14.462 Advanced Macroeconomics
Spring 2004

Problem Set 6 Solution

Problem 1

1. The guess

$$k_i = \beta x_i + (1-\beta)y.$$

implies

$$K = \beta\theta + (1-\beta)y.$$

Combining this with the relationship $y = K + \sigma_y u$ yields

$$y = \theta + \frac{1}{\beta}\sigma_u u$$

Thus as a signal of θ the precision is $\beta^2\pi_y$. Define

$$\phi = \frac{\pi_x}{\pi_x + \beta^2\pi_y}$$

Then we have

$$\mathbb{E}[\theta|x_i, y] = \phi x_i + (1-\phi)y.$$

Optimal investment is given by

$$\begin{aligned} k_i = \mathbb{E}[A|x_i, y] &= (1-\alpha)\mathbb{E}[\theta|x_i, y] + \alpha\mathbb{E}[K|x_i, y] \\ &= [(1-\alpha) + \alpha\beta]\mathbb{E}[\theta|x_i, y] + \alpha(1-\beta)y \end{aligned}$$

Substituting for $\mathbb{E}[\theta|x_i, z, y]$ we have

$$k_i = [(1-\alpha) + \alpha\beta]\phi x_i + [[(1-\alpha) + \alpha\beta](1-\phi) + \alpha(1-\beta-\gamma)]\, y$$

Matching coefficients we get

$$\beta = [(1-\alpha) + \alpha\beta]\phi$$

or equivalently

$$\phi = \frac{\beta}{(1-\alpha) + \alpha\beta}$$

Combining this with the equation definition ϕ gives the condition

$$\frac{\beta}{(1-\alpha)+\alpha\beta} = \frac{\pi_x}{\pi_x + \beta^2 \pi_y}$$

The left hand side is increasing in β with a range $[0,1]$ while the right hand side is decreasing with a range $\left[\frac{\pi_x}{\pi_x+\pi_y}, 1\right]$. Thus there is a unique solution for β. The left hand side is increasing in α, so β is decreasing in α. The right hand side is increasing in π_x and decreasing in π_y. It follows that β is increasing in π_x and decreasing in π_y, that is

$$\beta = B(\alpha, \pi_x, \pi_y)$$

with $B_\alpha < 0$, $B_{\pi_x} > 0$ and $B_{\pi_y} < 0$. For future purposes it is useful to compute the elasticities with respect to π_x and π_y explicitly. We get

$$\frac{B_{\pi_x}(\alpha, \pi_x, \pi_y)\pi_x}{B(\alpha, \pi_x, \pi_y)} = -\frac{B_{\pi_y}(\alpha, \pi_x, \pi_y)\pi_y}{B(\alpha, \pi_x, \pi_y)} = \frac{1}{\frac{(1-\alpha)}{[(1-\alpha)+\alpha\beta]^2}\frac{(\pi_x+\beta^2\pi_y)^2}{\beta\pi_x\pi_y} + 2} < \frac{1}{2}.$$

What is the intuition for these results. If α increases, then complementarities are stronger, and agents put more weight on the public signal since it helps predict what others will do. Higher precision of the private signal induces agents to put more weight on the private signal and higher precision of the signal about K induces agents to put more weight on this public signal. However, notice one difference to the paper by Angeletos and Pavan. If you increases α, this makes it more attractive to put more weight on the public signal. But if agents put more weight on the public signal, this makes the public signal less informative about θ, which makes it less attractive to put weight on the public signal, partially offsetting the initial effect. Thus all the effects on β are muted in comparison to Angeletos and Pavan. Why is the elasticity with respect to π_x and π_y less than $\frac{1}{2}$ in absolute value. Suppose we increase π_x by one percent and β increases by more that 0.5 percent. Then relative precision of the public signal y actually increases, in which case agents would not have wanted to put more weight on the private signal in the first place. Similarly, suppose we increase π_y by one percent. If β decreases by more than 0.5 percent, than relative precision of the public signal actually decreases, but in this case agents would not have wanted to put more weight on the public signal in the first place.
It is also instructive to consider how ϕ depends on the parameters. We have

$$\beta = \frac{(1-\alpha)\phi}{1-\alpha\phi} \tag{1}$$

Substituting into the definition of ϕ gives the condition

$$\phi = \frac{\pi_x}{\pi_x + \left(\frac{(1-\alpha)\phi}{1-\alpha\phi}\right)^2 \pi_y}.$$

Again there is a unique solution

$$\phi = \Phi(\alpha, \pi_x, \pi_y)$$

with $\Phi_\alpha > 0$, $\Phi_{\pi_x} > 0$ and $\Phi_{\pi_y} < 0$. Equation (1) implies

$$B(\alpha, \pi_x, \pi_y) \leq \Phi(\alpha, \pi_x, \pi_y)$$

with strict inequality if $\alpha > 0$ and clearly the wedge is increasing in α.

