14.581 International Trade

Class notes on 2/19/2013[1]

1 Overview

Assignment Models in the Trade Literature

- Small but rapidly growing literature using assignment models in an international context:
  - Trade: Grossman Maggi (2000), Grossman (2004), Yeaple (2005), Ohnsorge Trefler (2007), Costinot (2009), Costinot Vogel (2010), Sampson (2012)
  - Offshoring: Kremer Maskin (2003), Antras Garicano Rossi-Hansberg (2006), Nocke Yeaple (2008), Costinot Vogel Wang (2011)
- **What do these models have in common?**
  - Factor allocation can be summarized by an assignment function
  - Large number of factors and/or goods
- **What is the main difference between these models?**
  - *Matching:* Two sides of each match in finite supply (as in Becker 1973)
  - *Sorting:* One side of each match in infinite supply (as in Roy 1951)

This Lecture will restrict to sorting models, e.g. Ohnsorge and Trefler (2007), Costinot (2009), and Costinot and Vogel (2010)

- Objectives:
  1. Describe how these models relate to “standard” neoclassical models
  2. Introduce simple tools from the mathematics of complementarity
  3. Use tools to derive cross-sectional and comparative static predictions
- This is very much a methodological lecture. If you are interested in more specific applications, read the papers...

[1] The notes are based on lecture slides with inclusion of important insights emphasized during the class.

2 Log-Supermodularity

- **Definition 1** *A function* g: $X \to \mathbb{R}^+$ *is log-supermodular if for all* $x, x' \in X$, $g(\max(x,x')) \cdot g(\min(x,x')) \geq g(x) \cdot g(x')$

- **Bivariate example:**

  – If $g : X_1 \times X_2 \to \mathbb{R}^+$ is log-spm, then $x_1' \geq x_1''$ and $x_2' \geq x_2''$ imply

  $$g(x_1', x_2') \cdot g(x_1'', x_2'') \geq g(x_1', x_2'') \cdot g(x_1'', x_2',).$$

  – If g is strictly positive, this can be rearranged as

  $$g(x_1', x_2') / g(x_1'', x_2') \geq g(x_1', x_2'') / g(x_1'', x_2'') \ .$$

- **Lemma 1.** $g, h : X \to \mathbb{R}^+$ *log-spm* $\Rightarrow$ gh *log-spm*

- **Lemma 2.** $g : X \to \mathbb{R}^+$ *log-spm* $\Rightarrow G(x_{-i}) = \int_{X_i} g(x)\, dx_i$ *log-spm*

- **Lemma 3.** $g : T \times X \to \mathbb{R}^+$ *log-spm* $\Rightarrow x^*(t) \equiv \arg\max_{x \in X} g(t, x)$ *increasing in* t

Note: log-supermodular + g increasing $\Rightarrow$ supermodular

3 Sorting Models

3.1 Basic Environment

- Consider a world economy with:

  1. Multiple countries with characteristics $\gamma \in \Gamma$
  2. Multiple goods or sectors with characteristics $\sigma \in \Sigma$
  3. Multiple factors of production with characteristics $\omega \in \Omega$

- Factors are immobile across countries, perfectly mobile across sectors

- Goods are freely traded at world price $p(\sigma) > 0$

3.2 Technology

- Within each sector, factors of production are perfect substitutes

$$Q(\sigma,\gamma) = \int_{\Omega} A(\omega,\sigma,\gamma)L(\omega,\sigma,\gamma)d\omega,$$

- $A(\omega,\sigma,\gamma) \geq 0$ is productivity of ω-factor in σ-sector and γ-country, which is taken exogenous.
- **A1** *$A(\omega,\sigma,\gamma)$ is log-supermodular*
- If A1 holds, then pair by pair log-supermodular also holds
- A1 implies, in particular, that:
    1. High-γ countries have a comparative advantage in high-σ sectors
    2. High-ω factors have a comparative advantage in high-σ sectors

3.3 Factor Endowments

- $V(\omega,\gamma) \geq 0$ is inelastic supply of ω-factor in γ-country
- **A2** *$V(\omega,\gamma)$ is log-supermodular*
- A2 implies that:

