Chapter 7

Endogenous Growth II: R&D and Technological Change

7.1 Expanding Product Variety: The Romer Model

- There are three sectors: one for the final good sector, one for intermediate goods, and one for R&D.

- The final good sector is perfectly competitive and thus makes zero profits. Its output is used either for consumption or as input in each of the other two sector.

- The intermediate good sector is monopolistic. There is product differentiation. Each intermediate producer is a quasi-monopolist with respect to his own product and thus enjoys positive profits. To become an intermediate producer, however, you must first acquire a “blueprint” from the R&D sector. A “blueprint” is simply the technology or know-how for transforming final goods to differentiated intermediate inputs.

- The R&D sector is competitive. Researchers produce “blueprints”. Blueprints are protected by perpetual patents. Innovators auction their blueprints to a large number of potential buyers, thus absorbing all the profits of the intermediate good sector. But there is free entry in the R&D sector, which drive net profits in that sector to zero as well.

7.1.1 Technology

- The technology for final goods is given by a neoclassical production function of labor L and a composite factor $\mathcal{X}$:

$$Y_t = F(\mathcal{X}_t, L_t) = A(L_t)^{1-\alpha}(\mathcal{X}_t)^{\alpha}.$$

The composite factor is given by a CES aggregator of intermediate inputs:

$$\mathcal{X}_t = \left[\int_0^{N_t} (X_{t,j})^{\varepsilon} dj\right]^{1/\varepsilon},$$

where N_t denotes the number of different intermediate goods available in period t and $X_{t,j}$ denotes the quantity of intermediate input j employed in period t.

- In what follows, we will assume $\varepsilon = \alpha$, which implies

$$Y_t = A(L_t)^{1-\alpha} \int_0^{N_t} (X_{t,j})^{\alpha} dj.$$

Note that $\varepsilon = \alpha$ means the marginal product of each intermediate input is independent of the quantity of other intermediate inputs:

$$\frac{\partial Y_t}{\partial X_{t,j}} = \alpha A \left(\frac{L_t}{X_{t,j}} \right)^{1-\alpha}.$$

More generally, intermediate inputs could be either complements or substitutes, in the sense that the marginal product of input j could depend either positively or negatively on X_t.

- We will interpret intermediate inputs as capital goods and therefore let aggregate "capital" be given by the aggregate quantity of intermediate inputs:

$$K_t = \int_0^{N_t} X_{t,j} dj.$$

- Finally, note that if $X_{t,j} = X$ for all j and t, then

$$Y_t = A L_t^{1-\alpha} N_t X^\alpha \qquad \text{and} \qquad K_t = N_t X,$$

implying

$$Y_t = A(N_t L_t)^{1-\alpha}(K_t)^\alpha$$

or, in intensive form, $y_t = A N_t^{1-\alpha} k_t^\alpha$. Therefore, to the extent that all intermediate inputs are used in the same quantity, the technology is linear in knowledge N and capital K. Therefore, if both N and K grow at a constant rate, as we will show to be the case in equilibrium, the economy will exhibit long run growth, as in an AK model.

7.1.2 Final Good Sector

- The final good sector is perfectly competitive. Firms are price takers.

- Final good firms solve
$$\max \; Y_t - w_t L_t - \int_0^{N_t} (p_{t,j} X_{t,j}) dj$$
where w_t is the wage rate and $p_{t,j}$ is the price of intermediate good j.

- Profits in the final good sector are zero, due to CRS, and the demands for each input are given by the FOCs
$$w_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)\frac{Y_t}{L_t}$$
and
$$p_{t,j} = \frac{\partial Y_t}{\partial X_{t,j}} = \alpha A \left(\frac{L_t}{X_{t,j}}\right)^{1-\alpha}$$
for all $j \in [0, N_t]$.

7.1.3 Intermediate Good Sector

- The intermediate good sector is monopolistic. Firms understand that they face a downward sloping demand for their output.

- The producer of intermediate good j solves

$$\max \ \Pi_{t,j} = p_{t,j} X_{t,j} - \kappa(X_{t,j})$$

subject to the demand curve

$$X_{t,j} = L_t \left(\frac{\alpha A}{p_{t,j}} \right)^{\frac{1}{1-\alpha}},$$

where $\kappa(X)$ represents the cost of producing X in terms of final-good units.

- We will let the cost function be linear:

$$\kappa(X) = X.$$

  The implicit assumption behind this linear specification is that technology of producing intermediate goods is identical to the technology of producing final goods. Equivalently, you can think of intermediate good producers buying final goods and transforming them to intermediate inputs. What gives them the know-how for this transformation is precisely the blueprint they hold.