2. We have

$$\mathrm{Var}(k_i|\theta, y) = \mathrm{Var}(\beta x_i + (1-\beta)y|\theta, y) = \frac{\beta^2}{\pi_x}$$

so heterogeneity as a function of parameters is given by

$$H(\alpha, \pi_x, \pi_y) = \frac{B(\alpha, \pi_x, \pi_y)^2}{\pi_x}$$

It is decreasing in α and π_y. Both higher α and higher π_y induce agents to put less weight on the private signal, and less weight on the private signal translates into less heterogeneity. If π_x increases, this directly reduces heterogeneity. But agents also become more responsive to the private signal, which tends to increase heterogeneity. But since the elasticity is less than $\frac{1}{2}$, we know that this does not overturn the direct effect, and so heterogeneity falls. This differs from Angeletos and Pavan, where the overall effect is ambiguous.

We have

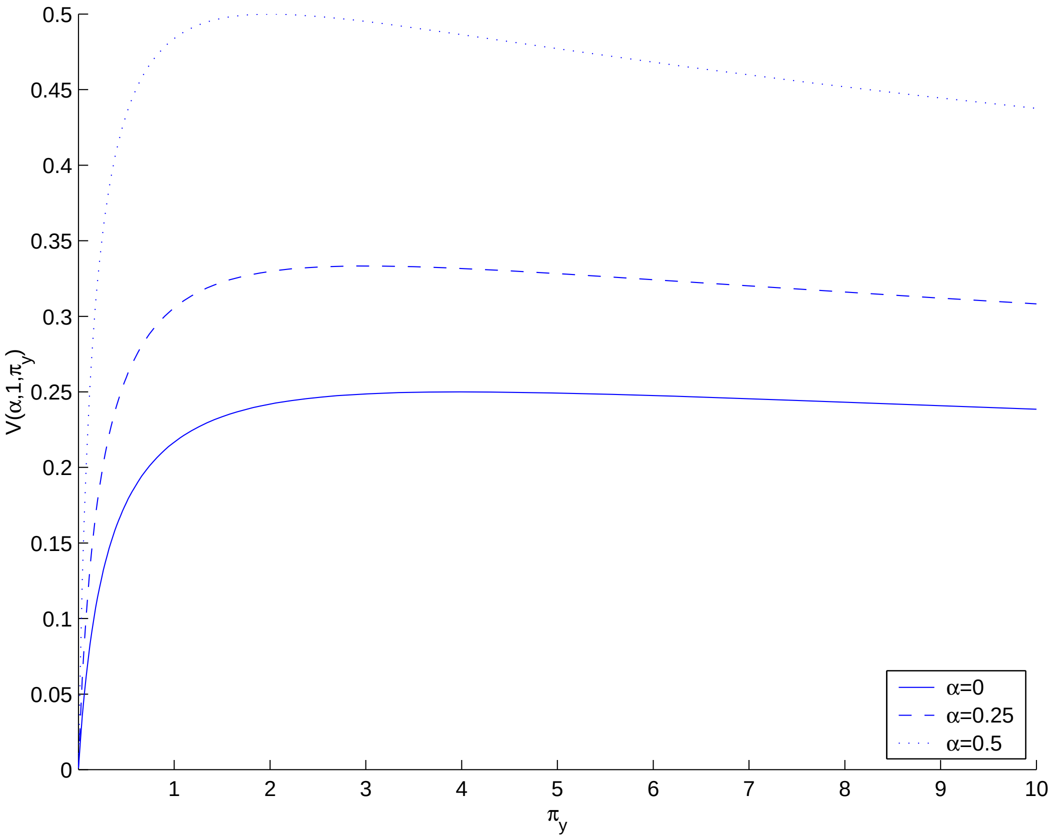
$$\mathrm{Var}(K|\theta) = \mathrm{Var}((1-\beta)y|\theta) = \mathrm{Var}\left(\frac{1-\beta}{\beta}\sigma_y u \,\middle|\, \theta\right) = \left(\frac{1-\beta}{\beta}\right)^2 \frac{1}{\pi_y}$$

Thus volatility as a function of the parameters is given by

$$V(\alpha, \pi_x, \pi_y) = \left(\frac{1 - B(\alpha, \pi_x, \pi_y)}{B(\alpha, \pi_x, \pi_y)}\right)^2 \frac{1}{\pi_y}$$

Clearly volatility is increasing in α and decreasing in π_x. Higher α induces agents to put more weight on the public signal, increasing volatility. Higher precision of the private signal does the opposite. The effect of an increase in the precision π_y is more complicated. The direct effect is to reduce volatility. There are two indirect effects, both related to the fact that agents become more responsive to the public and thus less responsive to the private signal. Higher responsiveness to the public signal increases volatility. This effect is also present in Angeletos and Pavan. In addition, less responsiveness to the private signal reduces the precision of y as a signal about θ, partially offsetting the increase in π_y and thus increasing in volatility. Volatility

Figure 1: Volatility as a function of π_y



as a function of π_y is analyzed in figure 1. Only the ratio of π_x and π_y matters for the shape, so I restrict attention to the case $\pi_x = 1$. Thus the figure shows volatility as function of π_y given $\pi_x = 1$, and the graph is plotted for different values of α. Of course one finds that higher α is associated with higher volatility. Volatility is initially increasing in π_y but eventually becomes decreasing.