  High-γ countries are relatively more abundant in high-ω factors
- Preferences will be described later on when we do comparative statics

4 Cross-Sectional Predictions

4.1 Competitive Equilibrium

- We take the price schedule $p(\sigma)$ as given [small open economy]
- In a competitive equilibrium, L and w must be such that:
    1. Firms maximize profit

       $p(\sigma)A(\omega,\sigma,\gamma) - w(\omega,\gamma) \leq 0$, for all $\omega \in \Omega$
       $p(\sigma)A(\omega,\sigma,\gamma) - w(\omega,\gamma) = 0$, for all $\omega \in \Omega$ s.t. $L(\omega,\sigma,\gamma) > 0$
    2. Factor markets clear

$$V(\omega,\gamma) = \int_{\sigma\in\Sigma} L(\omega,\sigma,\gamma)\, d\sigma, \text{ for all } \omega \in \Omega$$

4.2 Patterns of Specialization

Predictions

- Let $\Sigma(\omega, \gamma) \equiv \{\sigma \in \Sigma | L(\omega, \sigma, \gamma) > 0\}$ be the set of sectors in which factor ω is employed in country γ
- **Theorem** $\Sigma(\cdot, \cdot)$ *is increasing*
- **Proof:**
  1. Profit maximization $\Rightarrow \Sigma(\omega, \gamma) = \arg\max_{\sigma \in \Sigma} p(\sigma) A(\omega, \sigma, \gamma)$
  2. A1 $\Rightarrow p(\sigma) A(\omega, \sigma, \gamma)$ log-spm by Lemma 1
  3. $p(\sigma) A(\omega, \sigma, \gamma)$ log-spm $\Rightarrow \Sigma(\cdot, \cdot)$ increasing by Lemma 3
- **Corollary** *High-ω factors specialize in high-σ sectors*
- **Corollary** *High-γ countries specialize in high-σ sectors*

4.3 Relation to the Ricardian literature

- Ricardian model$\equiv$ Special case w/ $A(\omega, \sigma, \gamma) \equiv A(\sigma, \gamma)$
- Previous corollary can help explain:
  1. **Multi-country-multi-sector Ricardian model;** Jones (1961)
     - According to Jones (1961), efficient assignment of countries to goods solves $\max \sum \ln A(\sigma, \gamma)$
     - According to Corollary, $A(\sigma, \gamma)$ log-spm implies PAM of countries to goods; Becker (1973), Kremer (1993), Legros and Newman (1996).
  2. **Institutions and Trade;** Acemoglu Antras Helpman (2007), Costinot (2006), Cuñat Melitz (2006), Levchenko (2007), Matsuyama (2005), Nunn (2007), and Vogel (2007)
     - Papers vary in terms of source of "institutional dependence" σ and "institutional quality" γ
     - ...but same fundamental objective: providing micro-theoretical foundations for the log-supermodularity of $A(\sigma, \gamma)$

4.4 Aggregate Output, Revenues, and Employment

- Previous results are about the set of goods that each country produces
- **Question:** *Can we say something about how much each country produces? Or how much it employs in each particular sector?*
- **Answer:** *Without further assumptions, the answer is no*

Additional assumptions

- **A3.** *The profit-maximizing allocation L is unique (when one guy is working in some sector, all guys of same type will be working in that sector)*
- **A4.** *Factor productivity satisfies* $A(\omega,\sigma,\gamma) \equiv A(\omega,\sigma)$
- **Comments:**
  1. A3 requires $p(\sigma)\, A(\omega,\sigma,\gamma)$ to be maximized in a *single* sector
  2. A3 is an implicit restriction on the demand-side of the world-economy
     - ... but it becomes milder and milder as the number of factors or countries increases (see Figure below)
     - ... generically true if continuum of factors
  3. A4 implies no Ricardian sources of CA across countries
     - Pure Ricardian case can be studied in a similar fashion
     - Having multiple sources of CA is more complex (Costinot 2009)

