

Question about which other goods to use to form instruments.

Hausman: Use prices of goods in geographically distinct markets.

Problem with logit: Relative shares of any pair of cars depend only on prices for those cars.

$$\ln(s_j/s_k) = \delta_j - \delta_k = (x_j - x_k)'\beta - \alpha(p_j - p_k) + \xi_j - \xi_k.$$

Logit independence from independent alternatives (IIA) problem.

Example: Relative share of Corolla and Mercedes is not affected by price of BMW.

BLP try to deal with this problem by allowing for random coefficients in the logit.

Suppose $x_j$ is $L \times 1$. Then assume

$$\beta_\ell = \bar{\beta}_\ell + \sigma_\ell v_{i\ell},\ \ell = 1, \ldots, L.$$

$(v_{i1}, ..., v_{iL})$ standard normal.

Breaks IIA, but there are questions about how much additional substitutability this allows for. See Hausman and Harding (2007), "Using a Laplace Approximation to Estimate the Random Coefficients Logit Model by Non-linear Least Squares."

Random coefficients logit does not have closed form shares.

So simulate to get estimated share equations.

Also, BLP allow for income effects in their application.

Let $\bar{\beta} = (\bar{\beta}_1, ..., \bar{\beta}_L)'$, $y_i$ the income of the ith individual,.

$$\begin{aligned} U_{ij} &= \alpha \ln(y_i - p_j) + x_j'\bar{\beta} + \xi_j + \sum_{\ell=1}^{L} \sigma_\ell x_{j\ell} v_{i\ell} + \varepsilon_{ij}, j = 1, ..., J \\ U_{i0} &= \alpha \ln(y_i) + \sigma_0 v_{i0} + \varepsilon_{i0} \end{aligned}$$

Let

$$\delta_0 = 0, \delta_j = x_j'\bar{\beta} + \xi_j, (j = 1, ..., J), v = (v_{i0}, ..., v_{iJ}, y_i),$$

Also, let

$$\gamma = (\alpha, \sigma_0, \sigma_1, ..., \sigma_L),$$

Assume $\varepsilon' s$ independent of $v$. Then, conditional on $v' s$ still have logit form for choice probabilities:

$$P_j(\delta, v_i, \gamma)$$
$$= \frac{\exp\left(\delta_j + \alpha \ln(y_i - p_j) + \sum_{\ell=1}^{L} \sigma_\ell x_{j\ell} \nu_{i\ell}\right)}{\exp\left(\alpha \ln(y_i) + \sigma_0 v_{i0}\right) + \sum_{k=1}^{J} \exp\left(\delta_k + \alpha \ln(y_i - p_k) + \sum_{\ell=1}^{L} \sigma_\ell x_{k\ell} \nu_{i\ell}\right)}$$

Share equations are integral.

$$S_j(\delta, \gamma) = \int P_j(\delta, v_i, \gamma) G_0(dv_i).$$

No closed form. Estimate this by simulations:

$$S_j(\delta,\gamma) = \int P_j(\delta, v_i, \gamma) G_0(dv_i).$$

For $(i = 1, ..., I)$, let $v_{i0}, ..., v_{iL}, v_i^y$ be independent $N(0,1)$, and $v_i^* = (v_{i0}, ..., v_{iL}, \exp($
$\hat{\sigma}_y v_i^y))$.

This assumes that $y_i$ is log normally distributed.

Estimated share equations are

$$\tilde{S}_j(\delta,\gamma) = \sum_{i=1}^{I} P_j(\delta, v_i^*, \gamma)$$

To finish description, need to describe supply side.

Could estimate just using demand side, but get low precision.

This is a case where some of parameters enter supply and demand side, and using both improves precision substantially.

Assume that marginal cost of $j$ commodity is

$$\ln(mc_j) = w_j'\alpha + \eta_j$$

$w_j$ observed cost characteristics

$\eta_j$ unobserved characteristics

$F$ firms, $\mathcal{F}_f$ set of products produced by firm $f$. Profits of firm $f$:

$$\Pi_f = \sum_{r \in F_f} (p_r - mc_r) M s_r(p, x, \xi; \theta)$$

where $m$ is total consumers.

Assume Bertrand competition.

First-order conditions for profit maximization

$$s_j(p,x,\xi;\theta)+\sum_{t\in F_f}(p_t-mc_t)\frac{\partial s_t(p,x,\xi;\theta)}{\partial p_j}=0, t\in F_f$$

Define

$$\Delta_{jt}=\begin{cases}-\frac{\partial s_t}{\partial p_j}, & \text{if } t \text{ and } j \text{ produced by same firm}\\ 0, & \text{otherwise.}\end{cases}$$

Matrix form:

$$s(p,x,\xi;\theta)-\Delta(p,x,\xi;\theta)[p-mc]=0$$

Solve for marginal cost:

$$\eta_j=\ln(p-b(p,x,\xi;\theta))_j-w_j'\alpha; b=\Delta^{-1}s.$$

Basic Assumption: Let $z_j = [x_j, w_j]$, $z = [z_1, \ldots, z_J]$. Assume

$$E[\xi_j \mid z] = E[\eta_j \mid z] = 0.$$

Observed demand and cost characteristics mean independent of unobserved demand and cost characteristics.

Use functions of $z$ as instruments.

Form moment conditions as

$$\hat{g}(\theta) = \sum_{j=1}^{J} \begin{pmatrix} a_j(z)\hat{\xi}_j(\theta) \\ d_j(z)\hat{\eta}_j(\theta) \end{pmatrix} / J$$

Do GMM.

Need to estimate residuals.

To form $\hat{\xi}_j(\theta)$, replace $s_j(p, x, \xi, \theta)$ by $\hat{S}_j(\delta, \gamma)$, set equal to shares $\hat{s}_j$ in data, and solve for $\hat{\delta}_j(\hat{s}, p, x; \theta)$.

Solve

$$\hat{s}_j = \tilde{S}_j(\delta, \gamma)$$

for $\hat{\delta}_j(\hat{s}, \gamma)$. $\hat{S}(\delta, \gamma)$ is a contraction mapping in $\delta$; see paper.

$$\begin{aligned} \hat{\xi}_j(\hat{s}, \theta) &= \hat{\delta}_j(\hat{s}, \gamma) - x_j'\bar{\beta} \\ \hat{\eta}_j(\hat{s}, \theta) &= \ln(p_j - b_j(p, x, \hat{\xi}; \theta)) - w_j'\alpha \end{aligned}$$

Note that moment conditions will be linear in $\bar{\beta}$ and $\alpha$.

Can concentrate them out.

## Optimal Instrument

$z$ is *includes all* $j$.

Can show under symmetric Nash equilibrium $D_j(z)$ depends on

$$z_j, \sum_{\ell \in F_j} z_\ell, \sum_{\ell \notin F_j} z_\ell.$$

Do nonlinear IV using these as instruments (collinear with higher-order powers).