3. By definition

$$w = \int_0^1 u_i di.$$

Substituting the formula for $u_i = Ak_i - \frac{1}{2}k_i^2$ yields

$$w = A\int_0^1 k_i di - \frac{1}{2}\int_0^1 k_i^2 di = AK - \frac{1}{2}\int_0^1 k_i^2 di$$

Since $\int_0^1 k_i^2 di = \int_0^1 (k_i - K)^2 di + K^2$ this can be written as

$$w = AK - \frac{1}{2}\left[\int_0^1 (k_i - K)^2 di + K^2\right].$$

Substituting $A = (1-\alpha)\theta + \alpha K$ yields

$$\begin{aligned} w &= [(1-\alpha)\theta + \alpha K]\, K - \frac{1}{2}\left[\int_0^1 (k_i - K)^2 di + K^2\right] \\ &= (1-\alpha)\theta K - (1-2\alpha)\frac{1}{2}K^2 - \frac{1}{2}\int_0^1 (k_i - K)^2 di. \end{aligned}$$

Now notice that $k_i - K = \beta(x_i - \theta)$ and so

$$\int_0^1 (k_i - K)^2 di = \beta^2 \sigma_x^2 = \frac{\beta^2}{\pi_x}.$$

Thus

$$E[w|\theta] = (1-\alpha)\theta E[K|\theta] - (1-2\alpha)\frac{1}{2}E[K^2|\theta] - \frac{1}{2}\frac{\beta^2}{\pi_x}.$$

Using the facts that $E[K|\theta] = \theta$ and $E[K^2|\theta] = \text{Var}(K|\theta) + \theta^2$, this becomes

$$\begin{aligned} E[w|\theta] &= (1-\alpha)\theta^2 - (1-2\alpha)\frac{1}{2}\left[\text{Var}(K|\theta) + \theta^2\right] - \frac{1}{2}\frac{\beta^2}{\pi_x} \\ &= -\frac{1}{2}\theta^2 - \frac{1}{2}\left[(1-2\alpha)\text{Var}(K|\theta) + \frac{\beta^2}{\pi_x}\right]. \end{aligned}$$

Now recall from part 2. that $\text{Var}(k_i|\theta, y) = \frac{\beta^2}{\pi_x}$. Using this fact

$$E[w|\theta] = -\frac{1}{2}\theta^2 - \frac{1}{2}\left[(1-2\alpha)\text{Var}(K|\theta) + \text{Var}(k_i|\theta, y)\right].$$

So we can analyze welfare by looking at

$$\Omega(\alpha, \pi_x, \pi_y) = (1-2\alpha)V(\alpha, \pi_x, \pi_y) + H(\alpha, \pi_x, \pi_y).$$

Since both volatility and heterogeneity are decreasing in π_x, we immediately get that $\Omega(\alpha, \pi_x, \pi_y)$ is decreasing in π_x. Thus making private information more precise is unambiguously good for welfare. This is different from Angeletos and Pavan. There more precise private information meant less uncertainty at the expense of lower coordination, with ambiguous overall effects on welfare. But here precise private information is also vital for the informativeness of the public signal and thus for coordination. So it makes sense that here the effect is unambiguous.

$$\begin{aligned}
\frac{\beta}{(1-\alpha)+\alpha\beta} &= \frac{\pi_x}{\pi_x+\beta^2\pi_y} \\
\iff \beta\left[\pi_x+\beta^2\pi_y\right] &= \pi_x\left[(1-\alpha)+\alpha\beta\right] \\
\iff \beta\beta^2\pi_y &= \pi_x(1-\alpha)(1-\beta) \\
\iff \frac{\beta^2}{\pi_x} &= (1-\alpha)\frac{(1-\beta)}{\beta}\frac{1}{\pi_y}
\end{aligned}$$

Thus

$$\begin{aligned}
\Omega(\alpha,\pi_x,\pi_y) &= (1-2\alpha)\left(\frac{(1-\beta)}{\beta}\right)^2\frac{1}{\pi_y}+\frac{\beta^2}{\pi_x} \\
&= \frac{(1-2\alpha)}{1-\alpha}\left(\frac{(1-\beta)}{\beta}\right)\frac{\beta^2}{\pi_x}+\frac{\beta^2}{\pi_x} \\
&= \frac{\beta^2}{\pi_x}\left[\frac{(1-2\alpha)(1-\beta)+(1-\alpha)\beta}{(1-\alpha)\beta}\right] \\
&= \frac{\beta}{\pi_x}\left[\frac{(1-2\alpha)+\alpha\beta}{(1-\alpha)}\right] \\
&= \frac{\beta}{\pi_x}\left[1-(1-\beta)\frac{\alpha}{1-\alpha}\right]
\end{aligned}$$

The condition $\alpha < \frac{1}{2}$ is sufficient for $\Omega(\alpha,\pi_x,\pi_y)$ to be positive. Thus the last relationship implies that $\Omega(\alpha,\pi_x,\pi_y)$ is increasing in β for given π_x. Since β is decreasing in π_y, it follows that making public information more precise also increases welfare. Also notice that the right hand side is decreasing in α for given β and π_x. Since β is decreasing in α, it follows that $\Omega(\alpha,\pi_x,\pi_y)$ is also decreasing in α. Making complementarities stronger improves welfare.

