

# 14.54 International Economics

## Handout 5

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### 1 Current Account Adjustment and Exchange Rates

Now we enrich the model with 2 goods: a Home good and Foreign good.

Let the countries be US and Europe.

Consumption of the two goods is  $C_H, C_F$  at home and  $C_H^*, C_F^*$  abroad.

Production of home good is  $Y_H$ . Their prices are  $P_H$  (in dollars) and  $P_F$  (in euros). The nominal exchange rate is  $\mathcal{E}$ . The price of the foreign good in dollars is  $\mathcal{E}P_F$ .

The current account (in dollars) is

$$\begin{aligned} CA_t &= EXP_t - IMP_t \\ &= P_{H,t}(Y_{H,t} - C_{H,t}) - \mathcal{E}_t P_{F,t} C_{F,t} + r_t B_t \end{aligned}$$

where  $r_t$  is the return on net foreign assets and  $B_t = A_t - L_t$  is net foreign assets.

Suppose that the nominal price levels  $P_{H,t}$  and  $P_{F,t}$  are constant and equal to 1.

The consumer problem is now to choose how much to spend in periods 1 and 2 in each of the goods, home and foreign, that is to choose the vector  $(C_{H,1}, C_{F,1}, C_{H,2}, C_{F,2})$ . Suppose the utility function of the consumer is  $U(C_{H,1}, C_{F,1}) + \beta U(C_{H,2}, C_{F,2})$ , then the consumer's problem is:

$$\begin{aligned} \max \quad & U(C_{H,1}, C_{F,1}) + \beta U(C_{H,2}, C_{F,2}) \\ \text{s.t.} \quad & B_1 = Y_{H,1} - (C_{H,1} + \mathcal{E}_1 C_{F,1}) \\ & C_{H,2} + \mathcal{E}_2 C_{F,2} = Y_{H,2} + (1+r) B_1 \end{aligned} \tag{1}$$

where  $r$  is the interest rate (in dollars) and  $B_1$  is the financial position of country 1 (also, in dollars).

The general approach to this problem is the usual one: we have 3 prices  $(\mathcal{E}_1, \mathcal{E}_2, r)$ , and we can find the demand for the 4 goods  $(C_{H,1}, C_{F,1}, C_{H,2}, C_{F,2})$ . Then we can study the autarky equilibrium and find the prices  $(\mathcal{E}_1^a, \mathcal{E}_2^a, r^a)$  such that the economy does not trade:

$$(C_{H,1}, C_{F,1}, C_{H,2}, C_{F,2}) = (Y_{H,1}, 0, Y_{H,2}, 0).$$

Then we put together the two countries and find the world equilibrium prices  $(\mathcal{E}_1, \mathcal{E}_2, r)$  such that the four markets are clearing:

$$(C_{H,1}, C_{F,1}, C_{H,2}, C_{F,2}) + (C_{H,1}^*, C_{F,1}^*, C_{H,2}^*, C_{F,2}^*) = (Y_{H,1}, 0, Y_{H,2}, 0) + (0, Y_{F,1}^*, 0, Y_{F,2}^*).$$

However, this problem is not very easy to manage. To make the problem more manageable and derive useful expressions to think about the world equilibrium the idea is to split the problem in two: the financial side and the real side.

Think that the consumer solves problem (1) in two stages.

First he chooses how much to spend in each period (intertemporal problem), i.e. the expenditure levels  $EX_1$  and  $EX_2$ , where

$$\begin{aligned} EX_1 &= C_{H,1} + \mathcal{E}_1 C_{F,1}, \\ EX_2 &= C_{H,2} + \mathcal{E}_2 C_{F,2}. \end{aligned}$$

Then he decides how much to spend in each of the two goods in each separate period (spending composition problem).

The second problem in period  $t$  gives us

$$\begin{aligned} V(EX_t, \mathcal{E}_t) &= \max_{C_{H,t}, C_{F,t}} U(C_{H,t}, C_{F,t}) \\ s.t. \quad & C_{H,t} + \mathcal{E}_t C_{F,t} = EX_t. \end{aligned}$$

$V(EX_t, \mathcal{E}_t)$  is an “indirect utility function,” it gives the utility of a consumer that can spend  $EX_t$  if the exchange rate is  $\mathcal{E}_t$ .