Output predictions

- **Theorem** *If A3 and 4 hold, then* $Q(\sigma,\gamma)$ *is log-spm.*
- **Proof:**
  1. Let $\Omega(\sigma) \equiv \{\omega \in \Omega | p(\sigma)\, A(\omega,\sigma) > \max_{\sigma' \neq \sigma}\ p(\sigma')\, A(\omega,\sigma')\}$. A3 and A4 imply $Q(\sigma,\gamma) = \int \mathbb{1}_{\Omega(\boldsymbol{\sigma})}(\omega) \cdot A(\omega,\sigma) V(\omega,\gamma) d\omega$
  2. A1 $\Rightarrow \widetilde{A}(\omega,\sigma) \equiv \mathbb{1}_{\Omega(\boldsymbol{\sigma})}(\omega) \cdot A(\omega,\sigma)$ log-spm
  3. A2 and $\widetilde{A}(\omega,\sigma)$ log-spm + Lemma 1 $\Rightarrow \widetilde{A}(\omega,\sigma)V(\omega,\gamma)$ log-spm
  4. $\widetilde{A}(\omega,\sigma)V(\omega,\gamma)$ log-spm + Lemma 2 $\Rightarrow Q(\sigma,\gamma)$ log-spm
- **Intuition:**
  1. A1 $\Rightarrow$ high ω-factors are assigned to high σ-sectors
  2. A2 $\Rightarrow$ high ω-factors are more likely in high γ-countries

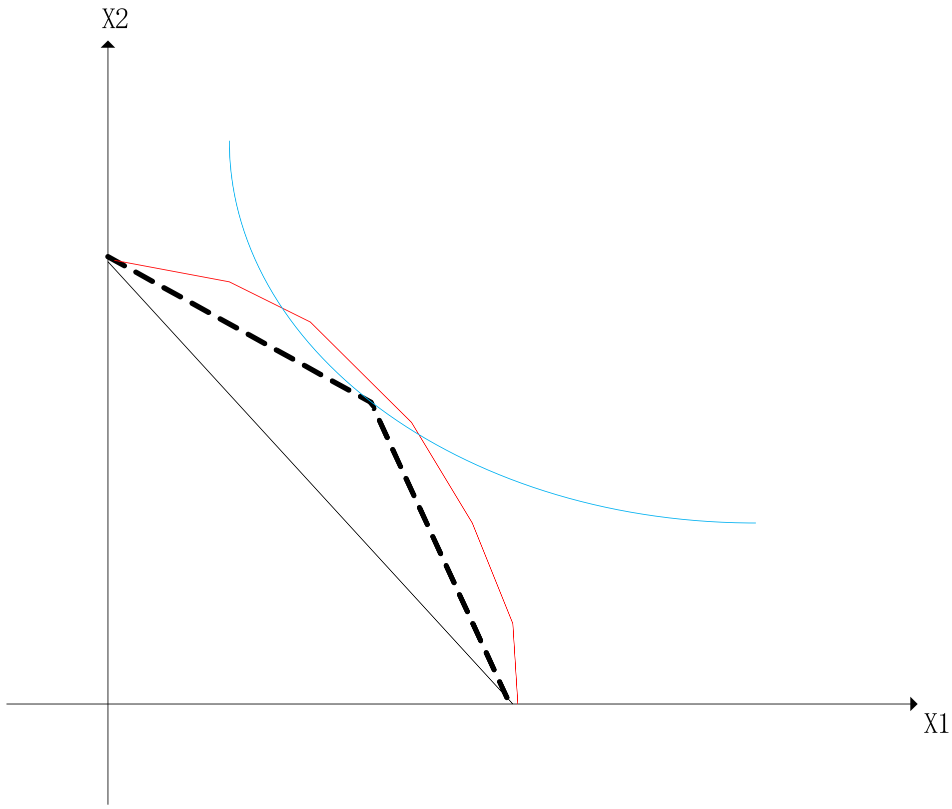

Figure 1: Consider a two sector economy with constant return to each factor. In one factor case, the production possibility frontier is a straight line (black solid line), so in equilibrium this factor must be used to produce goods in both sectors. In two factors case, the production frontier has a kink (black dotted line), at the kink, each factor is totally used to produce goods in a single sector. When the number of factors increase, it's more likely to have "kinks", therefore the allocation L is more likely to be unique. If there is a continuum of factors (as in the model), the unique allocation is generically true.