4. For this part I will not try to sign derivatives analytically. Instead I derive the relevant formulas and perform a limited numerical evaluation.
   Now start with the guess

   $$k_i = \beta x_i + \gamma z + (1-\beta-\gamma)y.$$

   This implies

   $$K = \beta\theta + \gamma z + (1-\beta-\gamma)y.$$

Combining this with the relationship $y = K + \sigma_y u$ yields

$$y = \frac{\beta}{\beta+\gamma}\theta + \frac{\gamma}{\beta+\gamma}z + \frac{1}{\beta+\gamma}\sigma_u u$$

To obtain the information provided by y beyond what is provided by z define

$$\tilde{y} = \frac{(\beta+\gamma)y - \gamma z}{\beta} = \theta + \frac{1}{\beta}\sigma_y u$$

Thus we get an additional signal of precision $\beta^2\pi_y$. Define

$$\begin{aligned} \delta &= \frac{\pi_z}{\pi_x + \pi_z + \beta^2\pi_y} \\ \phi &= \frac{\pi_x}{\pi_x + \pi_z + \beta^2\pi_y} \end{aligned}$$

Then we have

$$\begin{aligned} \mathbb{E}[\theta|x_i, z, y] &= \phi x_i + \delta z + (1-\phi-\delta)\tilde{y} \\ &= \phi x_i + \delta z + (1-\phi-\delta)\frac{(\beta+\gamma)y - \gamma z}{\beta} \\ &= \phi x_i + \left[\delta - (1-\phi-\delta)\frac{\gamma}{\beta}\right] z + (1-\phi-\delta)\frac{(\beta+\gamma)}{\beta}y \end{aligned}$$

Optimal investment is given by

$$\begin{aligned} k_i = \mathbb{E}[A|x_i, z, y] &= (1-\alpha)\mathbb{E}[\theta|x_i, z, y] + \alpha\mathbb{E}[K|x_i, z, y] \\ &= (1-\alpha)\mathbb{E}[\theta|x_i, z, y] + \alpha\mathbb{E}[K|x_i, z, y] \\ &= [(1-\alpha) + \alpha\beta]\mathbb{E}[\theta|x_i, z, y] + \alpha\gamma z + \alpha(1-\beta-\gamma)y \end{aligned}$$

Substituting for $\mathbb{E}[\theta|x_i, z, y]$ we have

$$\begin{aligned} k_i = &[(1-\alpha) + \alpha\beta]\phi x_i \\ &+ \left[[(1-\alpha) + \alpha\beta]\left[\delta - (1-\phi-\delta)\frac{\gamma}{\beta}\right] + \alpha\gamma\right] z \\ &+ \left[[(1-\alpha) + \alpha\beta]\left[(1-\phi-\delta)\frac{(\beta+\gamma)}{\beta}\right] + \alpha(1-\beta-\gamma)\right] y \end{aligned}$$

Matching coefficients we get

$$\begin{aligned} \beta &= [(1-\alpha) + \alpha\beta]\phi \\ \gamma &= \left[[(1-\alpha) + \alpha\beta]\left[\delta - (1-\phi-\delta)\frac{\gamma}{\beta}\right] + \alpha\gamma\right] \end{aligned}$$

Eliminating ϕ, we now get the following equation for β

$$\frac{\beta}{(1-\alpha)+\alpha\beta} = \frac{\pi_x}{\pi_x + \pi_z + \beta^2 \pi_y}$$

Again there is a unique solution

$$\beta = B(\alpha, \pi_x, \pi_z, \pi_y)$$

with $B_\alpha < 0$, $B_{\pi_x} > 0$, $B_{\pi_z} < 0$, $B_{\pi_y} < 0$.
From the condition defining γ we get

$$\gamma = \left[\frac{\beta}{\phi}\left[\delta - (1-\phi-\delta)\frac{\gamma}{\beta}\right] + \alpha\gamma\right]$$

$$\gamma\left[\phi(1-\alpha) + (1-\phi-\delta)\right] = \beta\delta$$

$$\gamma = \frac{\beta\pi_z}{\pi_x(1-\alpha) + \beta^2\pi_y}$$

Thus

$$\gamma = C(\alpha, \pi_x, \pi_y, \pi_z) = \frac{B(\alpha, \pi_x, \pi_z, \pi_y)\pi_z}{\pi_x(1-\alpha) + B(\alpha, \pi_x, \pi_z, \pi_y)^2\pi_y}$$

I will not try to sign the derivatives but instead do some limited numerical evaluation. This is done in Figure 2, and the results contain nothing unexpected. An increase in the degree of complementarity leads to an increase in the weight on z, as does an increase in its own precision, while higher precision of the other signals reduces the weight on z. Finally the coefficient on y is given by

$$1 - \beta - \gamma = D(\alpha, \pi_x, \pi_y, \pi_z) \equiv= 1 - B(\alpha, \pi_x, \pi_y, \pi_z) - C(\alpha, \pi_x, \pi_y, \pi_z)$$