The intertemporal problem is then

$$\begin{aligned} & \max V(EX_1, \mathcal{E}_1) + \beta V(EX_2, \mathcal{E}_2) \\ s.t. \quad B_1 &= Y_{1,H} - EX_1, \\ EX_2 &= Y_{2,H} + (1+r)B_1. \end{aligned}$$

Note one thing: this problem looks a lot like the problem that we solved in the case of one good.

A special case that can be solved easily is the case where

$$U(C_{H,t}, C_{F,t}) = \alpha \log C_{H,t} + (1 - \alpha) \log C_{F,t}$$

in this case

$$V(EX_t, \mathcal{E}_t) = \alpha \log \alpha + (1 - \alpha) \log (1 - \alpha) + \log (EX_t) - (1 - \alpha) \log \mathcal{E}_t \quad (2)$$

and the intertemporal problem is:

$$\begin{aligned} \max \quad & \log (EX_1) + \beta \log (EX_2) \\ \text{s.t.} \quad & B_1 = Y_{1,H} - EX_1, \\ & EX_2 = Y_{2,H} + (1 + r) B_1. \end{aligned}$$

(remember: the constants in the utility function do not matter for finding the optimal levels of expenditure) and this is exactly the problem we solved above.

**Exercise 1** Check that this (2) is correct. Find  $V(EX_t, \mathcal{E}_t)$  in the case where the utility function is instead given by:

$$U(C_{H,t}, C_{F,t}) = \left(1 - \frac{1}{\eta}\right) \left(C_{H,t}^{1-\frac{1}{\eta}} + C_{F,t}^{1-\frac{1}{\eta}}\right),$$

where  $\eta$  is a positive constant.

### The financial side

The problem gives us the optimal expenditure path

$$\begin{aligned} EX_1 &= \frac{1}{1 + \beta} \left[ Y_{1,H} + \frac{1}{1 + r} Y_{2,H} \right] \\ EX_2 &= \frac{\beta(1 + r)}{1 + \beta} \left[ Y_{1,H} + \frac{1}{1 + r} Y_{2,H} \right] \end{aligned}$$

This looks similar to the problem we solved before.

The current account (in dollars) of the home country is given by

$$CA_1 = B_1 = Y_{1,H} - \frac{1}{1 + \beta} \left[ Y_{1,H} + \frac{1}{1 + r} Y_{2,H} \right].$$

### The real side (the return of the Transfer Problem)

Once we have figured the optimal path for expenditure we want to figure out the demand for the two goods in period 1.

Remember that

$$EX_1 = Y_{H,1} - CA_1.$$

Now we have the demand of the home good

$$C_{H,1} = \alpha (Y_{H,1} - CA_1)$$

We can do the same for the foreign country and find

$$C_{H,1}^* = (1 - \alpha) (\mathcal{E}_1 Y_{F,1}^* - CA_1^*).$$

Equilibrium in the financial markets requires, as usual, that

$$CA_1 + CA_1^* = 0,$$

so  $CA_1 > 0$  is very much like a "transfer".

Assume now that there is home bias in consumption, that is

$$\alpha > 1/2.$$

then the equilibrium in the goods markets in period 1 requires

$$\alpha Y_{H,1} + (1 - \alpha) \mathcal{E}_1 Y_{F,1}^* + (2\alpha - 1) (-CA_1) = Y_{H,1}.$$

### **From financial autarky to open world capital markets.**

Now we make the following experiment. Start from a situation of *financial autarky*. Countries cannot borrow or lend from each other. However, there is free trade of goods. The financial markets are closed but the goods markets are open.

