

1 Overview

Income Fluctuation problem:

- – Quadratic-CEQ
→ Permanent Income
- CARA
→ precautionary savings
- CRRA
→ steady state inequality
- borrowing constraints
- General Equilibrium:
steady state capital and interest rate

2 Certainty Equivalence and the Permanent Income Hypothesis(CEQ-PIH)

2.1 Certainty

- assume $\beta R = 1$
 $T = \infty$ for simplicity
- no uncertainty:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$A_{t+1} = (1+r)(A_t + y_t - c_t)$$

- solution:

$$c_t = \frac{r}{1+r} \left[A_t + y_t + \sum_{j=1}^{\infty} R^{-j} y_{t+j} \right]$$

2.2 Uncertainty: Certainty Equivalence and PIH

- tempting...

$$c_t = \frac{r}{1+r} \left[A_t + y_t + \mathbb{E}_t \sum_{j=1}^{\infty} \left(\frac{1}{1+r} \right)^t y_{t+j} \right]$$

- Permanent Income Hypothesis (PIH)
- Certainty Equivalence:

$$x \rightarrow \mathbb{E}(x)$$

- valid iff:

– preferences: $u(c)$ quadratic and $c \in R$

- main insight:

given “permanent” income

$$y_t^p \equiv y_t + \mathbb{E}_t \sum_{j=1}^{\infty} \left(\frac{1}{1+r} \right)^t y_{t+j}$$

- c_t function of y_t^p and not independently of y_t
- innovations

$$\Delta c_t \equiv c_t - c_{t-1} = \frac{r}{1+r} \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j [\mathbb{E}_t y_{t+j} - \mathbb{E}_{t-1} y_{t+j}]$$

→ revisions in permanent income

- implications:

– random-walk:

$$\mathbb{E}_{t-1} [\Delta c_t] = 0$$

– no insurance...

...consumption smoothing → minimize Δc

- marginal propensity to consume from wealth:

$$\frac{r}{1+r}$$

- marginal propensity to consume from innovation to current income depends on persistence of income process

- example: $\{y_t\}$ is $MA(2)$

$$y_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} = \beta(L) \varepsilon_t$$

$$\begin{aligned} \Delta c_t &= \frac{r}{1+r} \sum_{j=1}^{\infty} R^{-j} [\mathbb{E}_t y_{t+j} - \mathbb{E}_{t-1} y_{t+j}] \\ &= \frac{r}{1+r} \{y_t - \mathbb{E}_{t-1} y_t + R^{-1} (\mathbb{E}_t y_{t+1} - \mathbb{E}_{t-1} y_{t+1})\} \\ &= \frac{r}{1+r} \varepsilon_t + \frac{r}{1+r} R^{-1} \beta_1 \varepsilon_t \\ &= \frac{r}{1+r} [1 + R^{-1} \beta_1] \varepsilon_t \end{aligned}$$

where $y_t = \varepsilon_t + \beta_1 \varepsilon_{t-1}$, $\mathbb{E}_{t-1} y_t = \beta_1 \varepsilon_{t-1}$ and $\mathbb{E}_{t-1} y_{t+j} = 0$ for $j \geq 1$ and $\mathbb{E}_t y_{t+1} = \beta \varepsilon_t$

- ARMA

$$\begin{aligned} \alpha(L) y_t &= \beta(L) \varepsilon_t \\ \rightarrow \Delta c_t &= \frac{r}{1+r} \frac{\beta(R^{-1})}{\alpha(R^{-1})} \varepsilon_t \end{aligned}$$

- persistence $\rightarrow \frac{\partial}{\partial \varepsilon_t} c_t > \frac{r}{1+r}$
- with a unit root in y_t
 \rightarrow mg propensity to consume may be greater than 1

3 Estimation and Tests

3.1 CEQ-PIH

- “random walk” (martingale):

$$\begin{aligned} \Delta c_t &= u_t \\ \mathbb{E}_{t-1} u_t &= 0 \end{aligned}$$

- u_t perfectly correlated with news arriving at t about the expected present value of future income:

$$\Delta c_t = u_t = \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} [\mathbb{E}_t y_{t+j} - \mathbb{E}_{t-1} y_{t+j}]$$

