Workers ex ante identical.
no utility discounting
zero interest rate
intertemporally additive preferences
no age variation in preferences
work a zero-one decision.
No private savings
4 period model:
- period 1 – everyone works
- period 2 – probability θ of being able to work
- period 3 – no one works
- period 4 – probabilities π and π' of survival to this period for able/not able

Two assumptions: Moral hazard constraint (normality)
Want maximal work in period 2

Notation

V_j	Lifetime expected utility assuming j periods of planned work if able
$u[x_z]$	flow utility at age z if working
x_z	consumption when working in period z
$v[c_z]$	flow utility if not working and had career of length z
w	wage - the same at all ages

Moral hazard constraint (normality)

$$u[x]=v[c]\Rightarrow u'[x]<v'[c] \quad (1)$$

Labor supply

Two possible plans: work 1 period or work 2 if able:

$$V_1=u[x_1]+v[c_1]+v[c_1]+(\theta\pi+(1-\theta)\pi')v[c_1] \quad (2)$$

$$V_2=u[x_1]+\theta u[x_2]+(1-\theta)v[c_1]+\theta v[c_2]+(1-\theta)v[c_1]+\theta\pi v[c_2]+(1-\theta)\pi' v[c_1] \quad (3)$$

Equating lifetime utilities implies:

$$V_1 = V_2 \Rightarrow \\ u[x_2] + (1+\pi)v[c_2] = (2+\pi)v[c_1] \tag{4}$$

SWF maximization

$$\text{Max } V_2 = u[x_1] + \theta u[x_2] + (1-\theta)v[c_1] + \theta v[c_2] + (1-\theta)v[c_1] + \theta\pi v[c_2] + (1-\theta)\pi' v[c_1] \\ \text{s. t. } \mathrm{E} + x_1 - w + \theta(x_2 + (1+\pi)c_2 - w) + (1-\theta)(2+\pi')c_1 = 0 \\ V_2 \geq V_1 \tag{5}$$

Two possibilities: incentive compatibility constraint binds or not. If not, would violate the combination of want work in period 2 and the moral hazard constraint. Therefore it binds.

FOC

$$u'[x_1] = \lambda \\ \theta u'[x_2] = \lambda\theta - \mu u'[x_2] \\ (1-\theta)(2+\pi')v'[c_1] = \lambda(1-\theta)(2+\pi') + \mu(2+\pi')v'[c_1] \\ \theta(1+\pi)v'[c_2] = \lambda\theta(1+\pi) - \mu(1+\pi)v'[c_2] \\ u[x_2] + (1+\pi)v[c_2] = (2+\pi)v[c_1] \\ \mathrm{E} + x_1 - w + \theta(x_2 + (1+\pi)c_2 - w) + (1-\theta)(2+\pi')c_1 = 0 \tag{6}$$

From the FOC, we have $u'[x_2] = v'[c_2]$. From this and the MH and work assumptions, we have $c_2 > c_1$ and $x_2 > x_1$.

Note that workers would save in the first period if they could

$$u'[x_1]-\theta u'[x_2]-(1-\theta)v'[c_1]=\lambda-(\lambda\theta-\mu u'[x_2])-(\lambda(1-\theta)+\mu v'[c_1])=\mu(u'[x_2]-v'[c_1])<0 \quad (7)$$

if they did, they would then retire early. Thus the same model with private savings has a more stringent incentive to work for the government to create.

Notice that π' does not appear in the incentive compatibility constraint.

The implicit tax on work, T, is

$$T=w-x_2-(1+\pi)c_2+(2+\pi)c_1 \quad (8)$$

To sign this we will need to examine the implications of the desirability of maximal work. Workers are indifferent to work. The government can have some of them retire. If that is not optimal that gives us an inequality. Assume some work in the second period. Then the impact of some more work in the second period (if not everyone is working) is

$$\{u[x_2]+(1+\pi)v[c_2]-(2+\pi)v[c_1]\}+\lambda\{w-x_2-(1+\pi)c_2+(2+\pi)c_1\}\geq 0 \quad (9)$$

The first $\{\ \}$ is zero by the binding of the incentive compatibility constraint, while the second is the implicit tax on work. If having everyone work is strictly better than having some able retire early, then this is strictly positive and so is the implicit tax.