Figure 3 provides a limited numerical evaluation of the properties of D. Notice that more complementarity does not necessarily lead to an increase in the weight on y. This makes sense, since now there is an alternative public signal available. As α increases, more weight is put on public information, but as the weight on private information shrinks, y becomes less and less precise as a signal about θ, and thus at high levels of α the signal z is the more attractive public signal.
Heterogeneity is once again

$$H(\alpha, \pi_x, \pi_y, \pi_z) = \frac{B(\alpha, \pi_x, \pi_y, \pi_z)^2}{\pi_x},$$

but volatility is more complicated

$$\mathrm{Var}(K|\theta) = \mathrm{Var}(\gamma z + (1-\beta-\gamma)y|\theta)$$

Substituting y yields

$$\begin{aligned}\mathrm{Var}(K|\theta) &= \mathrm{Var}\left(\gamma z + (1-\beta-\gamma)\left[\frac{\beta}{\beta+\gamma}\theta + \frac{\gamma}{\beta+\gamma}z + \frac{1}{\beta+\gamma}\sigma_u u\right]\middle|\,\theta\right) \\ &= \mathrm{Var}\left(\frac{\gamma}{\beta+\gamma}z + \frac{1-\beta-\gamma}{\beta+\gamma}\sigma_u\,\middle|\,\theta\right) \\ &= \left(\frac{\gamma}{\beta+\gamma}\right)^2\frac{1}{\pi_z} + \left(\frac{1-\beta-\gamma}{\beta+\gamma}\right)^2\frac{1}{\pi_y}\end{aligned}$$

Thus

$$V(\alpha,\pi_x,\pi_y,\pi_z) = \left(\frac{C(\alpha,\pi_x,\pi_y,\pi_z)}{1-D(\alpha,\pi_x,\pi_y,\pi_z)}\right)^2\frac{1}{\pi_z} + \left(\frac{D(\alpha,\pi_x,\pi_y,\pi_z)}{1-D(\alpha,\pi_x,\pi_y,\pi_z)}\right)^2\frac{1}{\pi_y}$$

Figure 4 provides a limited evaluation of volatility. Here it turns out that π_y reduces volatility.

Again we can analyze welfare by looking at

$$\Omega(\alpha,\pi_x,\pi_y,\pi_z) = (1-2\alpha)V(\alpha,\pi_x,\pi_y,\pi_z) + H(\alpha,\pi_x,\pi_y,\pi_z).$$

Figure 5 provides a limited numerical evaluation. Notice that an increase in π_z can reduce welfare.

Problem 2 (A simple Model of Savings)

1. The specification of the problem should of course include $\gamma > 0$ and $\gamma \neq 1$. Combining the budget constraints, future wealth is

$$w' = R'(w-c) + e'$$

and so the Bellman equation is

$$V(w) = \max_c\left\{\frac{c^{1-\gamma}}{1-\gamma} + \beta\mathbb{E}[V(R'(w-c)+e')]\right\}.$$

2. The first order condition is

$$c^{-\gamma} = \beta\mathbb{E}[R'V'(R'(w-c)+e')]$$

and the envelope condition is

$$V'(w) = \beta\mathbb{E}[R'V'(R'(w-c)+e')].$$

Thus $V'(w) = c^{-\gamma}$ and we obtain the Euler equation

$$1 = \beta\mathbb{E}\left[R'\left(\frac{c'}{c}\right)^{-\gamma}\right].$$

3. A guess that will work is

$$V(w) = a\frac{w^{1-\gamma}}{1-\gamma}$$

for some $a > 0$.

The objective of the recursive problem then reduces to

$$\frac{c^{1-\gamma}}{1-\gamma} + \beta a \bar{R}^{1-\gamma}\frac{(w-c)^{1-\gamma}}{1-\gamma}$$

where $\bar{R} = \mathbb{E}[(R')^{1-\gamma}]^{\frac{1}{1-\gamma}}$ is the certainty equivalent of R'. The first order condition becomes

$$c^{-\gamma} = \beta a \bar{R}^{1-\gamma}(w-c)^{-\gamma}$$

and so

$$c = \frac{1}{1+(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}}w$$

which yields

$$w - c = \frac{(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}}{1+(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}}w$$

Replacing into the objective yields the maximized value

$$\begin{aligned}
&\left[\frac{1}{[1+(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}]^{1-\gamma}} + \beta a \bar{R}^{1-\gamma}\frac{[(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}]^{1-\gamma}}{[1+(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}]^{1-\gamma}}\right]\frac{w^{1-\gamma}}{1-\gamma} \\
&= \left[\frac{1}{[1+(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}]^{1-\gamma}} + [(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}]^{\gamma}\frac{[(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}]^{1-\gamma}}{[1+(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}]^{1-\gamma}}\right]\frac{w^{1-\gamma}}{1-\gamma} \\
&= \left[\frac{1}{[1+(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}]^{1-\gamma}} + \frac{(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}}{[1+(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}]^{1-\gamma}}\right]\frac{w^{1-\gamma}}{1-\gamma} \\
&= [1+(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}]^{\gamma}\frac{w^{1-\gamma}}{1-\gamma}
\end{aligned}$$

For our guess to be correct we need

$$a = [1+(\beta a)^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}]^{\gamma}.$$

This equation has a unique positive solution if $\beta\bar{R}^{1-\gamma} < 1$, and then it is given by

$$a = \left[1 - \beta^{\frac{1}{\gamma}}\bar{R}^{\frac{1-\gamma}{\gamma}}\right]^{-\gamma}.$$

4. With $R\beta = 1$ the Euler equation becomes

$$c_t^{-\gamma} = \mathbb{E}_t[c_{t+1}^{-\gamma}]$$

and as marginal utility is strictly convex and endowment shocks are nondegenerate

$$\mathbb{E}_t[c_{t+1}] > c_t.$$

Thus $R\beta < 1$ is needed to obtain zero expected consumption growth.