- The FOCs give

$$p_{t,j} = p \equiv \frac{1}{\alpha} > 1$$

for the optimal price, and

$$X_{t,j} = xL$$

for the optimal supply, where

$$x \equiv A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}.$$

- The resulting maximal profits are

$$\Pi_{t,j} = \pi L$$

where

$$\pi \equiv (p-1)x = \tfrac{1-\alpha}{\alpha} x = \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}.$$

- Note that the price is higher than the marginal cost ($p = 1/\alpha > \kappa'(X) = 1$), the gap representing the mark-up that intermediate-good firms charge to their customers (the final good firms). Because there are no distortions in the economy other than monopolistic competition in the intermediate-good sector, the price that final-good firms are willing to pay represents the social product of that intermediate input and the cost that intermediate-good firms face represents the social cost of that intermediate input. Therefore, the mark-up $1/\alpha$ gives the gap between the social product and the social cost of intermediate inputs.

- *Hint:* The social planner would like to correct for this distortion. How?

7.1.4 The Innovation Sector

- The present value of profits of intermediate good j from period t and on is given by

$$V_{t,j} = \sum_{\tau=t} \frac{q_\tau}{q_t} \Pi_{\tau,j} \qquad \text{or} \qquad V_{t,j} = \Pi_{t,j} + \frac{V_{t+1,j}}{1+R_{t+1}}$$

- We know that profits are stationary and identical across all intermediate goods: $\Pi_{t,j} = \pi L$ for all t, j. As long as the economy follows a balanced growth path, we expect the interest rate to be stationary as well: $R_t = R$ for all t. It follows that the present value of profits is stationary and identical across all intermediate goods:

$$V_{t,j} = V = \frac{\pi L}{R/(1+R)} \approx \frac{\pi L}{R}.$$

Equivalently, $RV = \pi L$, which has a simple interpretation: The opportunity cost of holding an asset which has value V and happens to be a "blueprint", instead of investing in bonds, is RV; the dividend that this asset pays in each period is πL; arbitrage then requires the dividend to equal the opportunity cost of the asset, namely $RV = \pi L$.

- New blueprints are produced using the same technology as final goods: innovators buy final goods and transform them to blueprints at a rate $1/\eta$. It follows that producing an amount ΔN of new blueprints costs $\eta \cdot \Delta N$, where $\eta > 0$ measures the cost of R&D in units of output.

- On the other hand, the value of these new blueprints is $V \cdot \Delta N$, where $V = \pi L/R$.

- It follows that net profits for a research firm are thus given by

$$profits_{\text{R\&D}} = (V - \eta) \cdot \Delta N$$

- Free entry in the sector of producing blueprints imposes $profits_{\text{R\&D}} = 0$, or equivalently

$$V = \eta.$$

7.1.5 Households

- Households solve

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t. \quad c_t + a_{t+1} \leq w_t + (1 + R_t)a_t$$

- As usual, the Euler condition gives

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta(1 + R_{t+1}).$$

And assuming CEIS, this reduces to

$$\frac{c_{t+1}}{c_t} = \left[\beta(1 + R_{t+1})\right]^{\theta}.$$

7.1.6 Resource Constraint

- Final goods are used either for consumption by households (C_t), or for production of intermediate goods in the intermediate sector ($K_t = \int_j X_{t,j}$), or for production of new blueprints in the innovation sector ($\eta \Delta N_t$). The resource constraint of the economy is therefore given by

$$C_t + K_t + \eta \cdot \Delta N_t = Y_t,$$

where $C_t = c_t L$, $\Delta N_t = N_{t+1} - N_t$, and $K_t = \int_0^{N_t} X_{t,j} dj$.

- As always, the sum of the budgets across agents together with the market clearing conditions reduce to the resource constraint. Question: what are the market clearing conditions here? Related: what are the assets traded by the agents?

7.1.7 General Equilibrium

- Combining the formula for the value of innovation with the free-entry condition, we infer $\pi L/R = V = \eta$. It follows that the equilibrium interest rate is

$$R = \frac{\pi L}{\eta} = \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L/\eta,$$

  which verifies our earlier claim that the interest rate is stationary.

