Problem set 5. 14.461 Fall 2012.

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References:

1. Ljungqvist L., and Thomas J. Sargent (2000), "Recursive Macroeconomic Theory," – *sections 22.13* for Problem 1.
2. Mailath J. George and Larry Samuelson (2006), "Repeated Games and Reputations," – *section 3.6* for Problem 2.

1 Characterization of V.

Consider a government playing repeated game against public. At time t let y_t be government action and x_t – action chosen by the public. Action sets are compact. Government discounts at rate β stage game payoff $U(x, y)$. Agents could condition their actions on the outcome of public correlation device w_t.

Let σ be a pure strategy of the government, i.e. a mapping from history of actions and outcomes of correlation device at time t, h^t, to action y_t. We analyse pure SPE of the game and suppose that public chooses x_t myopically according to function h, $x_t = h(y_t)$.[1] Define by $H(x)$, myopic best response of govenrment to public action x. Govenment's expected[2] discounted utility is

$$v_0^\sigma = (1-\beta)\mathbb{E}^w \sum_{t=0}^{\infty} \beta^t U(x_t, y_t). \tag{1.1}$$

[1]Given particular environment it's usually easy to put restrictions on primitives, so that best response correspondence is indeed a function. Since this is not the focus of the problem we directly assume that h is a function.

[2]Expectation is with respect to correlation device outcomes.

Follow the steps below to characterize the set $V \equiv \{v : v = v_0^\sigma, \sigma \in \text{SPE}\}$.

1. Reduce the problem to the characterization of extreme points of V, i.e. show that $V = [\underline{v}, \bar{v}]$ for some $\underline{v}, \bar{v}$.

2. Write down the program for finding $\underline{v}$. In particular, show that $\underline{v} = U(h(\underline{y}), H(h(\underline{y}))$ for some $\underline{y}$. Does the solution to the program depend on $\bar{v}$? Did you use convexity of V at any step?

3. Write down the program for finding $\bar{v}$. In particular, show that $\bar{v} = U(h(\bar{y}), \bar{y})$.

4. Design a simple algorithm for computing V that works for sufficiently patient government. How do you adjust your algorithm to accomodate impatient govenrment?

2 Convexity without Public Correlation (*optional*).

Suppose that in previous problem no public correlation device is available. Take any v in convex hull of V and suppose that $v > v'$ where v' is utility from static Nash equilibrium of the game. We show that for sufficiently big β there is SPE σ with $v_0^\sigma = v$.[3]

1. Relate this fact to the solution to the first problem. In particular, we don't prove that V is convex but that it gets convex in the limit of β going to 1. Does the argument in problem 1 go through with this weaker statement?

2. Define $\zeta(h^t)$ by $\zeta^0 = 0$

$$\zeta^t(h^t) = (1-\beta)\sum_{\tau=0}^{t-1}(U(x^t, y^t) - v'). \tag{2.1}$$

   Interpret $\zeta^t(h^t)$. Specify strategy profile such that at each time t static Nash equilibrium profile giving v' is played if either deviation occured in past periods or $\zeta^t(h^t) \geq (1-\delta^{t+1})(v - v')$.

3. Show that for high β for all histories $\zeta^t(h^t) < v - v'$ and for on-equilibrium-path histories $\zeta^t(h^t) \geq (1-\delta^t)(v-v')$. What is the expected utility of such strategy for the government.

[3]The statement is true for all v in convex hull of V and reasoning for $v < v_1'$ is similar but more cumbersome.

4. Notice that given your interpretation of $\zeta^t(h^t)$ in part 2 it's natural to choose threshold for strategy $(1-\delta^t)(v-v')$ instead of $(1-\delta^{t+1})(v-v')$. Why this does not work.

5. Prove that strategy specified in part 2 is an SPE for high β.

3 Problem 22.3 from Ljungqvist, Sargent.

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