# 14.471: Fall 2012: Recitation 7: Application of linear taxation to intertemporal taxation

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*Questions: How to set optimal taxes on labor and capital in a dynamic infinite horizon economy? How do the results change when we allow for heterogeneity across agents and poll taxes?*

## 1 Review of lecture notes

### 1.1 Results and main intuition:

We have 3 results:

1. **At the steady state, the tax on capital is zero.**
2. Initial tax on capital and bonds (lump sum expropriation but time inconsistency)
3. Labor tax smoothing

**Intuition for the zero capital tax result (see Salanié p. 140)** Assume that at the steady state capital is paid a before-tax return $r$ and its tax rate is $\tau$. Then capital taxation changes the relative price of consumption at date $t$ and date $t+T$ by a factor:

$$\left(\frac{1+r}{1+r(1-\tau)}\right)^T$$

Indeed, without taxes consuming 1 today (at $t$) costs $(1+r)^T$ in terms of forgone consumption at $T+t$. But with a tax consuming 1 today costs$(1+r(1-\tau))^T$ of forgone consumption at $T+t$. Hence if the tax rate $\tau$ is positive and when $T$ tends to infinity, then the relative price of consuming today becomes zero! We get massive intertemporal distortion and incentives to consume today. Note: in reality, real-world consumers do not live infinite lives.

### 1.2 Setup

- Preferences: $\sum \beta^t u(c_t, L_t)$
- Resource constraint:

$$c_t + g_t + k_{t+1} \leq F(k_t, L_t) + (1-\delta)k_t$$

- Define the agent's period-by-period budget constraint affected by linear taxes:

$$c_t + k_{t+1} + q_{t,t+1}B_{t+1} \leq (1-\tau_t)w_tL_t + R_tK_t + (1-\kappa_t^B)B_t$$

where $q_{t,t+1}$ is the price of a bond at $t$ paying out \$1 at $t+1$, $R_t = 1 + (1-\kappa_t)(r_t - \delta)$ is the gross after tax return net of depreciation, consumption is not taxed (normalization) and WLOG we have a zero tax on bonds after the 1st period $\kappa_t^B = 0 \quad t > 0$ (if we were to tax bonds, then the bond prices would simply drop).

- The no-ponzi conditions:
  - $q_{0,t} = q_{0,1}q_{1,2}...q_{t-1,t}$: the cost of buying 1 unit of consumption at $t$ should be the same whether you buy a bond with maturity $t$ today or buy 1 period-bonds which you then reinvest every period until $t$
  - $\lim q_{0,T}B_T \geq 0$ : The discounted value of bond holdings at infinity cannot be negative.
- Budget constraint government: $g_t + B_t \leq \tau_t w_t L_t + \kappa_t K_t r_t + q_{t,t+1}B_{t+1}$
- Define an equilibrium where (i) agents maximize given prices and taxes, (ii) firms chose labor and capital inputs to maximize profits, (iii) government satisfies its B.C. and (iv) good-, capital- and bond markets clear

## 1.3 Methodology/Primal Approach:

- Write the following NPV budget constraint for the agent as a function of bond prices, initial holdings, consumption, labor but without capital (tricks: Solve bond holdings $B_t$ forward and eliminate capital with "no-arbitrage"):

$$\sum_{t=0}^{\infty} q_{0,t}\left(c_t - (1-\tau_t)w_t L_t\right) \leq R_0 K_0 + (1-\kappa_0^B)B_0 \tag{1}$$

- 2 Tricks to get (1):
  - Solving $B_t$ forward:

1. Use the budget constraint of the agent at $t=0$:

$$c_0 + k_1 + q_{0,1}B_1 - (1-\tau_0)w_0L_0 - R_0K_0 \leq (1-\kappa_0^B)B_0 \tag{2}$$

2. Use the budget constraint of the agent at $t=1$ to solve $B$ forward and use that $\kappa_1^B = 0$

$$c_1 + k_2 + q_{1,2}B_2 - (1-\tau_1)w_1L_1 - R_1K_1 = B_1 \tag{3}$$

3. Combining (2) and (3) gives:

$$c_0 + k_1 + q_{0,1}\left\{c_1 + k_2 + q_{1,2}B_2 - (1-\tau_1)w_1L_1 - R_1K_1\right\} - (1-\tau_0)w_0L_0 - R_0K_0 \leq (1-\kappa_0^B)B_0$$

4. Repeating this we get that the NPV of "net consumption and investment above earnings" cannot exceed initial bond holdings:

$$\sum_{t=0} q_{0,t}\left\{c_t + k_{t+1} - (1-\tau_t)w_tL_t - R_tK_t\right\} \leq (1-\kappa_0^B)B_0 \tag{4}$$

