1.3 Intro to q theory

- Hayashi (1982)

- Firm with initial stock of capital k_0

- Maximize expected present value of dividends

$$\mathsf{E}\sum_{t=0}^{\infty}\beta^t d_t$$

- Flow of funds constraint

$$d_t + G\left(k_{t+1}, k_t\right) = A_t F\left(k_t, l_t\right) - w_t l_t$$

- G is investment costs + adjustment costs, e.g.

$$G(k_{t+1}, k_t) = k_{t+1} - (1-\delta) k_t + \frac{\xi}{2} \frac{(k_{t+1} - k_t)^2}{k_t}$$

- Assumption:

  G and F are constant returns to scale

- Stochastic process for A_t and w_t

$$\begin{aligned} A_t &= A(s) \\ w_t &= w(s) \end{aligned}$$

$$s' = \Gamma\left(s, \epsilon'\right).$$

- Firm's problem

$$V(k,s) = \max_{d,l,k'} \quad d + \beta \mathsf{E}\left[V\left(k',s'\right)\right]$$
$$\text{s.t.} \quad d + G\left(k',k\right) \leq A(s)\,F(k,l) - w(s)\,l$$

- From sequence problem and CRS: $V(k,s)$ is linear

$$V(k,s) = R(s)\,k$$

- f.o.c.

$$1 = \lambda$$

$$\lambda G_1\left(k', k\right) = \beta \mathsf{E}\left[V_1\left(k', s'\right)\right]$$

- envelope

$$V_1(k, s) = \lambda\left(A(s) F_1(k, l) - G_2\left(k', k\right)\right)$$

1.3.1 Marginal q

$$q^m(s) = G_1\left(k', k\right) = \beta \mathsf{E}\left[V_1\left(k', s'\right)\right]$$

$$q^m(s) = G_1\left(\frac{k'}{k}, 1\right)$$

- one-to-one correspondence between q^m and investment

- it is also true that investment equalizes marginal return on capital to cost of funds

- $$\frac{\beta \mathsf{E}\left[V_1\left(k', s'\right)\right]}{q^m\left(s\right)} = 1$$

$$\frac{\mathsf{E}\left[\left(A\left(s'\right) F_1\left(k', l'\right) - G_2\left(k'', k'\right)\right)\right]}{q^m\left(s\right)} = 1/\beta$$

where $1/\beta$ interest rate

1.3.2 Average q

Define the value of the firm: total present value of future claims

$$p_t = \mathsf{E} \sum_{j=1}^{\infty} \beta^j d_{t+j}$$

$$p(k,s) = V(k,s) - d$$

From envelope+linearity of V function

$$V(k,s) = \left[A(s) F_1(k,l) - G_2\left(k',k\right)\right] k$$

From budget constraint

$$\begin{aligned} d &= A(s) F(k,l) - w(s) l - G\left(k',k\right) \\ &= A(s) F_1(k,l) k - G_1\left(k',k\right) k' - G_2\left(k',k\right) k \end{aligned}$$

$$p(k,s) = G_1\left(k', k\right) k'$$

Ratio of firms' value to the capital invested

$$q(s) = \frac{p(k,s)}{k'} = G_1\left(k', k\right) = q^m(s)$$

- average $q =$ marginal q

- average q sufficient statistic for investment

1.3.3 Used capital market

Market for used capital q^o

$$d + G\left(k', k^o\right) + q^o k^o \leq A(s) F(k, l) - w(s) l + q^o k$$

- Same as equilibrium above with

$$q^o = -G_2\left(k', k\right)$$

- Total liquidation value of the firm

$$A(s) F(k, l) - w(s) l + q^o k = R(s) k$$

- Total investment cost

$$G\left(k', k^o\right) + q^o k^o = q^m k'$$

- Then compact program

$$\begin{aligned} V(k, s) = \max \quad & d + \beta \mathsf{E}\left[V\left(k', s'\right)\right] \\ \text{s.t.} \quad & d + q^m(s)\, k' \leq R(s)\, k \end{aligned}$$

- $R(s)$ gross return on investment per unit of capital

- $q^m(s)$ shadow cost of new capital

- Immediate

$$V(k,s) = R(s)\,k$$

$$p(k,s) = R(s)\,k - d = q^m(s)\,k'$$

and

$$\frac{\mathsf{E}\left[R(s')\right]}{q^m} = \frac{1}{\beta}$$

1.4 Empirical performance of q theory

- Run regressions

$$(I/K)_{j,t} = ... + a_1 q_{j,t} + a_2 (CF/K)_{j,t} + e_{j,t}$$

- Approach I: Fazzari, Hubbard, Petersen (1988)

Discussion: maybe q is mis-measured and CF better predictor of future returns

- Gilchrist and Himmelberg (1995)

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- Construct artificial value of the firm

$$\tilde{p}_t = \mathsf{E} \sum_{j=1}^{\infty} d_{t+j}$$

and use it to get

$$\tilde{q}_t = \frac{\tilde{p}_t}{k_{t+1}}$$

- results do not change

Approach II: identify exogenous shocks to internal funds

- Blanchard, Lopez-de-Silanes, and Shleifer (1994), Lamont (1997), Rauh (2006)

- components of cash flow that are exogenous to investment opportunities

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