- The Euler condition combined with the equilibrium condition for the real interest rate implies that consumption grows at a constant rate, which is given by

$$\frac{C_{t+1}}{C_t} = 1+\gamma = \beta^\theta \left[1+R\right]^\theta = \beta^\theta \left[1 + \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L/\eta\right]^\theta$$

- Next, note that the resource constraint reduces to

$$\frac{C_t}{N_t} + \eta \cdot \left[\frac{N_{t+1}}{N_t} - 1 \right] + X = \frac{Y_t}{N_t} = AL^{1-\alpha} X^{\alpha},$$

where $X = xL = K_t/N_t$.

- It follows that C_t/N_t is constant along the balanced growth path, and therefore C_t, N_t, K_t, and Y_t all grow at the same rate, γ, where, again,

$$1 + \gamma = \beta^{\theta} \left[1 + \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L/\eta \right]^{\theta}$$

- The equilibrium growth rate of the economy decreases with η, the cost of producing new "knowledge."The growth rate is also increasing in L, or any other factor that increases the scale (size) of the economy, and thereby raises the profits of intermediate inputs and the demand for innovation. This is the (in)famous "scale effect" that is present in many models of endogenous technological change.

7.1.8 Efficiency and Policy Implications

- Consider now the problem of the social planner. He chooses $\{C_t, (X_{t,j})_{j\in[0,N_t]}, N_{t+1}\}_{t=0}^{\infty}$ so as to maximize lifetime utility subject to the resource constraint that the technologies.

- Obviously, due to symmetry in production, the social planner will choose the same quantity of intermediate goods for all varieties: $X_{t,j} = X_t = x_t L$ for all j. Using this, we can write the problem of the social planner as follows:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t),$$

subject to

$$C_t + N_t \cdot X_t + \eta \cdot (N_{t+1} - N_t) = Y_t = AL^{1-\alpha} N_t X_t^{\alpha},$$

where $C_t = c_t L$.

- The FOC with respect to X_t gives

$$X_t = x^* L,$$

where

$$x^* = A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}}$$

represents the optimal level of production of intermediate inputs.

- The Euler condition, on the other hand, gives the optimal growth rate as

$$1 + \gamma^* = \beta^\theta \left[1 + R^*\right]^\theta = \beta^\theta \left[1 + \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} L/\eta\right]^\theta ,$$

where

$$R^* = \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} L/\eta$$

represents that social return on savings.

- Note that

$$x^* = x \cdot \alpha^{-\frac{1}{1-\alpha}} > x$$

That is, the optimal level of production of intermediate goods is higher in the Pareto optimum than in the market equilibrium. This reflects simply the fact that, due to the monopolistic distortion, production of intermediate goods is inefficiently low in the market equilibrium. Naturally, the gap x^*/x is an increasing function of the mark-up $1/\alpha$.

- Similarly,

$$R^* = R \cdot \alpha^{-\frac{1}{1-\alpha}} > R.$$

That is, the market return on savings (R) falls short of the social return on savings (R^*), the gap again arising because of the monopolistic distortion in the intermediate good sector. It follows that

$$1 + \gamma^* > 1 + \gamma,$$

so that the equilibrium growth rate is too low as compared to the Pareto optimal growth rate.

- *Policy exercise:* Consider four different types of government intervention:
  - a subsidy on the production of final goods
  - a subsidy on the demand for intermediate inputs
  - a subsidy on the production of intermediate inputs
  - a subsidy on R&D.

  Which of these policies could achieve an increase in the market return and the equilibrium growth rate? Which of these policies could achieve an increase in the output of the intermediate good sector? Which one, or which combination of these policies, can implement the first best allocation as a market equilibrium?

7.1.9 Building on the Shoulders' of Giants

- In the original Romer (1990) model, the innovation sector uses a different technology than the one assumed here. In particular, the technology for producing a new blueprint is linear in the effective labor employed by the research firm, where "effective" means amount of labor (number of "researchers") times the existing stock of knowledge. Hence, for research firm j,

$$\Delta N_{j,t} = \theta L_{j,t} N_t$$

- The aggregate rate of innovation is thus given by

$$\Delta N_t = L_t^{\text{R\&D}} N_t$$

where $L_t^{\text{R\&D}}$ is the total amount of labor employed in the R&D sector. Market clearing in the labor market is now $L_t^{\text{final}} + L_t^{\text{R\&D}} = \bar{L}$. The private cost of innovation is now proportional to w_t, while the value of innovation remains as before. The rest of the model is also as before.

- The key modification here in that the aggregate N_t enters the technology of producing new blueprints—new researchers build on the shoulders of previous researchers.

- We have thus introduced a knowledge spillover that was absent in the previous model. When today's research firms decide how much to spend on R&D, they do not internalize how this will improve the know how of future innovators.

