

14.471/Fall 2012: PS 2: Solutions

October 30, 2012

Question 1

You are invited to a small tropical island nation to become a tax policy consultant, with particular responsibility for analyzing tax-induced distortions in labor supply. Sensing the opportunity for an interesting vacation, you accept. The republic's finance minister faxes you a copy of the current tax schedule, defined over total daily (labor plus non-labor) income, which is

$$T(y) = \begin{cases} 0 & y < 60 \\ 0.5(y - 60) & y \geq 60. \end{cases}$$

After arriving in the republic, you discover it is smaller than you thought. The Complete Population Survey (CPS) contains only four observations. Undaunted, you proceed with your proposal to estimate a linear hours-of-work model for the republic's population:

$$h = \alpha + \beta y_v + \gamma w,$$

where h denotes daily hours, y_v is virtual income, and w is the household's after-tax wage rate. The data you receive are shown in the table below.

Household	Non-labor income	Hours worked	Total pre-tax income
1	0	15.15	45.45
2	0	10.00	60.00
3	10	8.25	92.50
4	20	7.25	92.50

(a) Use these data to estimate (α, β, γ) . You should be able to solve for the parameters exactly (i.e., with no error terms in the hours equation) using three data points. (Hint: Remember to check whether each household is on a linear segment of the budget set, or at a kink point.)

Note that household two is at the kink induced by the exemption threshold. We will ignore it in estimating the model. Household 1 has total income less than the exemption amount and therefore will not be taxed. The pre-tax wages are

$$\begin{cases} w_1 = \frac{45.45}{15.15} = 3 \\ w_3 = \frac{82.5}{8.25} = 10 \\ w_4 = \frac{72.5}{7.25} = 10 \end{cases}$$

The after-tax wages equal:

$$\begin{cases} w_1^T = 3 \\ w_3^T = \frac{w_3}{2} = 5 \\ w_4^T = \frac{w_4}{2} = 5 \end{cases}$$

Virtual income is the post-tax income that the individual would get if his earnings were equal to zero was allowed to stay on the "virtual" linear schedule. For household 1, its virtual income is $y_1^v = 0$. Household three is taxed. Its total after-tax income is $y_3 = 92.5 - 0.5 * 32.5 = 76.25$. Virtual income is then $y_3^v = y_3 - h_3 w_3^T = 76.25 - 5 * 8.25 = 35$. For household four we get that the virtual income is $y_4^v = 40$. The data for "estimation" is then

$$\begin{bmatrix} 1 & 0 & 3 \\ 1 & 35 & 5 \\ 1 & 40 & 5 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 15.15 \\ 8.25 \\ 7.25 \end{bmatrix}.$$

Conveniently, we don't even need to use anything we learned in metrics, because the system has as many parameters as observations. The solution is given by

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 15 \\ -0.2 \\ 0.05 \end{bmatrix}.$$

Hence, labor supply for household not at the kink is given by:

$$h = \alpha + \beta y_v + \gamma w = 15 - 0.2y^v + 0.05w^T$$

Finally, we verify that household two's behavior is consistent with the model. With a virtual income of zero and after-tax wage of 6, household two would like to work

$$h = 15 - 0.2 \times 0 + 0.05 \times 6 = 15.3$$

hours, which would result in an income of 91.8, well in excess of the exemption amount. However, with virtual income of 30 and after-tax wage of 3 household two would like to work

$$h = 15 - 0.2 \times 30 + 0.05 \times 3 = 9.15$$

hours, which would result in an income of 54.9, less than the exemption amount. Thus household two's behavior is consistent with the estimated model.

(b) Compute the total amount of revenue currently collected by the tax system, and find the lump sum tax (equal across all households) that would be needed to raise the same amount of revenue. For household 1, find the equivalent variation of shifting to this tax.

Total revenue is $T(y_3) + T(y_4) = 2 \times 0.5 \times (92.5 - 60) = 32.5$. The lump sum tax required to raise the same amount of revenue is simply $32.5/4 = 8.125$. Recall that the equivalent variation is the amount of money an individual would be willing to pay to avoid a policy change. Since the only difference between the nonlinear income tax and the lump sum tax for household one is the lump sum, the individual would be willing to pay an amount equal to the lump sum to avoid it, i.e. 8.125.

Question 2

(a) Let us guess (and later verify) that consumer 1 will work less than 0.5 units and hence face the 25% marginal tax rate. Consumer 1 solves:

$$\max_{J,C,L} \log(J) + \log(C) + \log(1 - L)$$

s.t.

$$J + C \leq (1 - \tau)(Lw + e) = \frac{3}{4}(L + 1)$$

From the FOC's and the B.C., we get $L = \frac{1}{3} < \frac{1}{2}$, $J = C = \frac{1}{2}$.