  - Eliminating capital from (4) using no arbitrage:

1. Regrouping the terms in (4) with $k_2$ gives $q_{0,1}k_2 - q_{0,2}R_2k_2$
2. Now use that $q_{0,t} = \frac{q_{0,t-1}}{R_t}$ (the cost of obtaining 1 dollar at $t$ should not depend on whether you buy a long-term bond or buy a medium-term bond and reinvest it later in a 1 period bond)

- Combine the agent's NPV budget constraint (1) and his consumption and leisure FOC's into the implementability condition (trick: pricing equation $q_{0,t} = \beta^t$):

$$\sum_{t=0}^{\infty} \beta^t\left(u_{c,t}c_t + u_{L,t}L_t\right) \leq u_{c,t}\left(R_0K_0 + (1-\kappa_0^B)B_0\right) \tag{5}$$

1. FOC's wrt $c_t$ and $L_t$ when consumer maximizes $\sum \beta^t u(c_t, L_t)$ s.t. (1) give us:
   (a) $\beta^t u_c = \lambda q_{0,t}$
   (b) $\beta^t u_l = -\lambda q_{0,t}(1-\tau_t)w_t$
   (c) Combining we get the intratemporal condition for the agent:
   $$w_t(1-\tau_t) = -\frac{u_l}{u_c} \tag{6}$$
   (d) Using that $\frac{\beta^t u_c(c_t,L_t)}{q_{0,t}} = \lambda = \frac{\beta^{t+1} u_c(c_{t+1},L_{t+1})}{q_{0,t+1}}$ and the no-arbitrage condition, we get the intertemporal condition:
   $$\beta R_{t+1} u_c(c_{t+1}, L_{t+1}) = u_c(c_t, L_t) \tag{7}$$
2. Now you multiply (1) with $u_c$ to get (5)

- Let the planner maximize expected utility of the consumer s.t. implementability (5) and the resource constraint $F(k_t, L_t) + (1-\delta)k_t = c_t + g_t + k_{t+1}$ where $W(c, L; \mu) \equiv u(c, L) + \mu\left(u_c(c, l)c + u_L(c, L)L\right)$ and get intra and intertemporal "ish" conditions for $W$ where the social rate of return equals $R^*_{t+1} \equiv F_k(k_{t+1}, l_{t+1}) + 1 - \delta$
  - Creating the Lagrangian:
    * Indeed, the Lagrangian of the consumer (his objective function and his implementability condition) is:
    $$L = \sum \beta^t u(c_t, L_t) + \mu \left\{ \sum_{t=0}^{\infty} \beta^t \left(u_{c,t}c_t + u_{L,t}L_t\right) - u_{c,t}\left(R_0 K_0 + (1-\kappa_0^B)B_0\right) \right\}$$
    * Rewrite this as
    $$L = \sum \beta^t \left[u(c_t, L_t) + \mu\left(u_{c,t}c_t + u_{L,t}L_t\right)\right] - \mu u_{c,t}\left(R_0 K_0 + (1-\kappa_0^B)B_0\right)$$
    $$L = \sum \beta^t W(c, L; \mu) - \mu u_{c,t}\left(R_0 K_0 + (1-\kappa_0^B)B_0\right)$$
    * Hence the planner solves this maximization problem subject to the resource constraint:
    $$L^{planner} = \sum \beta^t W(c, L; \mu) - \mu u_c\left(R_0 K_0 + (1-\kappa_0^B)B_0\right) + \lambda_{RC}\left(F(k_t, L_t) + (1-\delta)k_t - c_t - g_t - k_{t+1}\right)$$
  - FOC's to the planner's maximization problem give:
    * Labor:
    $$\beta^t W_L = -\lambda_{RC} F_L$$
    * Capital:
    $$\lambda_{RC,t+1}\left(F_K + (1-\delta)\right) = \lambda_{RC,t}$$
    * Consumption:
    $$\beta^t W_{C,t} = \lambda_{RC,t}$$
  - Combining the planner's FOC conditions:
    * Combining the labor and consumption FOC's gives:
    $$\frac{W_L(c_t, l_t; \mu)}{W_C(c_t, l_t; \mu)} = -F_L(K_t, L_t) = -w_t$$

- ∗ Combining the capital and consumption FOC's gives:

$$\beta^t W_{C,t} = \beta^{t+1} W_{C,t+1} \left(F_K + (1-\delta)\right)$$

$$W_{C,t} = \beta W_{C,t+1} \left(F_K + (1-\delta)\right) = \beta W_{C,t+1} R^*_{t+1}$$

where $F_k(k_{t+1}, L_{t+1}) + 1 - \delta = R^*_{t+1}$ is the social rate of return