- How does this affect the efficiency/policy conclusions we derived in the previous model?

- Imagine that the technology faced by research firm j was given by

$$\Delta N_{j,t} = (L_{j,t})^{\eta_1}(K_{j,t})^{\eta_2}(N_t)^{\eta_3}$$

  What are the restrictions for η_1, η_2 and η_3 that are necessary for perpetual growth? What are the restrictions that are necessary for the individual firm to perceive constant returns to scale, and hence for the R&D sector to be competitive?

7.2 A Simple Variant of Aghion-Howitt

- The economy is populated by a large number of "entrepreneurs". Each entrepreneur lives (is present in the market) for $1+T$ periods, where T is random. Conditional on being alive in the present period, there is probability n that the entrepreneur will die (exit the market) by the end of that period. n is constant over time and independent of age. In each period, a mass n of existing entrepreneurs dies, and a mass n of new entrepreneurs is born, so that the population is constant.

- In the first period of life, the entrepreneur is endowed with the aggregate level of knowledge in the economy. In the first period of life, he also has a "fresh mind" and can engage in R&D activity. In later periods of life, instead, he is too old for coming up with good new ideas and therefore engages only in production, not innovation.

- Young producers engage in R&D in order to increase the profits of their own productive activities later in life. But individual innovation has spillover effects to the whole economy. When a mass of producers generate new ideas, the aggregate level of knowledge in the economy increases proportionally with the production of new ideas.

7.2.1 R&D Technology

- Let V_{t+1}^j denote the value of an innovation for individual j realized in period t and implemented in period $t+1$. Let z_t^j denote the amount of skilled labor that a potential innovator j employes in R&D and $q(z_t^j)$ the probability that such R&D activity will be successful. $q : \mathbb{R} \to [0,1]$ represents the technology of producing innovations and satisfies $q(0) = 0$, $q' > 0 > q''$, $q'(0) = \infty$, $q'(\infty) = 0$.

- The potential researcher maximizes

$$q(z_t^j) \cdot V_{t+1}^j - w_t \cdot z_t^j.$$

It follows that the optimal level of R&D is given by $q'(z_t^j)V_{t+1}^j = w_t$ or

$$z_t^j = g\left(V_{t+1}^j / w_t\right)$$

where the function $g(v) \equiv (q')^{-1}(1/v)$ satisfies $g(0) = 0$, $g' > 0$, $g(\infty) = \infty$. Note that z will be stationary only if both V and w grow at the same rate.

7.2.2 The Value of Innovation

- What determines the value of an innovation? For a start, let us assume a very simple structure. Let A_t^j represent the TFP of producer j in period t. The profits from his production are given by

$$\Pi_t^j = A_t^j \widehat{\pi}$$

  where $\widehat{\pi}$ represents normalized profits. We can endogenize π, but we won't do it here for simplicity.

- When a producer is born, he automatically learns what is the contemporaneous aggregate level of technology. That is, $A_t^j = A_t$ for any producer born in period t. In the first period of life, and only in that period, a producer has the option to engage in R&D. If his R&D activity fails to produce an innovation, them his TFP remains the same for the rest of his life. If instead his R&D activity is successful, then his TFP increases permanently by a factor $1 + \gamma$, for some $\gamma > 0$.

- That is, for any producer j born in period t, and for all periods $\tau \geq t+1$ in which he is alive,

$$A_\tau^j = \begin{cases} A_t & \text{if his R\&D fails} \\ (1+\gamma)A_t & \text{if his R\&D succeeds} \end{cases}$$

- It follows that a successful innovation generates a stream of "dividends" equal to $\gamma A_t \widehat{\pi}$ per period for all $\tau > t$ that the producer is alive. Therefore,

$$V_{t+1} = \sum_{\tau=t+1}^{\infty} \left(\frac{1-n}{1+R} \right)^{\tau} (\gamma A_t \widehat{\pi}) = \gamma \widehat{v} A_t \tag{7.1}$$

where where R is the interest rate per period and

$$\widehat{v} \equiv \sum_{\tau=1}^{\infty} \left(\frac{1-n}{1+R} \right)^{\tau} \widehat{\pi} \approx \frac{\widehat{\pi}}{R+n}.$$

Note that the above would be an exact equality if time was continuous. Note also that $\widehat{v}$ is decreasing in both R and n.