- **Corollary.** *Suppose that A3 and A4 hold. If two countries produce J goods, with* $\gamma_1 \geq \gamma_2$ *and* $\sigma_1 \geq ... \geq \sigma_J$*, then the high-*$\gamma$ *country tends to specialize in the high-*σ *sectors:*

$$\frac{Q(\sigma_1, \gamma_1)}{Q(\sigma_1, \gamma_2)} \geq ... \geq \frac{Q(\sigma_J, \gamma_1)}{Q(\sigma_J, \gamma_2)}$$

Employment and revenue predictions

- Let $L(\sigma, \gamma) \equiv \int_{\Omega(\sigma)} V(\omega, \gamma) d\omega$ be aggregate employment
- Let $R(\sigma, \gamma) \equiv \int_{\Omega(\sigma)} r(\omega, \sigma) V(\omega, \gamma) d\omega$ be aggregate revenues
- **Corollary.** *Suppose that A3 and A4 hold. If two countries produce J goods, with* $\gamma_1 \geq \gamma_2$ *and* $\sigma_1 \geq ... \geq \sigma_J$*, then aggregate employment and aggregate revenues follow the same pattern as aggregate output:*

$$\frac{L(\sigma_1, \gamma_1)}{L(\sigma_1, \gamma_2)} \geq ... \geq \frac{L(\sigma_J, \gamma_1)}{L(\sigma_J, \gamma_2)} \text{ and } \frac{R(\sigma_1, \gamma_1)}{R(\sigma_1, \gamma_2)} \geq ... \geq \frac{R(\sigma_J, \gamma_1)}{R(\sigma_J, \gamma_2)}$$

Relation to the previous literature

1. **Worker Heterogeneity and Trade**
   - Generalization of Ruffin (1988):
     - Continuum of factors, Hicks-neutral technological differences
     - Results hold for an arbitrarily large number of goods and factors
   - Generalization of Ohnsorge and Trefler (2007):
     - No functional form assumption (log-normal distribution of human capital, exponential factor productivity)
2. **Firm Heterogeneity and Trade**
   - Closely related to Melitz (2003), Helpman Melitz Yeaple (2004) and Antras Helpman (2004)
     - "Factors" $\equiv$ "Firms" with productivity ω
     - "Countries" $\equiv$ "Industries" with characteristic γ
     - "Sectors" $\equiv$ "Organizations" with characteristic σ
     - $Q(\sigma, \gamma) \equiv$ Sales by firms with "σ-organization" in "γ-industry"
   - In previous papers, $f(\omega, \gamma)$ log-spm is crucial, Pareto is not

5 Comparative Static Predictions

5.1 Closing The Model

Additional assumptions

- Assumptions A1-4 are maintained
- In order to do comparative statics, we also need to specify the demand side of our model:

$$U = \left\{ \int_{\sigma \in \Sigma} \left[C(\sigma, \gamma) \right]^{\frac{\varepsilon - 1}{\varepsilon}} d\sigma \right\}^{\frac{\varepsilon}{\varepsilon - 1}}$$

- For expositional purposes, we will also assume that:
  - $A(\omega, \sigma)$ is *strictly* log-supermodular
  - Continuum of factors and sectors: $\Sigma \equiv [\underline{\sigma}, \overline{\sigma}]$ and $\Omega \equiv [\underline{\omega}, \overline{\omega}]$

Autarky equilibrium
Autarky equilibrium is a set of functions (Q, C, L, p, w) such that:

1. Firms maximize profit

$$p(\sigma) A(\omega, \sigma) - w(\omega, \gamma) \leq 0, \text{ for all } \omega \in \Omega$$
$$p(\sigma) A(\omega, \sigma) - w(\omega, \gamma) = 0, \text{ for all } \omega \in \Omega \text{ s.t. } L(\omega, \sigma, \gamma) > 0$$

2. Factor markets clear

$$V(\omega, \gamma) = \int_{\sigma \in \Sigma} L(\omega, \sigma, \gamma) \, d\sigma, \text{ for all } \omega \in \Omega$$

3. Consumers maximize their utility and good markets clear

$$C(\sigma, \gamma) = I(\gamma) \times p(\sigma)^{-\varepsilon} = Q(\sigma, \gamma)$$

