1 One Sided Lack of Commitment

Planner

$$P(w) = \max - \sum_s \pi_s \left[c_s - y_s + \beta P(w'_s)\right]$$

$$u(c(s)) + \beta w_s \geq u(y_s) + \beta U_{aut}$$

$$\sum_s (u(c(s)) + \beta w_s)\, \pi_s \geq w$$

$$w \geq U_{aut}$$

FOC

$$\begin{aligned} (\mu_s + \lambda \pi_s)\, u'(c_s) &= \pi_s \\ \mu_s + \lambda \pi_s &= \pi_s P'(w_s) \end{aligned}$$

$$\Rightarrow u'(c_s) = \frac{1}{P'(w_s)}$$

- P increasing and convex

  $\Rightarrow c$ is increasing in w

- constraint not binding $\mu_s = 0$

  $\Rightarrow w_s = w$

- otherwise $w_s > w$

- dynamics: moving up

  long-run: participation constraint not binding (see Debraj Ray, Econometrica)

2 Two Sided / GE

sources:

- LS Chapter 15: good treatment but no long-run distribution

- Alvarez-Jermann (2000)

  persistence of income

  2 shocks

  dynamics

2.1 Dynamics

- environment:
  - symmetric

    two agents $i = 1, 2$
  - $y^1 > y^2$

    $y^1 + y^2 \equiv e$
  - $s = 1, 2$

    income for agent 1
  - $p = \Pr(s' = 2 \mid s = 1)$
- problem (recursive version)

$$T[V](w, s) = \max_{c^1, c^2, w'(\cdot)} [u(c^1) + \beta \sum_{s'} \pi(s' \mid s) V(w'(s'), s')]$$

$$\begin{aligned} c^1 + c^2 &= e(s) \\ u(c^2) + \beta \sum \pi(s'|s) w'(s') &\geq w \\ w'(s') &\geq U^2_{aut}(s') \\ V(w'(s'), s') &\geq U^1_{aut}(s') \end{aligned}$$

- take as given:

  $V(\cdot, s)$ is
  - decreasing
  - differentiable
  - concave
- last two constraints:

$$w'(s') \in [L(s'), H(s')]$$

  for some $L(s')$ and $H(s')$
- Pareto Frontier: first best
- second best

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Figure 1

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Figure 2

- FOCs:

$$\begin{aligned} u'\left(c^1\right) &= \lambda \\ \theta u'\left(c^2\right) &= \lambda \\ V_1\left(w'\left(s'\right), s'\right) &\lesseqgtr -\theta \end{aligned}$$

  with $=$ if $w'(s') \in (L(s'), H(s'))$
  with $\leq$ if $w'(s') = L(s')$
  and $\geq$ if $w'(s') = H(s')$

- Envelope

$$V_1(w, s) = -\theta$$

- **result 1:** $c^2(w, s)$ is increasing in w

  V is concave $\Rightarrow -V_1$ is increasing in w:

$$\frac{u'\left(e - c^2\right)}{u'\left(c^2\right)} = \theta = -V_1(w, s)$$

  $\Rightarrow c^2$ to increase with w

- **result 2:** if $s = s'$ then $w(s') = w$

  FOC

$$V_1\left(w'\left(s'\right), s'\right) \lesseqgtr -\theta = V_1(w, s)$$

  satisfied with $=$ at $(w'(s'), s') = (w, s)$ which is feasible since $w \in [L(s), H(s)]$

- **result 3:** 2 shocks if $s \neq s'$

$$V_1\left(w'\left(s'\right), s'\right) \lesseqgtr V_1(w, s)$$

- collecting results
  - $c^2(w, s)$ is increasing in w
  - $s' = s \to w'(s') = w$ (constraint not binding)
  - $s \neq s' \to$ binding $w'(s')$ closest value in $[L(s'), H(s')]$
  - figure
  - convergence (main result):

    stationary distribution is history independent and symmetric
  - GB attainable: converge to FB

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Figure 3

3 Private Information

Private information on:

- tastes, productivity or income
- insurance is smoother than with lack of commitment

  no bounds to hit or be slack

Some comments

- incentives $\rightarrow$ no perfect insurance

  static intuition

- dyanamic

  $\rightarrow$ use present and future consumption for incentives

  "intertemporal tie-ins" and "long-term contracting"

- infinite spreading of distribution

  $\rightarrow$ no invariant distribution (Atkeson-Lucas)

  $\rightarrow$ immiseration

Nice result

- Allen (1985) Cole-Kocherlakota (2000):

  model: private info on income + private savings (and borrowing)

  $\implies$ optimum is autarky

- microfound income fluctuations?