- *Remark:* We see that the probability of "death" reduces the value of innovation, simply because it reduces the expected life of the innovation. Here we have taken n as exogenous for the economy. But later we will endogenize n. We will recognize that the probability of "death" simply the probability that the producer will be displaced by another competitor who manages to innovate and produce a better substitute product. For the time being, however, we treat n as exogenous.

7.2.3 The Cost of Innovation

- Suppose that skilled labor has an alternative employment, which a simple linear technology of producing final goods at the current level of aggregate TFP. That is, if l_t labor is used in production of final goods, output is given by $A_t l_t$. Since the cost of labor is w_t, in equilibrium it must be that

$$w_t = A_t. \tag{7.2}$$

7.2.4 Equilibrium

- Combining (7.1) and (7.2), we infer that

$$\frac{V_{t+1}}{w_t} = \gamma \widehat{v}$$

It follows that the level of R&D activity is the same across all new-born producers:

$$z_t^j = z_t = g\left(\gamma \widehat{v}\right).$$

- The outcome of the R&D activity is stochastic for the individual. By the LLN, however, the aggregate outcome is deterministic. The aggregate rate of innovation is simply

$$\lambda_t = q(z_t) = \lambda\left(\gamma \widehat{v}\right)$$

where $\lambda\left(x\right) \equiv q\left(g\left(x\right)\right).$

- It follows that the aggregate level of technology grows at a rate

$$\frac{A_{t+1}}{A_t} = 1 + \gamma\lambda_t = 1 + \gamma\lambda\left(\gamma\widehat{v}\right).$$

- An increase in $\widehat{\pi}$ increases the incentives for R&D in the individual level and therefore results to higher rates of innovation and growth in the aggregate level. An increase in γ has a double effect. Not only it increases the incentive for R&D, but it also increase the spill over effect from individual innovations to the aggregate level of technology.

- What is aggregate output in the economy? It's the sum of the output of all entrepreneurs plus the output of workers not employed in *R&D*. Check that aggregate output grows at the same rate as aggregate knowledge.

7.2.5 Business Stealing

- Consider a particular market j, in which a producer j has monopoly power. Suppose now that there is an outside competitor who has the option to engage in R&D in an attempt to create a better product that is a close substitute for the product of producer j. Suppose further that, if successful, the innovation will be so "radical" that, not only it will increase productivity and reduce production costs, but it will also permit the outsider to totally displace the incumbent from the market.

- *Remark:* Here we start seeing how both production and innovation may depend on the IO structure. In more general versions of the model, the size of the innovation and the type of competition (e.g., Bertrand versus Cournot) determine what is the fraction of monopoly profits that the entrant can grasp and hence the private incentives for innovation.

- What is the value of the innovation for this outsider? Being an outsider, he has no share in the market of product j. If his R&D is successful, he expects to displace the incumbent and grasp the whole market of product j. That is, an innovation delivers a dividend equal to total market profits, $(1+\gamma)A_t\widehat{\pi}$, in each period of life. Assuming that the outsider also has a probability of death (or displacement) equal to n, the value of innovation for the outsider is given by

$$V_{t+1}^{out} = \sum_{\tau=t+1}^{\infty} \left(\frac{1-n}{1+R}\right)^{\tau} [(1+\gamma)A_t\widehat{\pi}] = (1+\gamma)\widehat{v}A_t$$

- Now suppose that the incumbent also has the option to innovate is later periods of life. If he does so, he will learn the contemporaneous aggregate level of productivity and improve upon it by a factor $1+\gamma$. The value of innovation in later periods of life is thus the same as in the first period of life:

$$V_{t+1}^{in} = \sum_{\tau=t+1}^{\infty} \left(\frac{1-n}{1+R}\right)^{\tau} [\gamma A_t\widehat{\pi}] = \gamma\widehat{v}A_t.$$

- Obviously, $V_{t+1}^{out} > V_{t+1}^{in}$. This is because the incumbent values only the potential increase in productivity and profits, while the outsider values in addition the profits of the incumbent. This "business-stealing" effect implies that, ceteris paribus, that innovation will originate mostly in outsiders.

- *Remark:* In the standard Aghion-Howitt model, as opposed to the variant considered here, *only* outsiders engage in innovation. Think why this is the case in that model, and why this might not be the case here. Then, find conditions on the technology q and the parameters of the economy that would ensure in our model a corner solution for the insiders and an interior solution for the outsiders. (Hint: you may need to relax the Inada condition for q.) We will henceforth assume that only outsiders engage in innovation.

- *Remark:* Things could be different if the incumbent has a strong cost advantage in R&D, which could be the case if the incumbent has some private information about the either the technology of the product or the demand of the market.