Properties of autarky equilibrium

- **Lemma** *In autarky equilibrium, there exists an increasing bijection $M : \Omega \to \Sigma$ such that $L(\omega, \sigma) > 0$ if and only if $M(\omega) = \sigma$*
- **Lemma** *In autarky equilibrium, M and w satisfy*

$$\frac{dM(\omega, \gamma)}{d\omega} = \frac{A[\omega, M(\omega, \gamma)] V(\omega, \gamma)}{I(\gamma) \times \{p[M(\omega), \gamma]\}^{-\varepsilon}} \tag{1}$$

$$\frac{d \ln w(\omega, \gamma)}{d\omega} = \frac{\partial \ln A[\omega, M(\omega)]}{\partial \omega} \tag{2}$$

with $M(\underline{\omega}, \gamma) = \underline{\sigma}$, $M(\overline{\omega}, \gamma) = \overline{\sigma}$, and $p[M(\omega, \gamma), \gamma] = w(\omega, \gamma) / A[\omega, M(\omega, \gamma)]$.

5.2 Changes in Factor Supply

- **Question:** *What happens if we change country characteristics from γ to $\gamma' \leq \gamma$?*
- If ω is worker "skill", this can be though of as a change in terms of "skill abundance":
$$\frac{V(\omega,\gamma)}{V(\omega',\gamma)} \geq \frac{V'(\omega,\gamma')}{V'(\omega',\gamma')}, \text{ for all } \omega > \omega'$$
- If $V(\omega,\gamma)$ was a normal distribution, this would correspond to a change in the mean

Consequence for factor allocation

- **Lemma** *$M(\omega,\gamma') \geq M(\omega,\gamma)$ for all $\omega \in \Omega$*
- **Intuition:**
  - If there are relatively more low-ω factors, more sectors should use them
  - From a sector standpoint, this requires *factor downgrading*

- **Proof:** If there is ω s.t. $M(\omega,\gamma') < M(\omega,\gamma)$, then there exist:
  1. $M(\omega_1,\gamma') = M(\omega_1,\gamma) = \sigma_1$, $M(\omega_2,\gamma') = M(\omega_2,\gamma) = \sigma_2$, and $\frac{M_\omega(\omega_1,\gamma')}{M_\omega(\omega_2,\gamma')} \leq \frac{M_\omega(\omega_1,\gamma)}{M_\omega(\omega_2,\gamma)}$
  2. Equation (1) $\Longrightarrow \frac{V(\omega_2,\gamma')}{V(\omega_1,\gamma')}\frac{C(\sigma_1,\gamma')}{C(\sigma_2,\gamma')} \geq \frac{V(\omega_2,\gamma)}{V(\omega_1,\gamma)}\frac{C(\sigma_1,\gamma)}{C(\sigma_2,\gamma)}$
  3. V log-spm $\Longrightarrow \frac{C(\sigma_1,\gamma')}{C(\sigma_2,\gamma')} \geq \frac{C(\sigma_1,\gamma)}{C(\sigma_2,\gamma)}$
  4. Equation (2) + zero profits $\Longrightarrow \frac{d\ln p(\sigma,\gamma)}{d\sigma} = -\frac{\partial \ln A\left[M^{-1}(\sigma,\gamma),\sigma\right]}{\partial\sigma}$
  5. $M^{-1}(\sigma,\gamma) < M^{-1}(\sigma,\gamma')$ for $\sigma \in (\sigma_1,\sigma_2)$ + A log-spm $\Rightarrow \frac{p(\sigma_1,\gamma)}{p(\sigma_2,\gamma)} < \frac{p(\sigma_1,\gamma')}{p'(\sigma_2,\gamma')}$
  6. $\frac{p(\sigma_1,\gamma)}{p(\sigma_2,\gamma)} < \frac{p(\sigma_1,\gamma')}{p'(\sigma_2,\gamma')}$ + CES $\Rightarrow \frac{C(\sigma_1,\gamma')}{C(\sigma_2,\gamma')} > \frac{C(\sigma_1,\gamma)}{C(\sigma_2,\gamma)}$. A contradiction