- Two main tests: (generally on aggregate data)
 - random walk \rightarrow unpredictability of consumption
violations = ‘excess sensitivity’ to predictable current income
 - propensity to consume
too small given income persistence = “excess smoothness”
- both tests rely on persistence of income \rightarrow controversial
- aggregation issues:
 - across goods
 - agents: Euler equation typically non-linear
Attanasio and Weber \rightarrow leads to rejection on aggregate data
 - time aggregation:
data averaged over continuous time
 \rightarrow introduces serial correlation

3.2 Euler Equations

- Hall: revolutionary idea:
forget consumption function
find property it satisfies
 \rightarrow Euler equation!

$$u'(c_t) = \beta(1+r) \mathbb{E}_t [u'(c_{t+1})]$$

- Attanasio et al

4 Precautionary Savings

- idea: break CEQ

4.1 Two Periods

- two period savings problem:

$$\max u(c_0) + \beta \mathbb{E}U(\tilde{c}_1)$$

$$a_1 + c_0 = Ra_0 + y_0 = x$$

$$\tilde{c}_1 = Ra_1 + \tilde{y}_1$$

- substituting:

$$\max_{a_1} \{u(x_0 - a_1) + \beta \mathbb{E}U(Ra_1 + \tilde{y}_1)\}$$

f.o.c. (Euler equation)

$$u'(x_0 - a_1) = \beta R \mathbb{E}U'(Ra_1 + \tilde{y}_1)$$

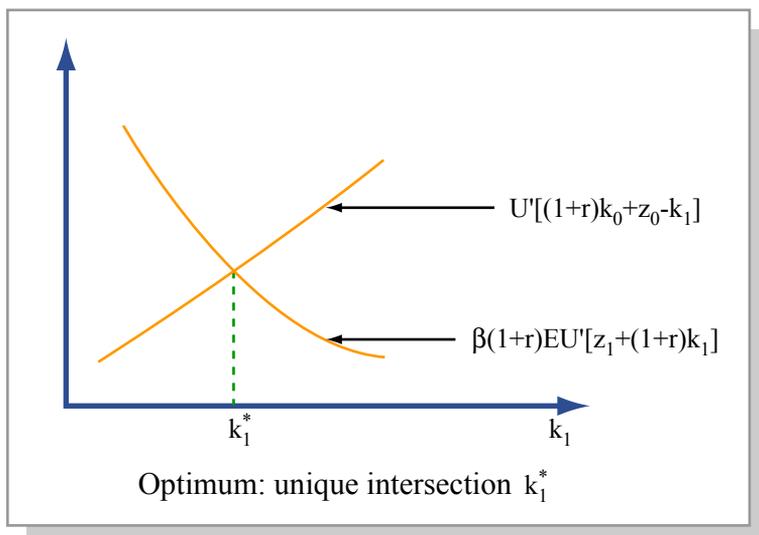
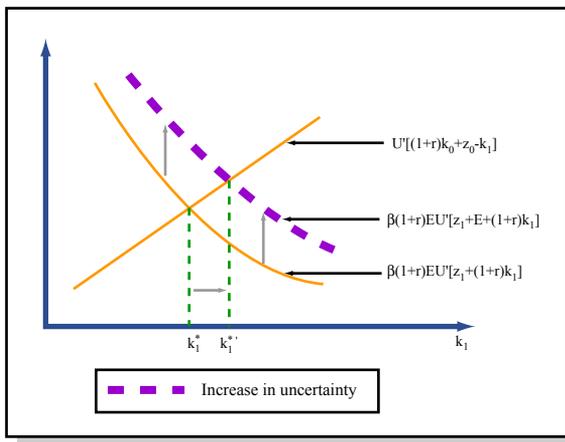


