

# 14.581 International Trade

Class notes on 3/4/2013<sup>1</sup>

## 1 Factor Proportion Theory

- The law of comparative advantage establishes the relationship between relative autarky prices and trade flows
  - But where do relative autarky prices come from?
- Factor proportion theory emphasizes **factor endowment differences**
- **Key elements:**
  1. Countries differ in terms of factor abundance [i.e. *relative* factor supply]
  2. Goods differ in terms of factor intensity [i.e. *relative* factor demand]
- Interaction between 1 and 2 will determine differences in relative autarky prices, and in turn, the pattern of trade
- In order to shed light on factor endowments as a source of CA, we will assume that:
  1. Production functions are identical around the world
  2. Households have identical homothetic preferences around the world
- We will first focus on two special models:
  - **Ricardo-Viner** with 2 goods, 1 “mobile” factor (labor) and 2 “immobile” factors (sector-specific capital)
  - **Heckscher-Ohlin** with 2 goods and 2 “mobile” factors (labor and capital)
- The second model is often thought of as a long-run version of the first (Neary 1978)
  - In the case of Heckscher-Ohlin, what is the time horizon such that one can think of total capital as fixed in each country, though freely mobile across sectors?

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<sup>1</sup>The notes are based on lecture slides with inclusion of important insights emphasized during the class.

## 2 Ricardo-Viner Model

### 2.1 Basic environment

- Consider an economy with:
  - Two goods,  $g = 1, 2$
  - Three factors with endowments  $l$ ,  $k_1$ , and  $k_2$

- Output of good  $g$  is given by

$$y_g = f^g(l_g, k_g),$$

where:

- $l_g$  is the (endogenous) amount of labor in sector  $g$
- $f^g$  is homogeneous of degree 1 in  $(l_g, k_g)$

- **Comments:**

- $l$  is a “mobile” factor in the sense that it can be employed in all sectors
- $k_1$  and  $k_2$  are “immobile” factors in the sense that they can only be employed in one of them
- Model is isomorphic to DRS model:  $y_g = f^g(l_g)$  with  $f_{ll}^g < 0$
- Payments to specific factors under CRS  $\equiv$  profits under DRS

### 2.2 Equilibrium (I): small open economy

- We denote by:
  - $p_1$  and  $p_2$  the prices of goods 1 and 2
  - $w$ ,  $r_1$ , and  $r_2$  the prices of  $l$ ,  $k_1$ , and  $k_2$
- For now,  $(p_1, p_2)$  is exogenously given: “**small open economy**”
  - So no need to look at good market clearing

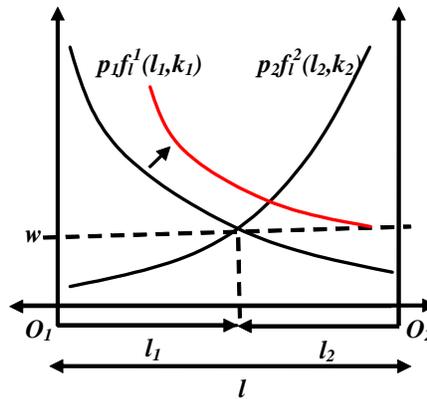
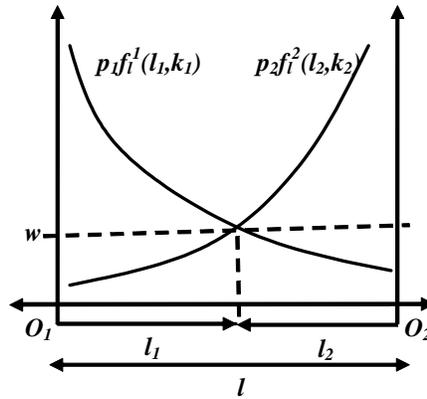
- **Profit maximization:**

$$p_g f_l^g(l_g, k_g) = w \tag{1}$$

$$p_g f_k^g(l_g, k_g) = r_g \tag{2}$$

- **Labor market clearing:**

$$l = l_1 + l_2 \tag{3}$$



### 2.3 Graphical analysis

- Equations (1) and (3) jointly determine labor allocation and wage

### 2.4 Comparative statics

- Consider a TOT shock such that  $p_1$  increases:
  - $w \nearrow$ ,  $l_1 \nearrow$ , and  $l_2 \searrow$
  - Condition (2)  $\Rightarrow r_1/p_1 \nearrow$  whereas  $r_2$  (and a fortiori  $r_2/p_1$ )  $\searrow$
- One can use the same type of arguments to analyze consequences of:
  - Productivity shocks
  - Changes in factor endowments
- In all cases, results are intuitive:

