14.452 Macroeconomic Theory II
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Problem Set 2

Due Date: **April 20th**
We encourage you to work together, as long as you write your own solutions. Please, write short answers!

Log-Linear RBC

This question is based on the paper "Notes on RBC" by T. Philippon and R. Segura-Cayuela in the class website (in particular sections 7 and 8). Please read this paper. The paper has some typos. I'll mention below one that is important for this question.

Consider the following problem. Consumers maximize

$$\max E_t \left[\sum_{j=0}^{\infty} b^j U(\widetilde{C}_{t+j}, N_{t+j}) \right],$$

subject to

$$\widetilde{K}_{t+1} = R_t \widetilde{K}_t + \widetilde{W}_t N_t - \widetilde{C}_t$$

$$U(\widetilde{C}_t, N_t) = \log\left(\widetilde{C}_t\right) + \phi \log(1 - N_t)$$

while firms solve

$$\left\{N_t, \widetilde{K}_t\right\} = \arg \max_{N_t, \widetilde{K}_t} Z_t F(A_t N_t, \widetilde{K}_t) - (R_t + \delta - 1)\widetilde{K}_t - \widetilde{W}_t N_t$$

$$F(A_t N_t, K_t) = Y_t = Z_t \widetilde{K}_t^{1-\alpha} (A_t N_t)^{\alpha}$$

and we assume

$$A_{t+1} = \gamma A_t.$$

This is the standard RBC model. Trending variables are those with a hat.

1. De-trend the model (i.e. express the variables in terms of efficiency units).
2. Solve for the $F.O.C.$ of the consumer problem, either using a Lagrangian or a Bellman Equation. You should find two equations: the Euler equation and the labor supply.
3. Solve the firm problem to find the labor and capital demands. Give the law of motion for capital (rewrite $\widetilde{K}_{t+1} = R_t \widetilde{K}_t + \widetilde{W}_t N_t - \widetilde{C}_t$ to get the usual equation)
4. We have a total of 5 equations. Discuss how would you proceed to log-linearize our system (you don't need to do all the math, just apply the method to one or two equations).
5. Our log-linearized system is the following (small variables denote logs, $\bar{X}$ denotes the

steady state value of *X*).

a. The law of motion and the market clearing conditions:

(Notice that there is a mistake in the log-linearization of the labor supply equation in the paper. Here, I present the correct equation.)

$$-\gamma k_{t+1} + \left(1 - \delta + (1-\alpha)\frac{\bar{Y}}{\bar{K}}\right)k_t - \frac{\bar{C}}{\bar{K}}c_t + \alpha\frac{\bar{Y}}{\bar{K}}n_t + \frac{\bar{Y}}{\bar{K}}z_t = 0$$

$$(1-\alpha)(k_t - n_t) - w_t + z_t = 0$$

$$\frac{n\bar{N}}{(1-\bar{N})} - w_t + c_t = 0$$

$$\alpha k_t + \frac{\bar{R}}{\bar{R}+\delta - 1}r_t - \alpha n_t - z_t = 0$$

b. The Euler Equation (or any other asset pricing equation):

$$E_t[r_{t+1} - c_{t+1}] + c_t = 0.$$

We assume:

$$z_t = \rho z_{t-1} + \varepsilon_t.$$

We want to write the system of six equations in order to solve it numerically using the process in Uhlig (1997). Let s_t be the vector of endogenous state variables (capital stock), $x_t = [c_t; r_t; n_t; w_t]'$ the endogenous control variables, and z_t the stochastic processes. To avoid confusion, matrices are named using two capitalized letters. There are two conceptually different sets of equations. The first set describes the law of motion of the economy, together with the market equilibrium conditions: Philippon and Segura-Cayuela use $M\_$ for these matrices (Motion and Market). In the second set, the equations are forward looking and they involve the expectations of the agents: Philippon and Segura-Cayuela use $F\_$ for these equations (Forward). Finally, we have the equation for the stochastic process z_t. Write the system in the following way:

$$MS_1 * s_{t+1} + MS * s_t + MX * x_t + MZ * z_t = 0$$

$$E_t[FS_2 s_{t+2} + FS_1 s_{t+1} + FS s_t + FX_1 x_{t+1} + FX x_t + FZ_1 z_{t+1} + FZ z_t] = 0$$

$$z_{t+1} - ZZ * z_t - \varepsilon_{t+1} = 0.$$

In other words, what does each matrix contain?

