

14.471: Fall 2012: Recitation 10: Diamond (1980): Income Taxation with fixed hours of work.

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Question: What are the trade-offs of income tax systems when the number of hours of work is fixed?

1 Quick Motivation

- Empirical distributions of labor show strong clustering around +/-1800 hours/year (cfr. referee report Heim (2010))
- Extensive margin is particularly relevant for the poor and the old (fixed costs of work, different wage rates for full time and part time work)
- Real world is combination of intensive margin (Mirrlees) and extensive margin (Diamond) as in Saez

2 General Model

- Number of hours of work is either 1 or zero -> now working reveals my type!¹
- Utility over consumption c and disutility of labor $my = m$
- Marginal product of labor/skill n
- For a given skill level n , a consumption level $c(n)$ and an unemployment benefit b , the cut-off disutility of labor m^* is given by:

$$u(b, 0) = u(c(n), m^*(n)) \quad (1)$$

- IC is different from intensive margin case:
 - We are not comparing with allocation of neighbor
 - Now comparing with unemployed
- Utilitarian welfare maximization amounts to choosing a consumption schedule $c(n)$ and an unemployment benefit b
 - to maximize the sum of utilities of workers and non-workers at different skill and disutility levels
 - where the net resources A are available for consumption;

¹We assume agents will not destroy output in case consumption would drop with skill levels. In that ruled out scenario, skills would not be revealed.

$$\begin{aligned}
& \max_{c(n), b} \int_{\underline{n}}^{\bar{n}} \left[\int_{\underline{m}(n)}^{m^*(c(n), b)} U(c(n), m) f(m, n) dm + \int_{m^*(c(n), b)}^{\bar{m}(n)} U(b, 0) f(m, n) dm \right] dn \\
& \text{s.t.} \quad \int_{\underline{n}}^{\bar{n}} \left[\int_{\underline{m}(n)}^{m^*(c(n), b)} \{c(n) - n\} f(m, n) dm + \int_{m^*(c(n), b)}^{\bar{m}(n)} b f(m, n) dm \right] dn = A \quad (2)
\end{aligned}$$

- By assumption $c(n)$ only affects those of skill n (because no intensive margin)
- Forming a Lagrangian, we can differentiate wrt $c(n)$ (focusing on skills with both workers and non-workers):

$$\frac{\partial L}{\partial c(n)} = \left[\int_{\underline{m}(n)}^{m^*(n)} \{U_1(c(n), m) - \lambda\} f(m, n) dm \right] - \lambda (c(n) - n - b) \frac{\partial m^*(n)}{\partial c} f(m^*(n), n) = 0 \quad (3)$$

- Intuition:
 - Gain from switching a unit of consumption to workers with skill n net of resource cost λ : $U_1(c(n), m) - \lambda$
 - Cost in net resources of the induced increase in the number of workers ($\frac{\partial m^*(n)}{\partial c}$ being positive)
- If the return to working $c - b$ is smaller than the output produced n , then the marginal utility of consumption must be smaller than λ
- The optimal level of consumption $c(n)$ only depends on the rest of the economy through b and λ
 - 2 economies with *different* skill distributions but with same conditional disutility of labor distribution for a given skill level n , unemployment benefits and shadow price of resources \rightarrow same tax structure
- Differentiate wrt unemployment benefit:

$$(U_1(b, 0) - \lambda) \int_{\underline{n}}^{\bar{n}} \int_{m^*}^{\bar{m}(n)} f(m, n) dm dn = \lambda \int_{\underline{n}}^{\bar{n}} (c(n) - n - b) \frac{\partial m^*(n)}{\partial b} f(m^*(n), n) dn \quad (4)$$

- Intuition:
 - LHS: Gain from switching a unit of consumption to non-workers (net of resource cost)
 - RHS: Cost in net resources of the induced decrease in work at all skill levels (where there is an internal disutility work cut-off)
- Assume that $U_1(b, 0) - \lambda > 0$ and then since $\frac{\partial m^*(n)}{\partial b} < 0$, we have that the RHS of (4) is positive:
 - Aggregating, the workers have a negative net cost of resources $(c(n) - n - b) < 0$ BUT this does not have to be true at every working skill level
 - If we increase b , then $U_1(b, 0)$ will drop and since $\frac{\partial m^*(n)}{\partial b} < 0$, the higher number of non workers will represent a resource cost \rightarrow Consumption of non workers is limited by disincentive effect of discouraging work
- Consumption does not have to increase with income (will get this result later by assuming same distribution of disutility of labor):
 - Professors might have very high marginal products and low disutilities of work and hence it might not be necessary to pay them very much to induce work

- But, if 2 groups of the same size have the same distribution of disutility of labor then the group with higher skills consumes at least as much as the other group. Proof by contradiction:
 - Assume that low-skilled group consumes more
 - Now switch high consumption to high-skilled group
 - This will not affect the number of individuals working (same disutility distribution)
 - This will not affect total utility
 - But output goes up because we have more higher skilled working and less relatively lower skilled working.