5. The way the budget constraints are written the price of the asset is normalized to one. Then it is more convenient to write $w_t = c_t + p_t b_{t+1}$ and $w_t = e_t + d_t b_t$ where now $d_t \geq 0$ is a given i.i.d. dividend and the price p_t has to adjust in equilbrium. The Euler equation is then

$$e^{-\gamma} p = \beta \mathbb{E}[d'(e')^{-\gamma}]$$

and so the price is a function of the current endowment:

$$p(e) = e^{\gamma} \beta \mathbb{E}[d'(e')^{-\gamma}]$$

A high endowment today implies low expected consumption growth, which makes transferring resources into the future attractive, so the price of the asset has to be high for no trade to be an equilibrium.
If there is only idiosyncratic risk, then we have an endowment economy version of Aiyagari. No trade is then of course not an equilibrium and the interest rate will depend on the wealth distribution. In steady state we of course must have $R\beta < 1$.

6. Here we have a special case of part 3. If $\beta R^{1-\gamma} < 1$, then

$$c_t = \frac{1}{1 + (\beta a)^{\frac{1}{\gamma}} \bar{R}^{\frac{1-\gamma}{\gamma}}} w_t = (1 - \beta^{\frac{1}{\gamma}} \bar{R}^{\frac{1-\gamma}{\gamma}}) w_t$$

and

$$w_{t+1} = R(w_t - c_t) = (\beta R)^{\frac{1}{\gamma}} w_t,$$

so

$$w_t = \left[(\beta R)^{\frac{1}{\gamma}} \right]^t w_0$$

and

$$c_t = (1 - \beta^{\frac{1}{\gamma}} \bar{R}^{\frac{1-\gamma}{\gamma}}) \left[(\beta R)^{\frac{1}{\gamma}} \right]^t w_0.$$

Substituing into the utility function yields

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} &= \sum_{t=0}^{\infty} \left[\beta (\beta R)^{\frac{1-\gamma}{\gamma}} \right]^t (1 - \beta^{\frac{1}{\gamma}} \bar{R}^{\frac{1-\gamma}{\gamma}})^{1-\gamma} \frac{w_0^{1-\gamma}}{1-\gamma} \\
&= \frac{(1 - \beta^{\frac{1}{\gamma}} \bar{R}^{\frac{1-\gamma}{\gamma}})^{1-\gamma}}{(1 - \beta^{\frac{1}{\gamma}} \bar{R}^{\frac{1-\gamma}{\gamma}})} \frac{w_0^{1-\gamma}}{1-\gamma} \\
&= \left[1 - \beta^{\frac{1}{\gamma}} \bar{R}^{\frac{1-\gamma}{\gamma}} \right]^{1-\gamma} \frac{w_0^{1-\gamma}}{\gamma} \\
&= a \frac{w_0^{1-\gamma}}{1-\gamma}.
\end{aligned}$$