6. Solving the log-linearized version of the model. We want to find numerically a solution of the type:

$$s_{t+1} = \widehat{S} * s_t + \widehat{SZ} * z_t$$

$$x_t = \widehat{X} * s_t + \widehat{XZ} * z_t,$$

You have to solve that using MATLAB. I give you a group of codes in the file *RBC.zip*. (Copy all the files in the directory from which you are running the programs.) The program that you have to modify is *rbc.m*. It contains the function *linear, which* obtains the solution of the system, the matrices derived in system above, and the values of the parameters of the model. We will assume:

$$\delta = 0.025,\ \gamma = 1.004,\ \alpha = \tfrac{2}{3},\ \phi = 1,\ \rho = .979,$$

$$\bar{R} = 1.0163,\ \bar{K} = 10.67,\ \sigma_e = .0072$$

Before actually solving the system, notice that some important variables are endogenous to these parameters (so you *cannot* change them). What are the steady state values for $\frac{C}{Y}, \frac{K}{Y}$, and N?

7. Now we want to do impulse-response functions. For that, the last part of the program calls two other functions (*impulse* and *impulse-plot*). The first finds the reduced form model and the second hits it with a shock. Finally, the program also computes the correlation of capital, consumption, the interest rate, and employment with output. That is the end of your program. Explain the results. How do they depend on the values assumed in part 6? (Run the program for different values of at least two parameters and compare results). Interpret.

Standard RBC Model: Employment and Output correlation

1. Alternatively, you can use Dynare to solve an RBC problem in its non-log-linearized version (focusing on the planer's problem). Let's work with the example given in class:

$$\max E_t \left[\sum_{j=0}^{\infty} \beta^j (\ln C_{t+j} - \frac{\psi}{1+\phi} N_{t+j}^{1+\phi}) \right],$$

$$s.t.\ C_{t+j} + I_{t+j} = A_{t+j} K_{t+j}^{\alpha} N_{t+j}^{1-\alpha}$$

$$K_{t+j+1} = (1-\delta)K_{t+j} + I_{t+j}$$

$$A_{t+j} = e^{\epsilon_{t+j}}$$

Let's assume the following processes for the productivity shock ϵ :

$$\epsilon_t = \rho_\epsilon \epsilon_{t-1} + v_t, 0 < \rho_\epsilon < 1$$

Let's also assume the following parameters:

$$\delta = 0.025,\ \alpha = \tfrac{1}{3},\ \rho_\epsilon = 0.8,\ \beta = 0.98$$

$$\phi = 2,\ \psi = 2, \sigma_v = 0.0011$$

Similarly as it was done in recitation, find the equations that characterize the equilibrium of this problem, and using DYNARE with our 5 variables, find the steady state and the corresponding inpulse response functions. Interpret.

2. Do the same for $\rho = .95, \rho = .5$. Interpret.

3. Given this model specification above, is it possible to obtain a decrease in employment in response to a positive productivity shocks? If not, what is missing from the model above in order to obtain such effect? Implement it, if this is the case, by showing the equations used, plotting the impulse-response functions, and displaying the parameters chosen. Interpret the results. (Hint: think of permanent vs. temporary productivity shocks)

Practice exercise on the RBC models:

Effects of Government Spending (Final 2003)

Consider the economy described by:

$$\max E\left[\sum_{o}^{\infty} \beta^i (U(C_{t+i}) + V(L_{t+i})) \middle| \Omega_t \right]$$

$$s.t.$$

$$LS : N_{t+i} + L_{t+i} = 1$$

$$BC : C_{t+i} + S_{t+i} + G_{t+i} = Z_{t+i} F(K_{t+i}, N_{t+i})$$

$$CAP : K_{t+i+1} = (1 - \delta) K_{t+i} + S_{t+i}$$

where G is government spending, and it is assumed not to affect the marginal utility of consumption or leisure.

1. Write the first order conditions of the consumer as an intertemporal and an intratemporal condition.

2. Using your intuition, describe the initial effects of an unexpected, transitory, increase in government spending, on consumption, investment, and employment.

3. How do your answers change as we make the increase in government spending more persistent?

4. How do these predictions fit what we know empirically about the effects of government spending?

5. How would your answers to 2. and 3. be modified if the increase in government spending was financed not by current taxes, but by debt and later increase in taxes.