This means that the current account has to be zero in all periods:  $-CA_1 = 0$ , that is we want to have trade balance in each period. To put it another way the home country has to finance all its imports with exports:

$$\mathcal{E}_1 C_{F,1} = Y_{H,1} - C_{H,1}.$$

To put it yet another way, this means that expenditure always has to equal income

$$EX_1 = Y_{H,1}.$$

Then we obtain the equilibrium exchange rate in periods 1 and 2:

$$\alpha Y_{H,t} + (1 - \alpha) \mathcal{E}_t^{fa} Y_{F,t} = Y_{H,t}$$

$$\mathcal{E}_t^{fa} = \frac{Y_{H,t}}{Y_{F,t}}$$

the exchange rate depends on the relative supply of the two goods period by period (*fa* stands for financial autarky).

Now suppose the financial markets open. Suppose that in the new equilibrium country Home is borrowing at date 1 and repaying at date 2. This means that  $-CA_1 = B_1 < 0$  and

$$\begin{aligned} EX_1 &= Y_{1,H} + (-CA_1), \\ EX_2 &= Y_{2,H} - (1+r)(-CA_1), \\ EX_1^* &= \mathcal{E}_1 Y_{1,F}^* - (-CA_1), \\ EX_2^* &= \mathcal{E}_2 Y_{2,F}^* + (1+r)(-CA_1). \end{aligned}$$

Now we can derive the equilibrium in the two periods and get

$$\alpha Y_{H,1} + (1 - \alpha) \mathcal{E}_1 Y_{F,1} + (2\alpha - 1) (Y_{H,1} - \alpha Y_{H,2} + (1 - \alpha) \mathcal{E}_2 Y_{F,2} - (2\alpha - 1)(1 + r)(-CA_1)) = Y_{H,2}$$

$$\mathcal{E}_1 = \frac{Y_{H,1}}{Y_{F,1}} - \frac{(2\alpha - 1)}{(1 - \alpha)} \frac{-CA_1}{Y_{F,1}} < \mathcal{E}_1^{fa} \quad (3)$$

$$\mathcal{E}_2 = \frac{Y_{H,2}}{Y_{F,2}} + \frac{(2\alpha - 1)}{(1 - \alpha)} (1 + r) \frac{-CA_1}{Y_{F,1}} > \mathcal{E}_2^{fa} \quad (4)$$

We can summarize this as follows:

- When we open capital markets and country Home runs a current account deficit ( $CA_1 < 0$ ) this brings about:
  - (i) An appreciation of the currency in the short run.
  - (ii) A depreciation in the long run.

**Exercise 2** Suppose that the countries have identical endowments, constant over time  $Y_{H,1} = Y_{H,2} = Y_{F,1}^* = Y_{F,2}^* = \bar{Y}$  and suppose that  $\beta < \beta^*$ .

(i) Show that the savings of country foreign are given by

$$B_1^* = \mathcal{E}_1 \bar{Y} - \frac{1}{1 + \beta^*} \left[ \mathcal{E}_1 \bar{Y} + \frac{1}{1 + r} \mathcal{E}_2 \bar{Y} \right].$$

(ii) Substitute (3) and (4) and use the fact that  $B_1 = -B_1^*$  to write  $B_1^*$  as a function of  $r$  alone (Hint: you will get something like

$$\frac{B_1^*}{\bar{Y}} = \phi \frac{1}{1 + \beta^*} \left[ \beta^* - \frac{1}{1 + r} \right]$$

where  $\phi$  is a constant that depends on  $\alpha$ , if  $\alpha = 1/2$  you should check that  $\phi = 1$ .)

(iii) Show that the savings of country home are given by

$$\frac{B_1}{\bar{Y}} = \frac{1}{1 + \beta} \left( \beta - \frac{1}{1 + r} \right).$$

(iv) Write down the equilibrium condition for international capital markets and show that it is given by an equation like

$$\phi \frac{1}{1 + \beta^*} [\beta^* (1 + r) - 1] = -\frac{1}{1 + \beta} [\beta (1 + r) - 1],$$

show that in a world equilibrium  $r^{fa*} < r < r^{fa}$ .

(v) Derive the equilibrium level of  $r$ , show that a larger  $\alpha$  implies that the interest rate is higher (i.e. closer to  $r^{fa}$ ) and  $-B_1$  is smaller.