- Remember the intra- and intertemporal conditions (6) and (7) for the agent:

$$w_t(1-\tau_t) = -\frac{u_l}{u_c}$$

$$\beta R_{t+1} u_c(c_{t+1}, L_{t+1}) = u_c(c_t, L_t)$$

- Hence, we can combine the planner's and the agent's optimality conditions to get insights on optimal taxes:

$$1 - \tau_t = -\frac{u_L}{u_c}\frac{1}{w_t} = \frac{W_c}{W_L}\frac{u_L}{u_c} \tag{8}$$

$$\frac{R_{t+1}}{R^*_{t+1}} = \frac{u_c(c_t, l_t)}{u_c(c_{t+1}, l_{t+1})}\frac{W_c(c_{t+1}, l_{t+1}; \mu)}{W_c(c_t, l_t; \mu)} \tag{9}$$

### 1.4 Results

- At the steady state, the tax on capital is zero (i.e. $\kappa_t = 0$ ) since:

$$\frac{R_{t+1}}{R^*_{t+1}} = 1 = \frac{1 + (1-\kappa_t)(r_t - \delta)}{F_k(k_{t+1}, L_{t+1}) + 1 - \delta}$$

- Initial tax on capital and bonds (lump sum expropriation but time inconsistency)
- Labor tax smoothing: no special role for current $g_t$ conditional on current allocation but expenditures affect $\mu$

## 2 Highlights of Werning (2007): Extension to heterogeneous agents

### 2.1 Introduction

- Standard Ramsey model adopts a representative-agent framework and derives optimal taxes on labor and capital (Chamley-Judd) where the reason for distortionary taxation is the ruling out of lump-sum taxes.
- But poll taxes are realistic (e.g. tax deductions or transfers from welfare programs) and a more natural rationale for distortionary taxation is distributional concerns (Mirrlees 1971): for instance non observable differences in productivity.
- Here focus on linear taxation with a poll tax: summarize the labor-income tax schedule with the lump-sum tax $T_t$ and the slope or marginal tax rate $\tau_t$.

## 2.2 Differences and similarities in set-up (2)

### Differences

- Finite types $i$ with weight $\pi_i$ and with different preferences $U^i(c_t, L_t)$:
  - typically differences in productivity $U^i(c, L) = U(c, \frac{L}{\theta^i})$:
- Type of workers is private information
- Uncertainty captured by a publicly observed state $s_t$ where the probability of a history $s^t$ is denoted $Pr(s^t)$
- $p(s^t)$ is the Arrow-Debreu price of consumption in period $t$ after history $s^t$
- Allow for a lump-sum tax (poll tax)

### Similarity

- Definition of a competitive equilibrium

## 2.3 Differences in solution methodology

### Fictitious agent

- Also primal approach: formulate planning problem in terms of aggregate allocation that can be implemented with taxes and prices (remember: in dual tax rates and prices are not eliminated but are the planner's controls)
- With linear taxes, all workers face the same after-tax prices for consumption and labor:
  - marginal rates of substitution are equated across workers
  - all inefficiencies due to distortive taxation are confined to the determination of aggregate consumption and aggregate labor
- NEW: Equilibrium after-tax prices can be computed as if the economy were populated by a fictitious representative agent with the utility function

$$U^m(c, L, \varphi) \equiv \max_{\{c_i, L_i\}} \sum \varphi^i U^i(c^i, L^i)\pi_i$$

  where the weighted sums of individuals' consumptions and labor levels equal the aggregate levels.
- Then, compute fictitious agent's intertemporal and intratemporal optimality conditions, combine with budget constraint and get implementability condition

### Planning problem

- Set of competitive equilibrium defines a set of attainable lifetime utilities
- Planner:
  - choses aggregate levels of consumption, labor, capital, market weights and the lump-sum tax
  - wants to reach the northeastern frontier of the set of attainable lifetime utitlies
- More equal weights imply a more equal consumption allocation and hence higher tax rates
- Also "pseudo-Lagrangian"

### 2.4 Some differences in terms of results

- The Chamley-Judd zero capital tax result is quite robust
- NEW 1: Distortionary taxation is a redistribution mechanism (since poll taxes are allowed)
  - A positive tax rate makes high-skilled, rich workers pay more taxes than low-skilled poor workers
  - The optimal tax rate balances distributional concerns against efficiency
- NEW 2: In some cases initial wealth taxation is unnecessary. If all workers start with the same capital holdings, then the effect of the initial capital levy is equivalent to a lump-sum tax
- NEW 3: Since capital levies can be become unnecessary, the time inconsistency problem result is not robust.
- NEW 4: If skill distribution changes over time, then tax smoothing results could fail since the trade-off between efficiency and distributional concerns becomes time-varying

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