- A decrease form γ to γ' implies *pervasive rise in inequality*:
$$\frac{w(\omega,\gamma')}{w(\omega',\gamma')} \geq \frac{w(\omega,\gamma)}{w(\omega',\gamma)}, \text{ for all } \omega > \omega'$$

- The mechanism is simple:

  1. Profit-maximization implies

$$\frac{d\ln w(\omega,\gamma)}{d\omega} = \frac{\partial \ln A[\omega, M(\omega,\gamma)]}{\partial\omega}$$
$$\frac{d\ln w(\omega,\gamma')}{d\omega} = \frac{\partial \ln A[\omega, M(\omega,\gamma')]}{\partial\omega}$$

  2. Since A is log-supermodular, task upgrading implies

$$\frac{d\ln w(\omega,\gamma')}{d\omega} \geq \frac{d\ln w(\omega,\gamma)}{d\omega}$$

Comments

- In Costinot Vogel (2010), we also consider changes in diversity
  - This corresponds to the case where there exists $\widehat{\omega}$ such that $V(\omega,\gamma)$ is log-supermodular for $\omega > \widehat{\omega}$, but log-*sub*modular for $\omega < \widehat{\omega}$
- We also consider changes in factor demand (Computerization?):

$$U = \left\{ \int_{\sigma\in\Sigma} B(\sigma,\gamma)\left[C(\sigma,\gamma)\right]^{\frac{\varepsilon-1}{\varepsilon}} d\sigma \right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

5.3 North-South Trade

Free trade equilibrium

- Two countries, Home (H) and Foreign (F), with $\gamma_H \geq \gamma_F$
- A competitive equilibrium in the world economy under free trade is s.t.

$$\frac{dM(\omega,\gamma_T)}{d\omega} = \frac{A[\omega, M(\omega,\gamma_T)]\,V(\omega,\gamma_T)}{I_T \times \{p[M(\omega,\gamma_T),\gamma_T]\}^{-\varepsilon}},$$

$$\frac{d\ln w(\omega,\gamma_T)}{d\omega} = \frac{\partial \ln A[\omega, M(\omega,\gamma_T)]}{\partial\omega},$$

where:

$$M(\underline{\omega},\gamma_T) = \underline{\sigma} \text{ and } M(\overline{\omega},\gamma_T) = \overline{\sigma}$$
$$p[M(\omega,\gamma_T),\gamma_T] = w(\omega,\gamma_T)\,A[\omega, M(\omega,\gamma_T)]$$
$$V(\omega,\gamma_T) \equiv V(\omega,\gamma_H) + V(\omega,\gamma_F)$$

Free trade equilibrium

- **Key observation:**
$\frac{V(\omega,\gamma_H)}{V(\omega',\gamma_H)} \geq \frac{V(\omega,\gamma_F)}{V(\omega,\gamma_F)}$, for all $\omega > \omega' \Rightarrow \frac{V(\omega,\gamma_H)}{V(\omega',\gamma_H)} \geq \frac{V(\omega,\gamma_T)}{V(\omega',\gamma_T)} \geq \frac{V(\omega,\gamma_F)}{V(\omega,\gamma_F)}$
- Continuum-by-continuum extensions of two-by-two HO results:

    1. *Changes in skill-intensities:*

    $$M(\omega,\gamma_H) \leq M(\omega,\gamma_T) \leq M(\omega,\gamma_F)\,,\text{ for all }\omega$$

    2. *Strong Stolper-Samuelson effect:*

    $$\frac{w(\omega,\gamma_H)}{w(\omega',\gamma_H)} \leq \frac{w(\omega,\gamma_T)}{w(\omega',\gamma_T)} \leq \frac{w(\omega,\gamma_F)}{w(\omega',\gamma_F)},\text{ for all }\omega > \omega'$$

Other Predictions

- North-South trade driven by factor demand differences:
    - Same logic gets to the exact opposite results
    - Correlation between factor demand and factor supply considerations matters
- One can also extend analysis to study "North-North" trade:
    - It predicts wage polarization in the more diverse country and wage convergence in the other

What's next?

- Dynamic issues:
    - Sector-specific human capital accumulation
    - Endogenous technology adoption
- Empirics:
    - Revisiting the consequences of trade liberalization

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