- “Dutch disease” (Boom in export sectors, Bids up wages, which leads to a contraction in the other sectors)
- Useful political-economy applications (Grossman and Helpman 1994)
- Easy to extend the analysis to more than 2 sectors:
  - Plot labor demand in one sector vs. rest of the economy

## 2.5 Equilibrium (II): two-country world

- Predictions on the pattern of trade in a two-country world depend on whether differences in factor endowments come from:
  - Differences in the relative supply of specific factors
  - Differences in the relative supply of mobile factors
- Accordingly, any change in factor prices is possible as we move from autarky to free trade (see Feenstra Problem 3.1 p. 98)

## 3 Two-by-Two Heckscher-Ohlin Model

### 3.1 Basic environment

- Consider an economy with:
  - Two goods,  $g = 1, 2$ ,
  - Two factors with endowments  $l$  and  $k$
- Output of good  $g$  is given by

$$y_g = f^g(l_g, k_g),$$

where:

- $l_g, k_g$  are the (endogenous) amounts of labor and capital in sector  $g$
- $f^g$  is homogeneous of degree 1 in  $(l_g, k_g)$

### 3.2 Back to the dual approach

- $c_g(w, r) \equiv$  unit cost function in sector  $g$

$$c_g(w, r) = \min_{l, k} \{wl + rk \mid f^g(l, k) \geq 1\},$$

where  $w$  and  $r$  the price of labor and capital

- $a_{fg}(w, r) \equiv$  unit demand for factor  $f$  in the production of good  $g$

- Using the Envelope Theorem, it is easy to check that:

$$a_{lg}(w, r) = \frac{dc_g(w, r)}{dw} \text{ and } a_{kg}(w, r) = \frac{dc_g(w, r)}{dr}$$

- $A(w, r) \equiv [a_{fg}(w, r)]$  denotes the matrix of total factor requirements

### 3.3 Equilibrium conditions (I): small open economy

- Like in RV model, we first look at the case of a “small open economy”
  - So no need to look at good market clearing

- **Profit-maximization:**

$$p_g \leq wa_{lg}(w, r) + ra_{kg}(w, r) \text{ for all } g = 1, 2 \quad (4)$$

$$p_g = wa_{lg}(w, r) + ra_{kg}(w, r) \text{ if } g \text{ is produced in equilibrium} \quad (5)$$

- **Factor market-clearing:**

$$l = y_1 a_{l1}(w, r) + y_2 a_{l2}(w, r) \quad (6)$$

$$k = y_1 a_{k1}(w, r) + y_2 a_{k2}(w, r) \quad (7)$$

### 3.4 Factor Price Equalization

- **Question:**

*Can trade in goods be a (perfect) substitute for trade in factors?*

- First classical result from the HO literature answers by the affirmative
- To establish this result formally, we’ll need the following definition:
- **Definition.** *Factor Intensity Reversal (FIR) does not occur if: (i)  $a_{l1}(w, r)/a_{k1}(w, r) > a_{l2}(w, r)/a_{k2}(w, r)$  for all  $(w, r)$ ; or (ii)  $a_{l1}(w, r)/a_{k1}(w, r) < a_{l2}(w, r)/a_{k2}(w, r)$  for all  $(w, r)$ .*

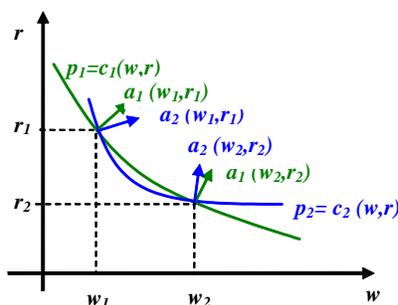
#### 3.4.1 Factor Price Insensitivity (FPI)

- **Lemma** *If both goods are produced in equilibrium and FIR does not occur, then factor prices  $\omega \equiv (w, r)$  are uniquely determined by good prices  $p \equiv (p_1, p_2)$*
- **Proof:** If both goods are produced in equilibrium, then  $p = A'(\omega)\omega$ . By Gale and Nikaido (1965), this equation admits a unique solution if  $a_{fg}(\omega) > 0$  for all  $f, g$  and  $\det[A(\omega)] \neq 0$  for all  $\omega$ , which is guaranteed by no FIR.
- **Comments:**

