

# 14.454 - Macroeconomics IV

## Problem Set I

04/02/2011

**Due: Monday 4/11/2011**

### 1 Question 1 - Kocherlakota (2000)

Take an economy with a representative, infinitely-lived consumer. The consumer owns a technology with which she produces output ( $Y$ ) using capital ( $K$ ) and land ( $L$ ) according to a production function:

$$Y = F(K, L)$$

where  $F(\cdot)$  is increasing, concave and differentiable. Capital fully depreciates after its use ( $\delta = 1$ )

The consumer is endowed with  $K_0$  units of capital and  $L_0 = 1$  units of land at  $t = 0$ , and has access to an internal land market; i.e. she can buy and sell land in the local market at a price  $Q_t$ . The consumer has also access to an international financial market: the consumer can borrow  $B_t$  units of consumption goods from international markets at time  $t$ , at an interest rate  $r > 0$ . However, they are constrained on how much they can borrow:

$$B_t \leq B^* \text{ for all } t \quad (1)$$

where  $B^*$  is an exogenously given borrowing constraint. The consumer is also born with a debt of  $B_0 < B^*$

The consumer has preferences over consumption streams according to the utility function

$$U = \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t \ln(C_t) \quad (2)$$

where we assume that  $\rho = r$ . The consumer chooses sequences of consumption  $\{C_t\}_{t=0}^{\infty}$ , capital  $\{K_{t+1}\}_{t=0}^{\infty}$  and land purchases  $\{L_t\}_{t=0}^{\infty}$  to maximize (2) given prices  $\{Q_t\}_{t=0}^{\infty}$  subject to the budget constraint:

$$C_t + K_{t+1} + Q_t L_{t+1} + B_t(1+r) \leq F(K_t, L_t) + B_{t+1} + Q_t L_t \quad (3)$$

and the borrowing constraint (1). So, the consumer must finance consumption  $C_t$ , investment  $K_{t+1}$ , interest payments on debt  $B_t(1+r)$  and land purchases  $Q_t L_{t+1}$  using output, borrowing more funds ( $B_{t+1}$ ) and selling the land they own at the price  $Q_t$ . To close the economy, assume that the total supply of land remains constant over time and equal to one; i.e.  $L_t = 1$  for all  $t = 1, 2, \dots$

(a) Define a competitive equilibrium for this economy

(b) A steady state equilibrium is an equilibrium in which  $C_t = C_{ss}$ ,  $B_t = B_{ss}$ ,  $Y_t = Y_{ss}$ ,  $Q_t = Q_{ss}$  and  $K_t = K_{ss}$ . Using the definition of a competitive equilibrium and the FOC's of the consumer's optimization problem, characterize such an equilibrium, given a borrowing level  $B_{ss} < B^*$ . Do  $K_{ss}$ ,  $Y_{ss}$  and  $Q_{ss}$  depend on the level of  $B_{ss}$ ?

(c) Assume that  $K_0 = K_{ss}$  and  $B_0 = B_{ss}$  (so the economy is at its steady state from period  $t = 0$ ). Suppose that the farmer has an unexpected negative shock on debt; i.e.  $B'_0 = B_0 - \Delta$  and  $\Delta = 0$  for all  $t \geq 1$ . Does this shock have any effect on consumption, output and land prices? What if  $B'_0 = B_0 + \Delta$  with  $\Delta \in (0, B^* - B_0)$ ? Would it change your results if there was no borrowing constraints?

(d) Suppose that the shock is positive, and that  $\Delta > B^* - B_0$  (so that the initial borrowing level exceeds the borrowing limit  $B^*$ ). What happens with equilibrium output  $Y_t$ ? Compare it with the situation in which there are no borrowing constraints. Explain.

(e) Suppose now that instead of the borrowing constraint (1), consumers must now collateralize their loans with the value of their land holdings. Namely

$$B_{t+1} \leq Q_t L_{t+1}$$

Moreover, assume that  $F(K, L) = K^{\alpha_1} L^{\alpha_2}$  where  $\alpha_1, \alpha_2 \geq 0$ . Repeat (a) and (b) on this new setting.