For type 2, guess that he will be facing the higher tax schedule (i.e. $L > \frac{1}{4}$). Hence his after tax income will be $\frac{3}{4}(1 + \frac{1}{2}) + (L - \frac{1}{4}) = \frac{7}{8} + L$. Consumer 2 solves:

$$\max_{J,C,L} \log(J) + \log(C) + \log(1 - L)$$

s.t.

$$J + C \leq \frac{7}{8} + L$$

From the FOC's and the B.C., we get $L = \frac{3}{8} > \frac{1}{4}$, $J = C = \frac{5}{8}$.

(b)

Revenue from type 1 is:

$$\frac{1}{4}(1 + \frac{1}{3}) = \frac{1}{3}$$

Revenue from type 2 is:

$$\frac{1}{4}(1 + \frac{1}{2}) + \frac{1}{2}(\frac{1}{8} * 2) = \frac{1}{2}$$

The per capita tax revenue is $\frac{5}{12}$

(c)

Taxable income is now $1 + wl - C$.

Let us guess (and later verify) that consumer 1 will face the 25% marginal tax rate. Consumer 1 solves:

$$\max_{J,C,L} \log(J) + \log(C) + \log(1 - L)$$

s.t.

$$J + \frac{3}{4}C \leq \frac{3}{4}(L + 1)$$

From the FOC's and the B.C., we get $L = \frac{1}{3}$, $J = \frac{1}{2}$ and $C = \frac{2}{3}$. And thus verify our conjecture: taxable income $1 + wl - C < \frac{3}{2}$.

Let us guess (and later verify) that consumer 2 now also faces the 25% marginal tax rate (since his taxable income drops in consumption). Consumer 2 solves:

$$\max_{J,C,L} \log(J) + \log(C) + \log(1 - L)$$

s.t.

$$J + \frac{3}{4}C \leq \frac{3}{4}(2L + 1)$$

From the FOC's and the B.C., we get $L = \frac{1}{2}$, $J = \frac{3}{4}$ and $C = 1$. And thus verify our conjecture: taxable income $1 + wl - C < \frac{3}{2}$.

Revenue from type 1 is:

$$\frac{1}{4}\left(1 + \frac{1}{3} - \frac{2}{3}\right) = \frac{1}{6}$$

Revenue from type 2 is:

$$\frac{1}{4}(1 + 1 - 1) = \frac{1}{4}$$

The per capita tax revenue is $\frac{5}{24}$ which is a drop of 50% triggered by:

1. A substitution towards the cheaper deductible consumption good
2. A switch of type 2 individuals towards the lowest tax bracket (higher deductions reduce taxable income)

On the other hand, because of the lower marginal rate, the high-skilled worker works more hours (counterbalancing the other 2 effects somewhat). Finally, we also have income effects resulting from lower taxes.

(d) The static revenue cost is

$$\begin{aligned} \Delta R &= -\frac{1}{2} \left(\frac{1}{4} \times \frac{1}{2} + \frac{1}{2} \times \frac{5}{8} \right) \\ &= -\frac{1}{2} \left(\frac{1}{8} + \frac{5}{16} \right) \\ &= -\frac{7}{32} = -0.219. \end{aligned}$$

The actual revenue cost was -0.208, which is slightly less than the static estimate. In this case the static approximation is reasonable, but in general this may not be the case. The static estimate picks up only the mechanical revenue loss, and does not include the effects from behavioral responses.

Question 3

(a)

We have the following tax schedule:

$$T(AGI) = \tau TI = \tau \max(0, AGI - 8,950)$$

Hence the marginal rates jump at four “kink” points:

- at $AGI = 8,950$ from 0 to 10%.
- at $AGI = 16,975$ from 10 to 15%.
- at $AGI = 32,550 + 8,950 = 41,500$ from 15 to 25%.
- at $AGI = 78,850 + 8,950 = 87,800$ from 25% to 28%.

We will thus test for bunching at those 4 kink points by adding variables called kink1, kink2, kink3 and kink4 for each bins in which these 4 points fall.

Figures (1) and () illustrate that the fit of the quadratic is decent while the cubic seems to perform even a little better. Table (1) shows that the only kink where we can reject the null of “no bunching” is the first kink. This is not surprising since this is the kink between paying taxes and no taxes. The level of significance drops from 0.01 to 0.05 when we move from the quadratic (column (1)) to the cubic specification (column (2)).

