There are two countries, denoted by $i = 1, 2$. Each country produces a mass (continuum) of traded goods N_i and of non-traded goods P_i. Each good is produced by an individual monopolist. There is no overlap between the goods produced by one country and those produced by the other country. All consumers have a utility given by

$$\left(\int_0^N c_k^\alpha dk\right)^{1/\alpha},$$

where $\alpha \in (0,1)$, and $N = N_1 + P_1 + N_2 + P_2$ is the total (maximum) number of goods. The goods are ordered as follows:

- $k \in [0, P_1] \Longrightarrow$ Non traded good produced by country 1
- $k \in [P_1, N_1 + P_1] \Longrightarrow$ Traded good produced by country 1
- $k \in [N_1 + P_1, N_1 + P_1 + N_2] \Longrightarrow$ Traded good produced by country 2
- $k \in [N_1 + P_1 + N_2, N] \Longrightarrow$ Traded good produced by country 2

The price charged by producer of good k is denoted by p_k. One will denote $\sigma = 1/(1-\alpha)$ and $\mu = \sigma/(\sigma - 1)$.

1. Compute the demand function for each of the four types of goods, as a function of its price, the aggregate nominal national income of each country Y_i, and other producers' prices. Show that the contribution of other producers' prices can be summarized using these two price indices=:

$$\bar{p}_1 = \left(\int_0^{N_1+P_1+N_2} p_k^{1-\sigma} dk\right)^{\frac{1}{1-\sigma}}$$

$$\bar{p}_2 = \left(\int_{P_1}^{N} p_k^{1-\sigma} dk\right)^{\frac{1}{1-\sigma}}$$

The production function for any good k is given by

$$y_k = q_k l_k,$$

where q_k is the quality of the manager hired by the firm (each firm uses exactly 1 manager), and l_k is its labor input. w_i is the wage of raw labor in country i.

2. Show that the price charged by a firm with managerial quality q_k in country i is

$$p_k = \mu \frac{w_i}{q_k}.$$

Compute the output and employment of a firm as a function of its managerial quality, wages, aggregate income and price indices, in the four cases.

3. Compute the profit of a firm in the four cases, denoted as functions $\pi_{P1}(q), \pi_{N1}(q), \pi_{N2}(q), \pi_{P2}(q)$.

We now try to characterize the wage schedule $\omega_i(q)$, which tells us how much a manager of quality q earns in country i. Firms chose their managerial quality by maximizing $\pi(q) - \omega(q)$, where $\pi(.)$ is the relevant profit function for the type of firm being considered.

4. Show that if two types q, q' are both employed by exporting firms in country i, then it must be that

$$\omega_i(q) - \omega_i(q') = \pi_{Ni}(q) - \pi_{Ni}(q'),$$

and that a similar equality holds if they are both employed by non-exporting firms.

We assume each individual is endowed with one unit of labor exactly, and q units of managerial quality. In country i, managerial quality is distributed over $[0, \bar{q}_i]$, with c.d.f $F_i(q)$, and density $F_i'(q) = f_i(q)$. Furthermore, total labor force in country i L_i is such that $L_i > N_i + P_i$. People have to fully specialize between being a manager or a worker.

5. Show that in equilibrium if a manager type q is employed in an exporting firm then any manager with $q' > q$ is also hired by an exporting firm.

We thus look for an equilibrium such that in country i, there exists two critical values q_{Pi}, q_{Ni} such that $q_{Pi} < q_{Ni}$ and
-if $q \in [0, q_{Pi}]$, the worker supplies labor
-if $q \in [q_{Pi}, q_{Ni}]$, the worker becomes a manager in the non-traded sector
-if $q \in [q_{Ni}, \bar{q}_i]$, the worker becomes a manager in the traded sector

6. What are the values of q_{Pi} and q_{Ni}? What are the values of the price indices $\bar{p}_i$ as a function of the wages w_i, the critical levels q_{Pi} and q_{Ni}, and the distributions of managerial quality?

7. So that the wage schedule for managers is given by

$$\begin{aligned} \omega(q_{Pi}) &= w_i + \pi_{P1}(q) - \pi_{P1}(q_{Pi}),\ q \in [q_{Pi}, q_{Ni}] \\ & \quad w_i + \pi_{P1}(q_{Ni}) - \pi_{P1}(q_{Pi}) + \pi_{N1}(q) - \pi_{N1}(q_{Ni}) \end{aligned}$$

How does the return to managerial quality evolve when one moves up the quality ladder, if $\sigma > 2$?

8. Show that $Y_i = \mu w_i L_i F(q_{Pi})$ and that the model can be closed either by
–Writing one of two (redundant) labor market clearing conditions and picking a price normalization

–Writing one of two (redundant) trade balance equilibrium conditions and picking a price normalization.

We normalize prices so that $w_1 = 1$

9. Show that the highest wage in country 1 is

$$\begin{aligned}\omega(\bar{q}_1) &= 1 + (\mu - 1)\mu^{-\sigma}(\bar{q}_1^{\sigma-1} - q_{N1}^{\sigma-1})\left[Y_1\bar{p}_1^{\sigma-1} + Y_2\bar{p}_2^{\sigma-1}\right] \\ &\quad + (\mu - 1)\mu^{-\sigma}(q_{N1}^{\sigma-1} - q_{P1}^{\sigma-1})\left[Y_1\bar{p}_1^{\sigma-1}\right].\end{aligned}$$

10. We now look at the effect of increases in international trade in country one, by assuming a marginal increase in N_1, $dN_1 > 0$, compensated by a fall in P_1, $dP_1 = -dN_1$, so that the total number of goods in country 1 remains constant. We measure inequality by the ration between the highest and the lowest wage.

Show that, *holding Y_i and $\bar{p}_i$ constant,* this shift increases inequality. Why?

11. How would you go about evaluating the indirect contribution of the induced changes in Y_i and $\bar{p}_i$ in country 1?