Figure 1: optimum: unique intersection k_1^*

- mean preserving spread: second order stochastic dominance

replace \tilde{y}_1 with $\tilde{y}'_1 = \tilde{y}_1 + \tilde{\varepsilon}$ with $\mathbb{E}(\tilde{\varepsilon} | y) = 0$



Comparative Static with $u''' > 0$: Mean Preserving Spread

- three possibilities:
 - $U'(\cdot)$ linear $\Rightarrow a_1^*$ constant
 - $U'(\cdot)$ convex: RHS rises $\Rightarrow a_1^*$ increases
 - $U'(\cdot)$ concave: RHS falls $\Rightarrow a_1^*$ decreases
- introspection: a_1^* increases $\Rightarrow U'(\cdot)$ is convex $U''' > 0$
- CRRA: $U'(c) = c^{-\sigma}$ for $\sigma > 0$ is convex
- somewhat unavoidable:
 - $U'(c) > 0$ and $c \geq 0$
 - $\Rightarrow U'(c)$ strictly convex near 0 and ∞

4.2 Longer Horizon

- i.i.d. income shocks

$$T = \infty$$

- Bellman equation

$$V(x) = \max \{u(x - a') + \beta \mathbb{E}V(Ra' + \tilde{y})\}$$

- FOC from Bellman

$$u'(c) = \beta R \mathbb{E}V'(Ra + \tilde{y})$$

- again: V' convex \rightarrow precautionary savings
- but V''' endogeneous!
- result: $u''' > 0$ then $v''' > 0$ (Sibley, 1975)

4.3 CARA

- CARA preferences

$$u(c) = -\exp(-\gamma c)$$

$$V(x) = \max_a \{u(x - a') + \beta \mathbb{E}V(Ra' + \tilde{y})\}$$

- no borrowing constraints (except No-Ponzi)
no non-negativity for consumption
- guess and verify:

$$V(x) = Au(\lambda x)$$

where $\lambda \equiv \frac{r}{1+r}$

- note with CARA

$$u(a + b) = -u(a)u(b)$$

- verifying

$$V(x) = \max \{u(x - a') + \beta A \mathbb{E}u(\lambda(Ra' + \tilde{y}))\}$$

$$V(x) = -u\left(\frac{r}{1+r}x\right) \max \left\{ u\left(-\left(a' - \frac{1}{R}x\right)\right) + \beta A \mathbb{E}u\left(r\left(a' - \frac{1}{R}x\right) + \frac{r}{R}\tilde{y}\right) \right\}$$

$$V(x) = -u\left(\frac{r}{R}x\right) \max \left\{ u(-\alpha') + \beta A \mathbb{E}u\left(r\alpha' + \frac{r}{R}\tilde{y}\right) \right\}$$

where

$$\alpha' = a' - x/R \quad \text{or equivalently} \quad c = \frac{r}{1+r}x - \alpha'$$

confirms guess. Solving for A :

$$A = \max \left\{ u(-\alpha') + \beta A \mathbb{E}u \left(r\alpha' + \frac{r}{R}\tilde{y} \right) \right\}$$

$$u'(-\alpha') = r\beta A \mathbb{E}u' \left(r\alpha' + \frac{r}{R}\tilde{y} \right)$$

$$u(-\alpha') = r\beta A \mathbb{E}u \left(r\alpha' + \frac{r}{R}\tilde{y} \right)$$

where we used $u'(c) = -\gamma u(c)$

$$\begin{aligned} A &= u(-\alpha') + \beta A \mathbb{E}u \left(r\alpha' + \frac{r}{R}\tilde{y} \right) \\ &= u(-\alpha') + \frac{u(-\alpha')}{r} = -\frac{1+r}{r}u(-\alpha') \end{aligned}$$

(note $A > 0$)

coming back...