3 Additive utility and independence of skill and disutility

- Assume disutility distribution is the same for all skills:

$$f(m, n) = g(m)h(n),$$

- Assume utility is additive which (if utility is continuously differentiable) should be equivalent to :

$$U_{12} = 0$$

- Assume that optimum involves some non workers (difference with Ivan's notes)
- We will show that there will be 3 zones:
 - Below n_1 : no one works (assumption)
 - Between n_1 and n_2 : some persons work and **consumption is increasing in n**
 - Above n_2 : everybody works and **consumption is constant**
- Equations (3) and (4) become:

$$\frac{\partial L}{\partial c(n)} = [\{U_1(c(n), m) - \lambda\} G(m * (n))] - \left[\lambda (c(n) - n - b) \frac{\partial m * (n)}{\partial c} g(m * (n)) h(n) \right] = 0 \quad (5)$$

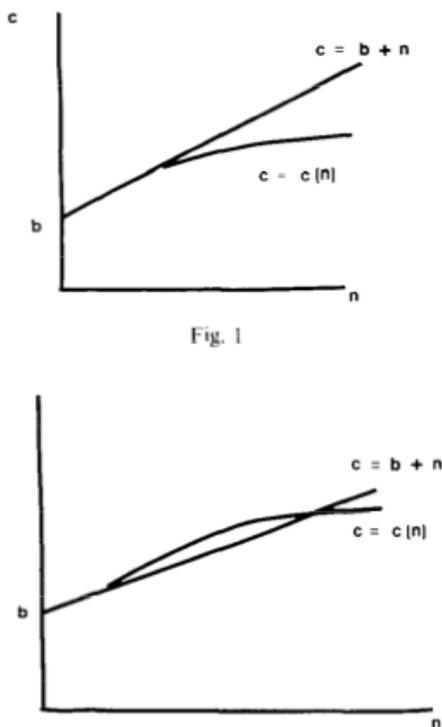
$$(U_1(b, 0) - \lambda) \int_{\underline{n}}^{\bar{n}} (1 - G(m * (n))) h(n) dn = \lambda \int_{\underline{n}}^{\bar{n}} (c(n) - n - b) \frac{\partial m * (n)}{\partial b} g(m * (n)) h(n) dn$$

- Why is consumption increasing in skills?
 - In equation (5) for $\frac{\partial L}{\partial c(n)}$, n only appears twice (i) via λn and (ii) via $h(n)$.
 - For $n' > n''$, we can evaluate $\frac{\partial L}{\partial c(n)}$ at the same consumption level c and use $\lambda n' > \lambda n''$ to show that:

$$h^{-1}(n') \frac{\partial L}{\partial c(n')} \Big|_{c(n')=c} \geq h^{-1}(n'') \frac{\partial L}{\partial c(n'')} \Big|_{c(n'')=c}$$
 - * Hence optimal consumption levels satisfy $c(n') \geq c(n'')$ (and strict if one of the two disutility cutoffs is interior)
 - Assuming that we would start from identical consumption for high and low skilled individuals, we would be better off increasing consumption of high types than of low types -> deviation from optimum.
 - With $c(n)$ increasing, we have by (1) that cut-off disutility of labor $m * (n)$ is also increasing:

- * The percentage of workers is higher among high skilled groups
- Why can we have both positive and negative marginal tax rates?
 - In (5), $\frac{G(m^*(n))}{g(m^*(n))}$ and $\frac{\partial m^*(n)}{\partial c}$ are both positive and hence the 2 following terms must have the same sign:
 - * The difference between the shadow value of resources and the marginal utility of consumption: $\lambda - U_1(c(n), m)$
 - * The tax on earned income: $n + b - c$
 - At medium skill levels n , we can have a negative tax on earned income $n + b - c < 0$ (a subsidy to work) iff $U_1(c(n), m) > \lambda$
 - But for high skill levels, there will be a positive tax.
 - Hence, we have 2 possibilities shown in figure (1):
 - * In figure 1.1, the medium skilled are sufficiently better off compared to the non-workers s.t. we can incentivize them to work without a work subsidy
 - * In figure 1.2, to incentivize medium skilled to work, we need a positive work subsidy but this work subsidy will be limited by the DWL it causes (the high skilled have to pay more taxes to pay for the subsidy of the middle skilled workers)

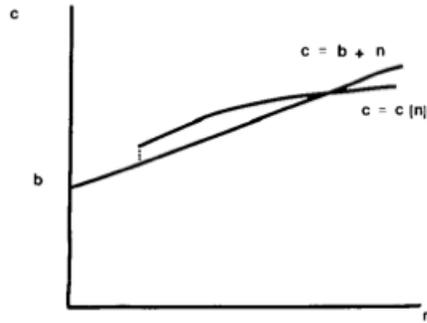
Figure 1: Positive and negative marginal tax rates are possible for medium skill workers



- Why can consumption $c(n)$ be discontinuous in skills?
 - Example in figure (2): Below the discontinuity, no one works (no work tax/subsidy)
 - At discontinuity, there is a work subsidy (some are induced to work at this skill)
 - * Start at skill level where the social return to inducing working becomes positive

- * If we increase skill marginally, then in order to incentivize to supply labor (which jumps from zero to one hour) need to offset jump in disutility of labor with jump in utility of consumption
- Justification of EITC: subsidize first, then higher marginal tax rates...

Figure 2: Discontinuity in consumption



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