(f) Suppose  $K_0 = K_{ss}$  and  $B_0 = Q_{ss} + \Delta$ , with  $\Delta > 0$ . Does your conclusions from (d) change?

**Note:** In Kocherlakota (2000)<sup>1</sup>, the author also explores how these two different models create different amplification mechanisms of the credit constraint channel. Which one do you expect to be the one that generates more amplification? Compare to Kiyotaki and Moore (1997)

## 2 Question 2 (Lorenzoni 2010)

This problem analyzes the welfare effects of a “capital injection” in a model with financial frictions. There are two periods, 0 and 1. Consumers and entrepreneurs have a linear utility function,  $c_0 + c_1$ . Consumers have a large endowment of consumption goods in each period and a unit endowment of labor in period 1, which they sell inelastically on a competitive labor market at the wage  $w_1$ .

Entrepreneurs have a given endowment of consumption goods  $E_0$  and no capital. Then they borrow  $b_1$  and invest  $k_1$ . In period 1 they hire workers at the wage  $w_1$  and produce consumption goods according to the Cobb-Douglas production function:

$$y_1 = k_1^\alpha l_1^{1-\alpha} \quad (4)$$

The entrepreneurs face the collateral constraint

$$b_1 \leq \lambda(y_1 - w_1 l_1)$$

where  $\lambda \in (0, 1)$  is a given scalar (think that in period 1 the entrepreneurs can run away with a fraction  $(1 - \lambda)$  of the firm’s profits). Assume the consumers endowment is large enough that the gross interest rate is always 1 in equilibrium.

The entrepreneur’s problem is then

$$s.t. : \begin{cases} \max_{(c_0, c_1) \geq 0, k_1, l_1, b_1} c_0 + c_1 \\ c_0 + k_1 \leq E_0 + b_1 : (t = 0 \text{ budget constraint}) \\ c_1 + b_1 \leq y_1 - w_1 l_1 : (t = 1 \text{ budget constraint}) \\ b_1 \leq \lambda(y_1 - w_1 l_1) : (\text{collateral constraint}) \end{cases}$$

(a) Argue that the entrepreneurs will always choose  $l_1$  to maximize profits in period 1 and that then profits are a linear function of the capital stock  $k_1$ , that is:

$$y_1 - w_1 l_1 = R(w_1) k_1$$

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<sup>1</sup>Kocherlakota, Narayana, "Creating Business Cycles Through Credit Constraints", *Federal Reserve Bank of Minneapolis Quarterly Review*, Vol. 24, No. 3, Summer 2000, pp. 2–10

where  $R(w_1)$  is some (potentially non-linear) function of  $w_1$ . Restate the entrepreneur's problem as a simpler linear problem.

(b) Argue that if

$$\lambda R(w_1) < 1 \leq R(w_1)$$

then the entrepreneur's problem is well defined and the entrepreneur's demand for capital is finite. Derive it.

What happens if  $\lambda R(w_1) \geq 1$ ? What if  $R(w_1) < 1$ ?

(c) Show that there is a cutoff  $\hat{E}$  such that if  $E_0 > \hat{E}$ , the entrepreneurs can finance the first-best level of capital  $k_1 = k^* = \alpha^{\frac{1}{1-\alpha}}$ , and the collateral constraint is not binding.

(d) Show that if  $E < \hat{E}$  there is an equilibrium where the entrepreneurs are constrained and the equilibrium value of  $k_1$  is an increasing function of  $E_0$ .

(e) Suppose the consumers pay a lump-sum tax  $\tau$  in period 0. The receipts from the tax are transferred directly to the entrepreneurs. Derive an expression for the expected utility of consumers and entrepreneurs as a function of  $\tau$

(f) Show, analytically or by numerical example, that there is a non-monotone relation between  $\tau$  and the expected utility of the consumers. In particular, if  $E_0$  is sufficiently small, a small positive tax can increase the utility of both consumers and entrepreneurs.

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