Figure 2: Properties of $C(\alpha, \pi_x, \pi_y, \pi_z)$

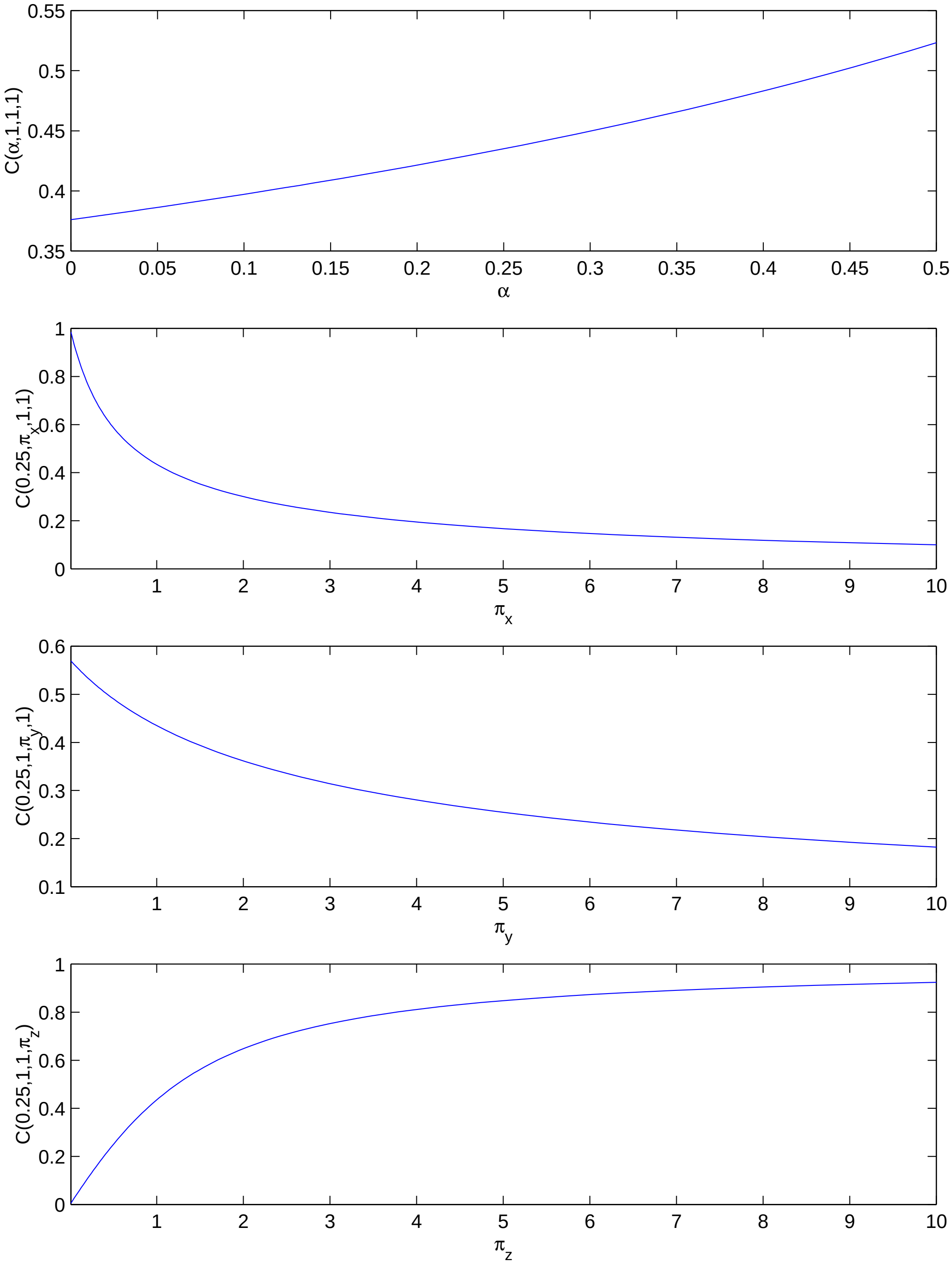


Figure 3: Properties of $D(\alpha, \pi_x, \pi_y, \pi_z)$

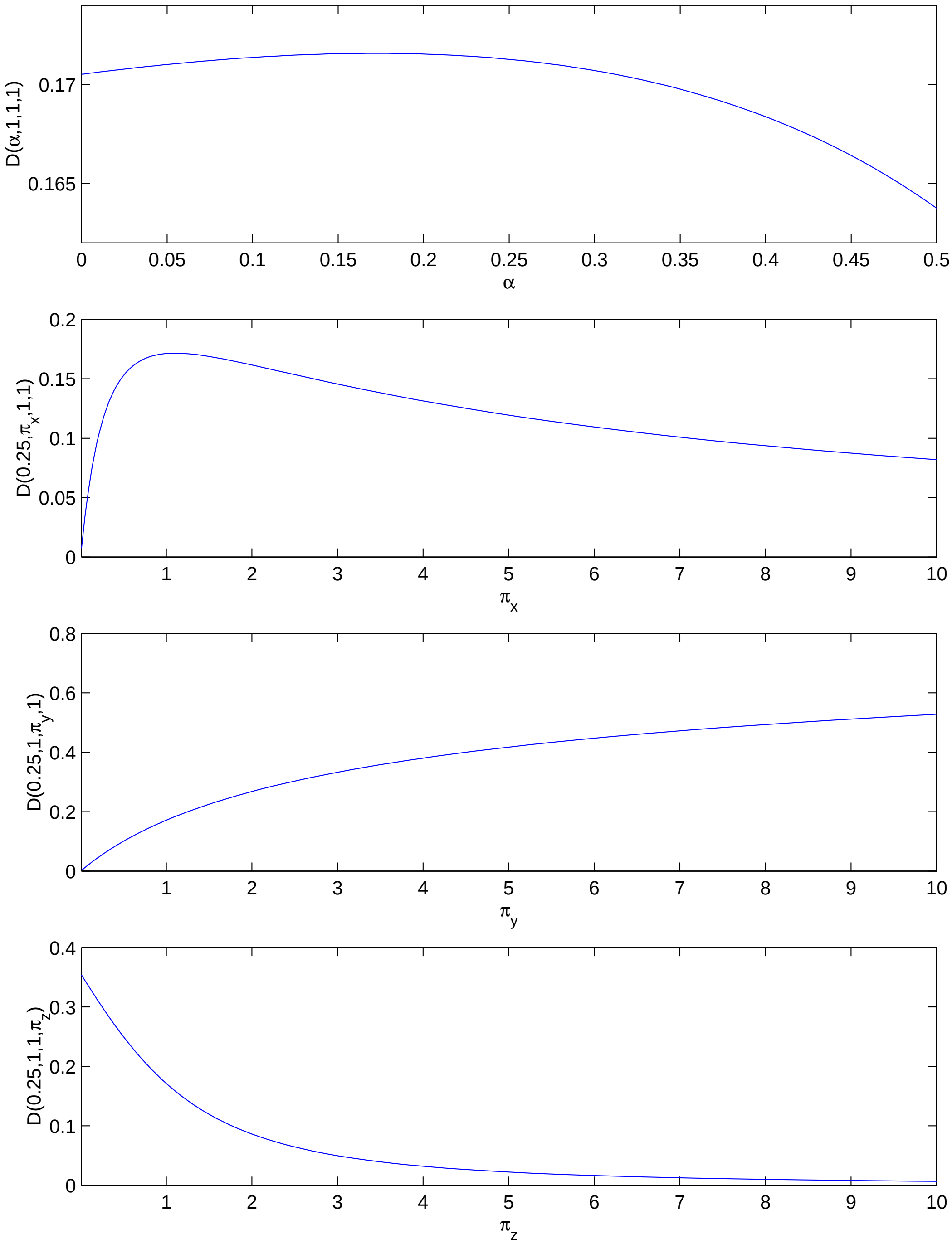