- Assuming that only outsiders engage in R&D, and using $V_{t+1}^{out}/w_t = (1+\gamma)\widehat{v}$, we infer that the optimal level of R&D for an outsider is

$$z_t^{out} = z_t = g\left((1+\gamma)\widehat{v}\right).$$

and therefore the aggregate rate of innovation is

$$\lambda_t = q(z_t) = \lambda\left((1+\gamma)\widehat{v}\right)$$

We conclude that the growth rate of the economy is

$$\frac{y_{t+1}}{y_t} = \frac{A_{t+1}}{A_t} = 1 + \gamma\lambda\left((1+\gamma)\widehat{v}\right).$$

- We can now reinterpret the probability of "death" as simply the probability of being displaced by a successful outside innovator. Under this interpretation, we have

$$n = \lambda \left((1+\gamma)\widehat{v} \right)$$

and $\widehat{v}$ solves

$$\widehat{v} = \frac{\widehat{\pi}}{R + \lambda \left((1+\gamma)\widehat{v} \right)}$$

- Note that an increase in $\widehat{\pi}$ will now increase $\widehat{v}$ by less than one-to-one, because the displacement rate will also increase.

7.2.6 Efficiency and Policy Implications

- Discuss the spillover effects of innovation... Both negative and positive...

- Discuss optimal patent protection... Trade-off between incentives and externalities...

7.3 Ramsey Meets Schumpeter: The Aghion-Howitt Model

notes to be completed

7.4 Romer Meets Acemoglu: Biased Technological Change

7.4.1 Definition

- Consider a two-factor economy, with

$$Y_t = F(L_t, H_t, A_t)$$

where L and H denote, respectively, unskilled labor and skilled labor (or any two other factors) and A denotes technology.

- We say that technology is $H-$biased if and only if

$$\frac{\partial}{\partial A}\left(\frac{\partial F(L,H,A)/\partial H}{\partial F(L,H,A)/\partial L}\right) > 0$$

- Note that this is different from saying that technology is $H-$augmenting.

7.4.2 A simple model of biased technological change

- We onsider a variant of the Romer model where we split the final good sector in two sub-sectors, one that is intensive in L and another that is intensive in H.

- Aggregate output is given by

$$Y_t = \left[\gamma \left(Y_{Lt}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(Y_{Ht}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where

$$\begin{aligned} Y_{Lt} &= L^{\beta} \int_0^{N_{Lt}} \left(x_{Lt}\right)^{1-\beta} dj \\ Y_{Ht} &= H^{\beta} \int_0^{N_{Ht}} \left(x_{Ht}\right)^{1-\beta} dj \end{aligned}$$

- The differentiated intermediate input firms use blueprints to transform 1 unit of the final good to one unit of the differentiated intermediate good.

- The R&D firms transform final goods to blueprints. Blueprints

- The resource constraint is given by

$$C_t + K_t + \eta_L \Delta N_{Lt} + \eta_H \Delta N_{Ht} \leq Y_t,$$

where

$$K_t = \int_0^{N_{Lt}} x_{Lt} dj + \int_0^{N_{Ht}} x_{Ht} dj.$$

- Given the technologies, the skill premium is given by

$$\omega \equiv \frac{w_H}{w_L} = const \cdot \left(\frac{N_H}{N_L}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}$$

where

$$\sigma \equiv \varepsilon - (\varepsilon - 1)(1 - \beta).$$

- The relative value of innovations is given by

$$\begin{aligned}\frac{V_H}{V_L} &= \left(\frac{p_H}{p_L}\right)^{\frac{1}{\beta}} \frac{H}{L} \\ &= const \cdot \left(\frac{N_H}{N_L}\right)^{-\frac{1}{\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-1}{\sigma}}.\end{aligned}$$

and the equilibrium innovation rates satisfy

$$\left(\frac{N_H}{N_L}\right) = const \cdot \left(\frac{H}{L}\right)^{\sigma-1}.$$

- Hence, once we take into account the endogeneity of technologies, the equilibrium skill premium is given by

$$\omega = const \cdot \left(\frac{H}{L}\right)^{\sigma-2}.$$

- Finally, it is easy to show that the growth rate is given by

$$\gamma - 1 \approx \frac{1}{\theta}\left(\beta\left[(1-\gamma)^{\varepsilon}(\eta_H H)^{\sigma-1} + \gamma^{\varepsilon}(\eta_L L)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} - \rho\right).$$