$$\begin{aligned} u(-\alpha') &= r\beta \frac{1+r}{r} (-u(-\alpha')) \mathbb{E}u \left(r\alpha' + \frac{r}{R}\tilde{y} \right) \\ u(-r\alpha') &= \beta(1+r) \mathbb{E}u \left(\frac{r}{R}\tilde{y} \right) \\ -\alpha' &= \frac{1}{r}u^{-1} \left(\beta(1+r) \mathbb{E}u \left(\frac{r}{R}\tilde{y} \right) \right) \\ &= \frac{1}{r}u^{-1}(\beta(1+r)) + \frac{1}{r}\mathbb{E}u \left(\frac{r}{R}\tilde{y} \right) \end{aligned}$$

- verifying $c(x) = \lambda x + \alpha$ using Euler...

$$\begin{aligned} u'(c_t) &= \beta R \mathbb{E}_t u'(c_{t+1}) \\ 1 &= \beta R \mathbb{E}_t u'(c_{t+1} - c_t) \\ 1 &= \beta R \mathbb{E}_t u'(c(x_{t+1}) - c(x_t)) \\ 1 &= \beta R \mathbb{E}_t u'(\lambda(x_{t+1} - x_t)) \end{aligned}$$

since $x_{t+1} = Ra_{t+1} + y_{t+1}$ and $a_{t+1} = \alpha' + x_t/R$

$$x_{t+1} - x_t = R(\alpha' + x_t/R) + y_{t+1} - x_t = R\alpha' + y_{t+1}$$

$$1 = \beta R \mathbb{E}_t u' \left(r\alpha' + \frac{r}{R} y_{t+1} \right)$$

same as before

- Verifying value function (again)

note that $u'(c) = -\gamma u(c)$

$$u'(c_t) = \beta R \mathbb{E}_t u'(c_{t+1}) \iff u(c_t) = \beta R \mathbb{E}_t u(c_{t+1})$$

$$\mathbb{E}_t u(c_{t+1}) = (\beta R)^{-t} u(c_t)$$

Then welfare \iff current consumption:

$$V_t \equiv \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t u(c_{t+s}) = \sum_{s=0}^{\infty} \beta^s (\beta R)^{-s} u(c_t) = \frac{1+r}{r} u(c_t)$$

Verifying

$$c_t = \lambda x_t \iff V(x) = \frac{1+r}{r} u(\lambda x - \alpha')$$

- consumption function

$$c(x) = \lambda \left[x + \frac{1}{r} y^* \right] - \frac{1}{r} \frac{\log(\beta(1+r))}{\gamma}$$

$$y^* \equiv \frac{1}{\lambda} u^{-1} [\mathbb{E} u(\lambda y)]$$

- suppose $\beta R = 1$
no CEQ...

...but simple deviation: constant y^*

- CARA, the good:

- tractable
- useful benchmark \rightarrow helps understand other cases
- good for aggregation (linearity)

- CARA, the bad:

- negative consumption
- unbounded inequality

5 Income Fluctuation Problem

- iid income y_t
- $c_t \geq 0$
- borrowing constraints

5.1 Borrowing Constraints: Natural and Ad Hoc

- natural borrowing constraint
maximize borrowing given $c_t \geq 0$
 $c_t \geq 0$ + No-Ponzi

$$\Rightarrow a_t \geq -\frac{y_{\min}}{r}$$

- ad hoc borrowing constraint:

$$a_t \geq -\phi$$

$$\phi = \min\{y_{\min}/r, b\}$$

- Bellman

$$V(x) = \max_{a' \geq -\phi} \{u(x - a') + \beta \mathbb{E}V(Ra' + \tilde{y})\}$$

- change of variables

$$\hat{a}_t = a_t + \phi \text{ and } \hat{a}_{t+1} \geq 0$$

$$z_t = R\hat{a}_t + y_t - r\phi$$

$$z_t = \hat{a}_t + c_t$$

- transformed problem

$$v(z) = \max_{\hat{a}' \geq 0} \{u(z - \hat{a}') + \beta \mathbb{E}v(R\hat{a}' + \tilde{y} - r\phi)\}$$

(dropping $\hat{\cdot}$ notation)

$$v(z) = \max_{a' \geq 0} \{u(z - a') + \beta \mathbb{E}v(Ra' + \tilde{y} - r\phi)\}$$

5.2 Properties of Solution

$\beta R = 1$

- CARA: $\mathbb{E}[a_{t+1}] > a_t$ and $\mathbb{E}[c_{t+1}] > c_t$

- Martingale Convergence Theorem:

If $x_t \geq 0$ and

$$x_t \geq \mathbb{E}[x_{t+1}]$$

then $x_t \rightarrow \tilde{x}$ (note: $\tilde{x} < \infty$ a.e.)