Figure 4: Properties of $V(\alpha, \pi_x, \pi_y, \pi_z)$

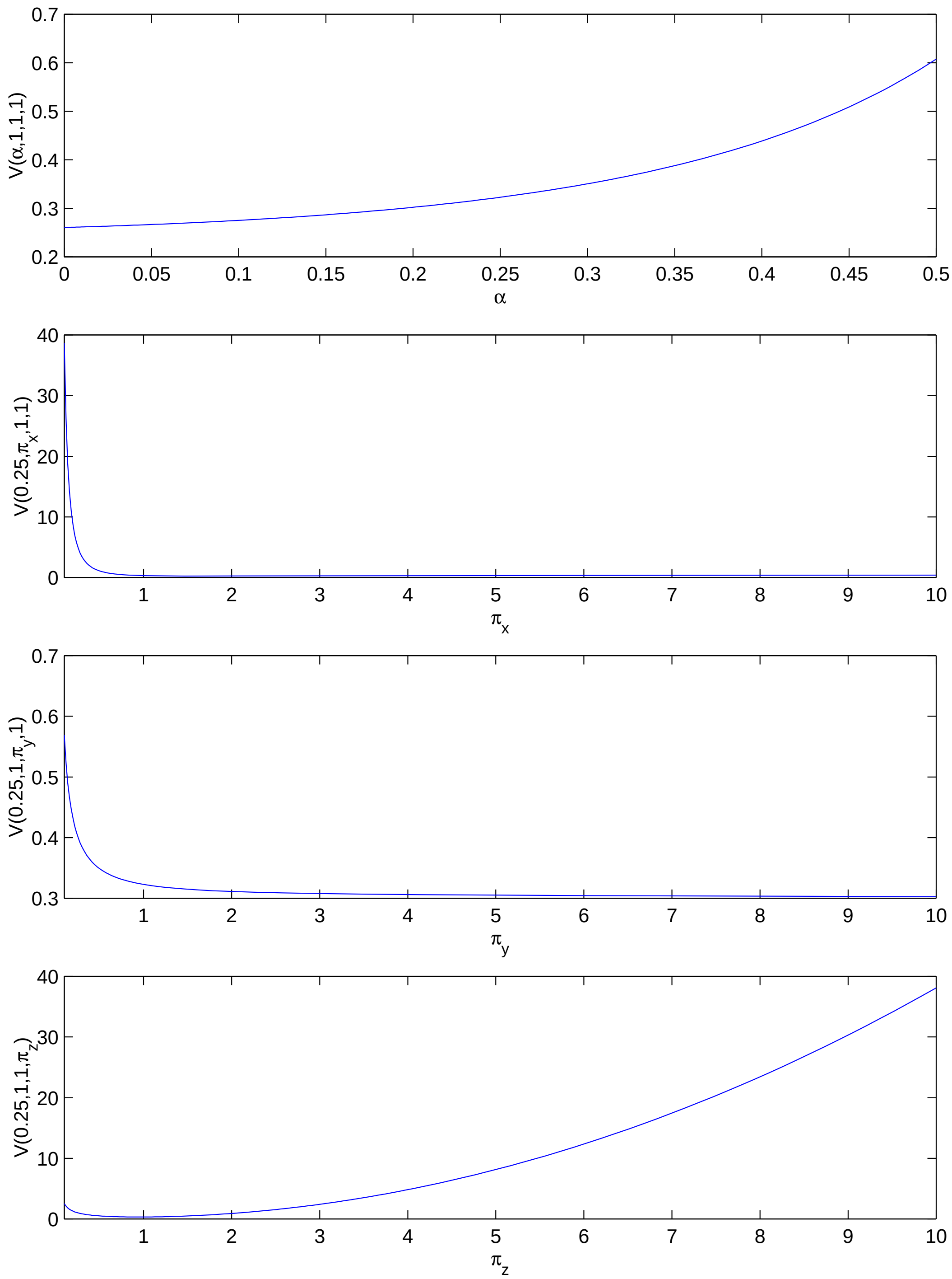


Figure 5: Properties of $\Omega(\alpha, \pi_x, \pi_y, \pi_z)$