14.462 Advanced Maroeconomics
Spring 2004

Problem Set 5

(due April 12)

Problem 1

There is a measure-one continuum of agents, indexed by $i \in [0,1]$. Each agent can choose between two actions. The action of agent i is denoted as $k_i \in \{0,1\}$, where $k_i = 0$ represents "not invest" and $k_i = 1$ represents "invest". All agents move simultaneously. The utility of agent i is given by

$$u_i = U(k_i, K, \theta) = e^{\theta} k_i (1 + K^{\gamma}) - c k_i$$

where θ reflects "fundamentals" and $K = \int k_i di$ denotes the mass of agents investing.

1. Suppose that θ is commonly known by all agents. What is the best response $g(K,\theta)$ (agents are assumed not invest in case of indifference)? Derive the thresholds $\underline{\theta}$ and $\bar{\theta}$ such that: (i) all agents not investing is the unique equilibrium for $\theta < \underline{\theta}$; (ii) all agents investing is the unique equilibrium for $\theta \geq \bar{\theta}$ and (iii) for intermediate values $\theta \in [\underline{\theta}, \bar{\theta})$ there are multiple equilibria.

From now on assume that agent i observes a private signal

$$x_i = \theta + \xi_i$$

with $\xi_i \sim \mathcal{N}(0, \sigma_x^2)$. All agents also observes an exogenous public signal

$$z = \theta + \varepsilon$$

where $\varepsilon \sim \mathcal{N}(0, \sigma_z^2)$. Let $\alpha_x = \sigma_x^{-2}$ and $\alpha_z = \sigma_z^{-2}$. Agents have a common prior about θ, which is uniform over the entire real line. Equilibrium is defined as follows: a strategy $k(\cdot)$ and an aggregate investment $K(\cdot)$ constitute an equilibrium if

$$k(x,z) \in \arg\max_k \mathbb{E}[U(k, K(\theta,z), \theta | x, z],$$

$$K(\theta, z) = \int k(x,z) \sqrt{\alpha_x} \phi(\sqrt{\alpha_x}[x - \theta]) dx.$$

The remainder of the exercise asks you to numerically compute monotone equilibria, that is, equilibria in which $k(x,z)$ is monotone in x. In a monotone equilibrium, for any realization of z, there is a threshold $x^*(z)$ such that an agent invests if and only if $x \geq x^*(z)$. Throughout, set $\gamma = 0.8$ and $c = 2$.

2. Set $\alpha_x = 10$ and $\alpha_z = 1$. Over a range of values for z (somewhat wider than $[\underline{\theta}, \bar{\theta}]$) plot the equilibrium thresholds $x^*(z)$. Are there values of z for which there are multiple equilibrium thresholds?

3. Repeat part 2. for the values $\alpha_x = 1$ and $\alpha_z = 10$. Compute the range $[\underline{z}, \bar{z})$ of values of z for which there is multiplicity.

4. Fix α_x at one. What happens to the range $[\underline{z}, \bar{z})$ as α_z becomes large?

Problem 2

Consider the following two stage version of Morris-Shin. The utility of agent i is given by

$$u_i = a_{1,i}(Rb - c) + \beta a_{2,i}(Rb - c)$$

Here $a_{1,i} \in \{0, 1\}$ is the action of agent i in the first stage and $a_{2,i} \in \{0, 1\}$ is the action of agent i in the second stage. For both actions, a value of one represents "attack" while a value of zero represents "not attack". The regime outcome is denoted as R, where $R = 0$ represents survival of the status quo and $R = 1$ represents collapse. The decision to attack is irreversible: $a_{1,i} = 1$ implies $a_{2,i} = 1$. Let $A_1 = \int a_{1,i} di$ be mass of agents attacking in the first stage. Let $A_2 = \int a_{2,i}(1 - a_{1,i}) di$ be the mass of agents that did not attack in the first stage but join the attack in the second stage. The timing is as follows. First agent i observes a private signal $x_i = \theta + \xi_i$ where $\xi_i \sim \mathcal{N}(0, \sigma_x^2)$ and $\alpha_x = \sigma_x^{-2}$. Then agents simultaneously choose their first stage actions $a_{1,i}$. Then agents observe the public signal $z = \Phi^{-1}(A_1) + v$ where $v \sim \mathcal{N}(0, \sigma_z^2)$ and $\alpha_z = \sigma_z^{-2}$. The status quo collapses ($R = 1$) if $A_1 + A_2 \geq \theta$. The goal is to characterize the monotone equilibria of this model.

1. First note that agent i will attack in the first period if $x_i \leq x_1^*$ for some threshold x_1^*. What is the mass of agents choosing to attack in the first period as a function of θ, denoted as $A_1(\theta)$? Plug the function $A_1(\theta)$ into the expression for the public signal z. What are the properties of z as a signal about θ?

2. Given a first period threshold x_1^*, characterize the second stage equilibrium thresholds $x_2^*(z)$. Under what condition on α_z and α_x is their multiplicity?

3. Now move back to the first stage and analyze the monotone equilibria of the model. Discuss multiplicity and how it is related to the precision parameters α_z and α_x and the discount rate β.