- Euler

$$u'(c_t) = \beta R \mathbb{E}[u'(c_{t+1})]$$

$\Rightarrow u'(c_t)$ converges

$\Rightarrow c_t \rightarrow c$

- if $c < \infty$ contradiction with budget constraint equality
- $a_t \rightarrow \infty$ and $c_t \rightarrow \infty$

$\beta R < 1$

- Bellman equation

$$v(z) = \max_{a'} \{u(x - a') + \beta \mathbb{E}v(Ra' + \tilde{y} - r\phi)\}$$

- v is increasing, concave and differentiable

- Preview of Properties

– monotonicity of $c(z)$ and $a'(z)$

– borrowing constraint is binding iff $z \leq z^*$

– if

$$\lim_{c \rightarrow 0} \frac{u''(c)}{u'(c)} = 0$$

then assets bounded

- if $u \in HARA$ class $\Rightarrow c(z)$ is concave (Carrol and Kimball)

Figures removed due to copyright restrictions.

See figures 1a and 1b on p. 667 in Aiyagari, S. Rao.
 "Uninsured Idiosyncratic Risk and Aggregate Savings."
Quarterly Journal of Economics 109, no. 3 (1994): 659-684.

- borrowing constraints
 - certainty: $[0, z^*]$ large
approached monotonically
 - uncertainty: $[0, z^*]$ relatively small
not approached monotonically
- concavity of v
 - \Rightarrow concavity of $\Phi(a') = \beta \mathbb{E}v(Ra' + \tilde{y})$
 - \Rightarrow standard consumption problem with two normal goods

$$v(z) = \max_{c, a'} \{u(c) + \beta v(Ra' + \tilde{y})\}$$

$$c + a' \leq x$$

$$a' \geq 0$$

$\Rightarrow c(z)$ and $a'(z)$ are increasing in z

- FOC (Euler)

$$u'(x - a') \geq \beta R \mathbb{E} v'(Ra' + \tilde{y})$$

equality if $a' > 0$

- define

$$u'(z^*) = \beta R \mathbb{E} v'(\tilde{y})$$

$$\begin{aligned} z \leq z^* &\Rightarrow c = z \\ &\Rightarrow a' = 0 \end{aligned}$$

Assets bounded above

- not a technicality...

...remember CARA case

- idea: take $a \rightarrow \infty$

income uncertainty unrelated to a (i.e. absolute risk)

$\frac{-u''}{u'} \rightarrow 0 \Rightarrow$ income uncertainty unimportant

βR bites $\Rightarrow a' < a$ falls

Proof

exist a z^* such that $z'_{\max} = (1 + r)a'(z) + y_{\max} \leq z$ for $z \geq z^*$

Euler

$$u'(c(z)) = \beta(1 + r) \frac{Eu'(c(z'))}{u'(\bar{c}(z))} u'(\bar{c}(z))$$

where $\bar{c}(z) = c(z'_{\max}(z)) = c(a'(z) + y_{\max} - r\phi)$

$$\text{IF } \lim_{z \rightarrow \infty} \frac{E[u'(c(z'))]}{u'(\bar{c}(z))} = 1 \Rightarrow \text{DONE}$$

$$1 \geq \frac{Eu'(c(z'))}{u'(\bar{c}(z))} \geq \frac{u'(\underline{c}(z))}{u'(\bar{c}(z))} \geq \frac{u'(\bar{c}(z) - (\bar{c}(z) - \underline{c}(z)))}{u'(\bar{c}(z))}$$

since a' is increasing

$$\bar{c}(z) - \underline{c}(z) = c(Ra'(z) + y_{\max} - r\phi) - c(Ra'(z) + y_{\min} - r\phi) < y_{\max} - y_{\min}$$

$$1 \geq \frac{Eu'(c(z'))}{u'(\bar{c}(z))} \geq \frac{u'(\bar{c}(z) - (y_{\max} - y_{\min}))}{u'(\bar{c}(z))}$$