Figure 1: Quadratic fit

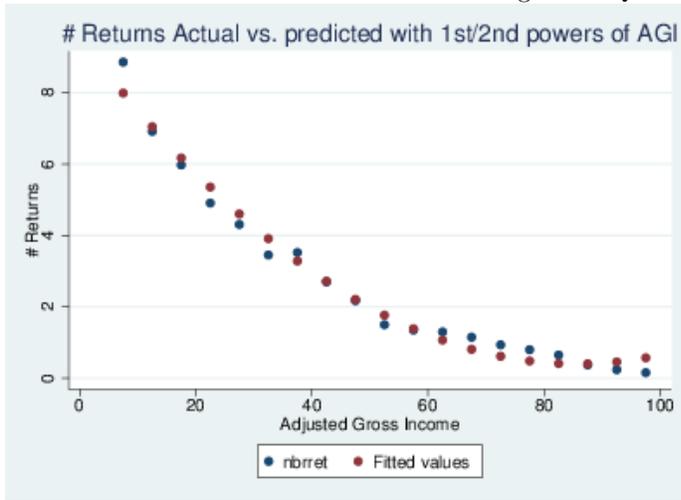


Figure 2: Cubic fit

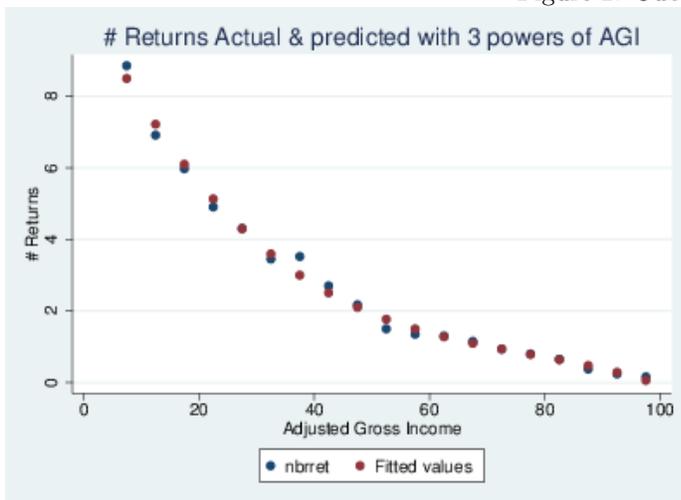


Table 1: Quadratic and Cubic regressions

	(1)	(2)
	nbrret	nbrret
AGI	-0.1827*** (0.0128)	-0.2796*** (0.0303)
AGI squared	0.0010*** (0.0001)	0.0030*** (0.0006)
kink1	1.4865*** (0.3561)	0.8453** (0.3236)
kink2	0.1820 (0.3055)	0.0706 (0.2267)
kink3	-0.0310 (0.2795)	0.1406 (0.2114)
kink4	-0.0121 (0.2885)	-0.0831 (0.2127)
AGI cube		-0.0000*** (0.0000)
Constant	8.6800*** (0.3220)	9.9410*** (0.4436)
<i>N</i>	19	19

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

(b)

Identifying bunching with aggregate data is difficult because (i) the bins are relatively large, (ii) the number of observations is limited (iii) data on taxable income are coarse, (iv) non-convexities of other social programs (e.g. EITC, means-tested benefits, ...) are omitted and (v) the number of kinkpoints is relatively large compared to the number of bins. Aggregate data might be helpful if (i) kink points fell on the boundaries of the cells, (ii) we had data on taxable income instead of AGI and/or if (iii) the bins were finer.

Question 4

(a)

The lifetime budget constraint is:

$$C_1 + \frac{C_2}{1+r} + \beta(A) \leq (1 - \tau_1)(Y_1 - A) + \frac{(1 - \tau_2)}{1+r}(Y_2 + A)$$

(b)

The optimal level of revenue shifting does not depend on the utility function because it does not affect utility directly. The optimal level of shifting is chosen to maximize the present value of wealth $(1 - \tau_1)(Y_1 - A) + \frac{(1 - \tau_2)}{1+r}(Y_2 + A) - \beta(A)$ which yields the following FOC:

$$\beta'(A) = \frac{(1 - \tau_2)}{1+r} - (1 - \tau_1)$$

(c)

From the FOC, we get:

$$A = \frac{1}{2\gamma} ((1 - \tau_2) - (1 - \tau_1))$$

The elasticity of tax avoidance is then given by:

$$\epsilon_A = -\frac{1 - \tau_1}{(1 - \tau_2) - (1 - \tau_1)}$$

Revenues are equal to:

$$R = \tau_1(Y_1 - A) + \tau_2(Y_2 + A)$$

Then:

$$\frac{\partial R}{\partial \tau_1} = Y_1 - 2A = Y_1 - \frac{\tau_1 - \tau_2}{\gamma}$$

which has an ambiguous sign. Indeed, raising the tax in the early period:

1. Has a mechanic positive effect on tax revenues (at constant reported income)
2. Has a negative effect by incentivizing to shift income from the early high-tax period to the late low-tax period.

The relative importance of the 2nd effect will depend on how much tax evasion is occurring which depends on the marginal cost (see γ) and benefit (see $\tau_1 - \tau_2$) of evading.

But:

$$\frac{\partial R}{\partial \tau_2} = Y_2 + 2A$$

which has an positive sign. Indeed, raising the tax in the early period:

1. Has a mechanic positive effect on tax revenues (at constant reported income)
2. Has a positive effect by disincentivizing to shift income from the early high-tax period to the late low-tax period.

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