Problem set 6. 14.461 Fall 2012.

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References:

1. Angeletos M., C. Hellwig and Alessandro Pavan (2006), "Signaling in a Global Game: Coordination and Policy Trap," – Problem 1.

2. Hassan T., and Thomas Mertens (2011), "Market Sentiments: A Tragedy of the Commons," – Problem 2.

1 Signaling in a Global Game.

Consider the following modification of Global Game discussed in class. At date 0 government observes fundamental θ and could "burn money"[1] in the amount $M > 0$. Starting from date 1 the game proceeds as in the benchmark model: agents observe signals $x_i = \theta + \sigma\xi_i$ where $\xi \sim [-1, 1]$ and choose whether to attack or not. At date 2 government decides whether to abandom status quo ($D = 1$) or not ($D = 0$) to maximize payoff $(1 - D)(\theta - A) - M$ where A is size of attack. Please, answer the following questions.

1. Is (unique) outcome of a benchmark model still an equilibrium in the modification? What beliefs support this outcome?

2. "Rationality of agents implies that burning money is not played when $\theta > 1 + 2\sigma$ or $\theta < -2\sigma$". True or False.

[1] Think of liqudity injections for example.

3. Provide an example of equilibrium in which government burns money? For what values of M your construction is possible. Be sure to specify beliefs of agents off-equilibrium path. Is (interim) utility of govenment monotone in M?

4. Is it possible to have the following outcome as an equilibrium: there are thresholds $\theta_1 < \theta_2 < \theta_3 < \theta_4$ such that government burns money whenever $\theta \in [\theta_1, \theta_2] \cup [\theta_3, \theta_4]$ and does not burn money otherwise. If yes, could you generalize the construction to infinite amount of intervals. Be sure to be precise about details of equilibrium construction or argument of why such equilibrium is not possible.[2]

2 Attention to Sentiments.

Consider the following simple version of Hassan-Mertens.

Agents indexed by $i \in [0,1]$ have endowment 1. Game is played in two stages. In the first stage each agent chooses μ_i, amount of attention to pay to sentiment $\epsilon \sim \mathcal{N}(0,1)$. In the second stage each agent chooses z_i, amount of asset to buy given private signal $s_i = x + \nu_i$ and price p that clears the market, where $x \sim \mathcal{N}(0, \sigma_x^2)$ is fundamental value of asset and $\nu_i \sim \mathcal{N}(0, \sigma_\nu^2)$ is idiosyncratic noise.

Agents are dynamically inconsistent. Let κ be cost of attention. In the second stage unless sufficient amount of self-control is exercised agent succumbs to sentiment which affects his estimate of fundamental. More precisely, in the second stage agent chooses z_i to maximize mean-variance utility

$$U_2(z_i, \mu_i) = (1 + z_i(\mathbb{E}[x|s_i, p] + \mu_i \epsilon - p))(1 - \kappa(1 - \mu_i)) - \frac{1}{2} z_i^2 \mathbb{V}[x|s_i, p](1 - \kappa(1 - \mu_i))^2$$

In the first stage agents are aware of this bias and decide on the amount of attention μ_i to pay to the sentiment to maximize expectation of

$$U_1(\mu_i) = (1 + z_i(\mathbb{E}[x|s_i, p] - p))(1 - \kappa(1 - \mu_i)) - \frac{1}{2} z_i^2 \mathbb{V}[x|s_i, p](1 - \kappa(1 - \mu_i))^2.$$

where z_i is chosen by agent in the second stage.[3] Follow the following steps to BNE of the game.

1. Solve for optimal value of z_i in the second stage.

[2] For constructions you may put any bounds on M and σ, for proof on impossibility of construction you should show that such construction is not possible for any M and σ.

[3] Think of this change of preferences as dual-selves model.

2. Characterize REE in the second stage.

3. Find best response of agent in the first stage and find equilibrium level of attention.

4. Does the game exhibit strategic complementarity or substitutability in choice of attention to sentiment?

5. Is equilibrium unique? Is there an equilibrium with zero attention to sentiments? Give intuition for uniqueness or multiplicity.

6. How do equilibrium level of attention and volatility change with parameters σ_ν^2, σ_x^2 and κ?

3 Investment, complementarities and beliefs.

Suppose we have the information structure:

$$\theta \sim \mathcal{N}(0, 1)$$

$$z = \theta + \nu$$

$$\nu \sim \mathcal{N}(0, \beta)$$

$$x_i = \theta + \epsilon_i$$

$$\epsilon_i \sim \mathcal{N}(0, \alpha)$$

and payoffs:

$$u(a_i, A) = a_i(\theta + A)$$

where $a_i \in \{0, 1\}$ and $A = \int_I a_i di$.

1. Derive the equations that characterize threshold equilibria and find sufficient condition for uniqueness.

2. With progam of your choice plot the equilibrium correspondense $x^*(z)$ for different values of α and β.

3. Compute the probabilities $P[x_j < x_i | x_i, z]$.

4. Show that probabilities computed in part 3 converge to 1/2 for any (x_i, z) when the precision of private information goes to infinity. That is, we generate "typicality" in beliefs.

5. Now assume that the agent's system of beliefs (which is common knowledge) is such that for all x_i

$$\mathbb{E}[\theta | x_i] = x_i$$

and

$$\mathbb{P}[x_j = x_i | x_i] = \lambda$$

$$\mathbb{P}[x_j < x_i | x_i] = \mathbb{P}[x_j > x_i | x_i] = \frac{1}{2}(1 - \lambda).$$

Show that you can construct two threshold equilibria, one with $x^* = -\lambda - \frac{1}{2}(1 - \lambda)$ and the other with $x^* = -\frac{1}{2}(1 - \lambda)$. What happens when $\lambda \to 0$? when $\lambda \to 1$? Interpret your results.

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