Since $z \rightarrow \infty \Rightarrow a'(z), c(z) \rightarrow \infty$ then $\bar{c}(z) = c(a'(z) + y_{\max} - r\phi) \rightarrow \infty$. Apply Lemma below. ■

Lemma. for $A > 0$

$$\frac{u'(c-A)}{u'(c)} \rightarrow 1$$

Proof. $1 \leq$

$$\begin{aligned} \frac{u'(c-A)}{u'(c)} &= 1 + \int_0^A \frac{u''(c-s)}{u'(c)} ds \\ &= 1 - \int_0^A \frac{u'(c-s) - u''(c-s)}{u'(c) u'(c-s)} ds \\ &= 1 - \int_0^A \frac{u'(c-s)}{u'(c)} \gamma(c-s) ds \\ &\leq 1 - \int_0^A \gamma(c-s) ds \end{aligned}$$

since $\frac{u'(c-s)}{u'(c)} > 1$ for all $t > 0$

$$\int_0^A \gamma(c-s) ds \rightarrow 0$$

so $\frac{u'(c-A)}{u'(c)} \rightarrow 1$. ■

6 Lessons from Simulations

From Deaton's "Saving and Liquidity Constraints" (1991) paper:

- important
borrowing constraint may bind infrequently
(wealth endogenous)
- marginal propensity to consume
higher than in PIH

Figure removed due to copyright restrictions.

See Figure 1 on p. 1228 in Deaton, Angus. "Saving and Liquidity Constraints."
Econometrica 59, no. 5 (1991): 1221-1248.

Figure removed due to copyright restrictions.

See Figure 2 on p. 1230 in Deaton, Angus.

“Saving and Liquidity Constraints.” *Econometrica* 59, no. 5 (1991): 1221-1248.

Figure removed due to copyright restrictions.

See Figure 4 on p. 1234 in Deaton, Angus.

“Saving and Liquidity Constraints.” *Econometrica* 59, no. 5 (1991): 1221-1248.

- consumption
 - smoother temporary shocks
 - harder with permanent shocks

7 Invariant Distributions

- initial distribution $F_0(z_0)$
- laws of motion

$$z' = Ra'(z) + y'$$

generate

$$\begin{aligned} F_0(z_0) &\rightarrow F_1(z_1) \\ F_1(z_1) &\rightarrow F_2(z_2) \\ &\vdots \end{aligned}$$

- steady state: invariant distribution

$$F(z) \rightarrow F(z)$$

- result:

1. exists
2. unique
3. stable

- key: bound on assets and monotonicity

- $A(r) \equiv E(a'(z))$

- continuous
- not necessarily monotonically increasing in r
income vs. substitution; and $w(r)$ effect
typically: monotonically increasing
- $A(r) \rightarrow \infty$ as $R \rightarrow \beta^{-1}$

8 General Equilibrium

- GE effects of precautionary savings?
→ more k , lower r
- how much?

8.1 Huggett: Endowment

- endowment economy
- no government
- zero net supply of assets
- idea: any precautionary saving translates to lower equilibrium interest rate
- computational GE exercise:
 - CRRA preferences
 - borrowing constraints

8.2 Aiygari

- adds capital
- $y_t = wl_t$ and l_t is random; w is economy-wide wage
- N is given by $N = \sum l^i p^i$
- define steady state equilibrium:
3 equations / 3 unknowns: (K, r, w)

$$\int A(z, r, w) dF(z; r, w) - \phi = K$$

$$r = F_k(K, N) - \delta$$

$$w = F_N(K, N)$$

- solve $w(r)$ and substitute:

$$A^{GE}(r) = \int a(z, r, w(r)) d\mu(z; r, w(r)) = K$$

intersect with

$$r = F_k(K, N) - \delta$$

- $A^{GE}(r)$
 - continuous
 - not necessarily monotonically increasing in r
 - (a) income vs. substitution; (b) $w(r)$ effect
 - typically: monotonically increasing
 - $A(r) \rightarrow \infty$ as $R \rightarrow \beta^{-1}$

Figures removed due to copyright restrictions.

