Problem set 7. 14.461 Fall 2012.

George-Marios Angeletos.

December 5, 2012

References:

1. Bergemann, Dirk, and Stephen Morris (2012), "Robust Predictions in Games with Incomplete Information," – Problem 1.
2. Mackowiak, Bartosz, and Mirko Wiederhold (2009), Optimal Sticky Prices under Rational Inattention," – Problem 3.

1 Bergemann, Morris.

Following Bergemann, Morris approach of characterizing BNE over all information structures do the following exercises (notations are as in the class). Consider special case $\mu_\theta = 0$ and $\sigma_\theta = 1$.

1. Consider the model

$$\begin{pmatrix} \theta \\ a \\ A \\ B_\theta \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ \mu_a \\ \mu_A \\ \mu_{B_\theta} \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & * & * & * \\ \rho_{\theta a}\sigma_a & \sigma_a^2 & * & * \\ \rho_{\theta A}\sigma_A & \rho_{Aa}\sigma_A\sigma_a & \sigma_A^2 & * \\ \rho_{\theta B_\theta}\sigma_{B_\theta} & \rho_{AB_\theta}\sigma_A\sigma_{B_\theta} & \rho_{aB_\theta}\sigma_a\sigma_{B_\theta} & \sigma_{B_\theta}^2 \end{pmatrix} \right)$$

where $B_\theta = \int_I \mathbb{E}_i[\theta]di$ is average first-order beliefs about fundamental. Best response for agent i is given by

$$a_i = (1-r)\mathbb{E}_i[\theta] + r\mathbb{E}_i[A].$$

What information structure maximizes volatility $\mathbb{V}[A]$? noise generated volatility $\mathbb{V}[A|\theta]$? non-fundamental fraction of volatility $\mathbb{V}[A|\theta]/\mathbb{V}[A]$? volatility generated by higher-order beliefs $\mathbb{V}[A|\theta, B_\theta]$ for a fixed precision of information about fundamental, $\rho_{\theta B_\theta}$? Interpret.

Does this modification of the model allow to capture effect of higher-order beliefs independent of first-order beliefs?

2. Consider the following information structure. Each agent gets private signal $x_i = \theta + u + \epsilon_i$ and public signal $y = \theta + \zeta$ where ϵ_i, u and ζ are independently normally distributed with zero means and precisions τ_ϵ, τ_u and τ_ζ, respectively. Find $\mathbb{V}[A|\theta, B_\theta]$ in this model. How is your answer related to your finding in question 1. (for this question you may assume that $\theta \sim \mathcal{N}(0, \infty)$.)

3. Consider two period version of continuous population model. Suppose that $\theta_t = \theta + \epsilon_t, t = 1, 2$ where $\epsilon_t \sim \mathcal{N}(0, 1)$ are independent across periods. Consider the following version of correlated equilibrium. Best-response in first period is

$$a_1 = (1 - r)\mathbb{E}[\theta_1|a_1] + r\mathbb{E}[A_1|a_1],$$

best-response in second period is

$$a_2 = (1 - r)\mathbb{E}[\theta_1|a_1, a_2] + r\mathbb{E}[A_2|a_1, a_2]$$

$i = 1, 2$. The model is

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ a_1 \\ a_2 \\ A_1 \\ A_2 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ \mu_{a_1} \\ \mu_{a_2} \\ \mu_{A_1} \\ \mu_{A_2} \end{pmatrix}, \begin{pmatrix} 2 & * & * & * & * & * \\ 1 & 2 & * & * & * & * \\ \rho_{\theta_1 a_1}\sigma_{a_1}\sqrt{2} & \rho_{\theta_1 a_1}\sigma_{a_1}\sqrt{2} & \sigma_{a_1}^2 & * & * & * \\ \rho_{\theta_1 a_2}\sigma_{a_2}\sqrt{2} & \rho_{\theta_2 a_2}\sigma_{a_2}\sqrt{2} & \rho_{a_1 a_2}\sigma_{a_1}\sigma_{a_2} & \sigma_{a_2}^2 & * & * \\ \rho_{\theta_1 A_1}\sigma_{A_1}\sqrt{2} & \rho_{\theta_1 A_1}\sigma_{A_1}\sqrt{2} & \rho_{A_1 a_1}\sigma_{A_1}\sigma_{a_1} & \rho_{A_1 a_2}\sigma_{A_1}\sigma_{a_2} & \sigma_{A_1}^2 & * \\ \rho_{\theta_1 A_2}\sigma_{A_2}\sqrt{2} & \rho_{\theta_2 A_2}\sigma_{A_2}\sqrt{2} & \rho_{A_2 a_1}\sigma_{A_2}\sigma_{a_1} & \rho_{A_2 a_2}\sigma_{A_2}\sigma_{a_2} & \rho_{A_2 A_1}\sigma_{A_2}\sigma_{A_1} & \sigma_{A_2}^2 \end{pmatrix}\right)$$

Denote by $\rho_{b_\theta^1} = \mathbb{E}[\theta|a_1]$ and $\rho_{b_\theta^2} = \mathbb{E}[\theta|a_1, a_2]$. One could think of $\rho_{b_\theta^2\theta}/\rho_{b_\theta^1\theta}$ as speed of learning and $\rho_{A_2\theta}/\rho_{A_1\theta}$ as inertia. Explore what is the maximal intertia one could get given speed of learning. What is the dependence of level of strategic complementarity/substitutability r? How is this model related to BNE of game with signals?

2 Optimal Monetary Policy with Heterogeneous Information.

Show that in a standard New Keynesian model with only i.i.d. TFP shocks and heterogeneous informaion about shocks price stabilization is an optimal monetary policy (carried after observation of TFP shock).[1]

3 Information Capacity Constratints.

Suppose agents have the following objective function:

$$\pi_i = -\mathbb{E}\left[k_i - ((1-\xi)\theta + \xi K + \eta_i)\right]^2$$

where $\eta_i \sim N(0, \alpha_\eta)$ is an idiosyncratic shock that is not directly observable by the agents. The prior on θ is $N(\mu, \alpha_\mu)$ and, before choosing k_i agents observe private signals about θ and η_i:

$$x_i = \theta + \epsilon_i$$

$$s_i = \eta_i + \chi_i$$

where $\epsilon_i \sim N(0, \alpha_\epsilon)$ and $\chi_i \sim N(0, \alpha_s)$.

1. Conjecture that the equilibrium decision rule is of the form:

$$k_i = \gamma_0\mu + \gamma_1 x_i + \gamma_2 s_i$$

   and solve for coefficients.

2. Now suppose the agent can choose the precision of the signals he receives subject to a capacity constraint:

$$\frac{1}{\alpha_x} + \frac{1}{\alpha_s} \geq \lambda.$$

   Solve for the equilibrium α_x and α_s.

3. How does the degree of complementarity ξ affect α_x and α_s. Interpret.

[1] Formulization is part of a problem.

14.161 Advanced Macroeconomics I
Fall 2012