See Figures IIa and IIb on p. 668 in Aiyagari, S. Rao.
 "Uninsured Idiosyncratic Risk and Aggregate Savings."
Quarterly Journal of Economics 109, no. 3 (1994): 659-684.

- comparative statics
 - $\frac{\partial}{\partial b} A(0, b) > 0$
 typically: $\frac{\partial}{\partial b} A(r, b) > 0$

$$- \uparrow \sigma_y^2 \Rightarrow \uparrow A$$

Table removed due to copyright restrictions.

See Table II on p. 678 in Aiyagari, S. Rao.

"Uninsured Idiosyncratic Risk and Aggregate Savings."

Quarterly Journal of Economics 109, no. 3 (1994): 659-684.

- wealth distribution: not as skewed
- transition? monotonic?

9 Inequality

- CEQ-PIH and CARA
inequality increases linearly
unbound inequality
- CRRA
inequality increases initially
bounded inequality

Figure removed due to copyright restrictions.

See Figure 2 on p. 444 in Deaton, Angus, and Christina Paxson. "Intertemporal Choice and Inequality." *Journal of Political Economy* 102, no. 3 (1994): 437-467.

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See Figure 4 on p. 445 in Deaton, Angus, and Christina Paxson.
"Intertemporal Choice and Inequality."
Journal of Political Economy 102, no. 3 (1994): 437-467.

Figure removed due to copyright restrictions.

See Figure 6 on p. 450 in Deaton, Angus, and Christina Paxson.
"Intertemporal Choice and Inequality."
Journal of Political Economy 102, no. 3 (1994): 437-467.

Figure removed due to copyright restrictions.

See Figure 1d) on p. 769 in Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante. "Two Views of Inequality Over the Life-Cycle." *Journal of the European Economic Association* 3, nos. 2-3 (2005): 765-775.

Deaton and Paxson

Revisionist (Heathcote, Storesletten, Violante)

Guvenen

Storesletten, Telmer and Yaron:

10 Life Cycle: Consumption tracks Income

Carroll and Summers:

11 Other Features and Extensions

- Social Security:

Hubbard-Skinner-Zeldes (1995): "Precautionary Savings and Social Security"

Scholz, Seshadri, and Khitatrakun (2006): "Are Americans Saving "Optimally" for Retirement?"

- Medical Shocks: Palumbo (1999)

Figure removed due to copyright restrictions.

See Figure 1 in Guvenen, Fatih.
"Learning Your Earning: Are Labor Income Shocks Really Very Persistent?"
American Economic Review. (Forthcoming)
http://www.econ.umn.edu/~econdept/learning_your_earning.pdf

Figure 9

Figure removed due to copyright restrictions.

See Figure 1 on p. 613 in Storesletten, Kjetil, Chris Telmer, and Amir Yaron.
"Consumption and Risk Sharing over the Life Cycle."
Journal of Monetary Economics 51, no. 3 (2004): 609-663.

Figure 10

Figure removed due to copyright restrictions.

See Figure 5 on p. 624 in Storesletten, Kjetil, Chris Telmer, and Amir Yaron.
"Consumption and Risk Sharing over the Life Cycle."
Journal of Monetary Economics 51, no. 3 (2004): 609-663.

Figure 11

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Figure 12

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Figure 13

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Figure 14

Figure removed due to copyright restrictions.

Figure 15

- Learning Income Growth: Guvenen (2006)
- Hyperbolic preferences: Harris-Laibson
- Leisure Complementarity
Aguiar-Hurst (2006): “Consumption vs. Expenditure”
- Attanasio-Weber: Demographics and Taste Shocks