- Good prices rather than factor endowments determine factor prices
- In a closed economy, good prices and factor endowments are, of course, related, but not for a small open economy
- All economic intuition can be gained by simply looking at Leontieff case
- Proof already suggests that “dimensionality” will be an issue for FIR

Factor Price Insensitivity (FPI): graphical analysis

- Link between no FIR and FPI can be seen graphically:



- If iso-cost curves cross more than once, then FIR must occur

### 3.4.2 Factor Price Equalization (FPE) Theorem

- The previous lemma directly implies (Samuelson 1949) that:
- **FPE Theorem** *If two countries produce both goods under free trade with the same technology and FIR does not occur, then they must have the same factor prices*
- **Comments:**
  - Trade in goods can be a “perfect substitute” for trade in factors
  - Countries with different factor endowments can sustain same factor prices through different allocation of factors across sectors
  - Assumptions for FPE are stronger than for FPI: we need free trade and same technology in the two countries...
  - For next results, we’ll maintain assumption that both goods are produced in equilibrium, but won’t need free trade and same technology

### 3.5 Stolper-Samuelson (1941) Theorem

- **Stolper-Samuelson Theorem** *An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduced the real return to the other factor*
- **Proof:** W.l.o.g. suppose that (i)  $a_{l1}(\omega)/a_{k1}(\omega) > a_{l2}(\omega)/a_{k2}(\omega)$  and (ii)  $\hat{p}_2 > \hat{p}_1$ . Differentiating the zero-profit condition (5), we get

$$\hat{p}_g = \theta_{lg}\hat{w} + (1 - \theta_{lg})\hat{r}, \quad (8)$$

where  $\hat{x} = d \ln x$  and  $\theta_{lg} \equiv wa_{lg}(\omega)/c_g(\omega)$ . Equation (8) implies

$$\hat{w} \geq \hat{p}_1, \hat{p}_2 \geq \hat{r} \text{ or } \hat{r} \geq \hat{p}_1, \hat{p}_2 \geq \hat{w}$$

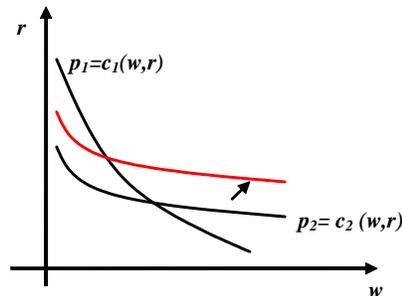
By (i),  $\theta_{l2} < \theta_{l1}$ . So (i) requires  $\hat{r} > \hat{w}$ . Combining the previous inequalities, we get

$$\hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}$$

- **Comments:**

- Previous “hat” algebra is often referred to “Jones’ (1965) algebra”
- The chain of inequalities  $\hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}$  is referred as a “magnification effect”
- SS predict both winners and losers from change in relative prices
- Like FPI and FPE, SS entirely comes from zero-profit condition (+ no joint production)
- Like FPI and FPE, sharpness of the result hinges on “dimensionality”
- In the empirical literature, people often talk about “Stolper-Samuelson effects” whenever looking at changes in relative factor prices (though changes in relative good prices are rarely observed)

Stolper-Samuelson (1941) Theorem: graphical analysis



- Like for FPI and FPE, all economic intuition could be gained by looking at the simpler Leontieff case:
  - In the general case, iso-cost curves are not straight lines, but under no FIR, same logic applies

### 3.6 Rybczynski (1965) Theorem

- Previous results have focused on the implication of *zero profit condition*, Equation (5), for *factor prices*
- Now turn our attention to the implication of *factor market clearing*, Equations (6) and (7), for *factor allocation*
- **Rybczynski Theorem** *An increase in factor endowment will increase the output of the industry using it intensively, and decrease the output of the other industry*
- **Proof:** W.l.o.g. suppose that (i)  $a_{l1}(\omega)/a_{k1}(\omega) > a_{l2}(\omega)/a_{k2}(\omega)$  and (ii)  $\hat{k} > \hat{l}$ . Differentiating factor market clearing conditions (6) and (7), we get

$$\hat{l} = \lambda_{l1}\hat{y}_1 + (1 - \lambda_{l1})\hat{y}_2 \quad (9)$$

$$\hat{k} = \lambda_{k1}\hat{y}_1 + (1 - \lambda_{k1})\hat{y}_2 \quad (10)$$

where  $\lambda_{l1} \equiv a_{l1}(\omega) y_1/l$  and  $\lambda_{k1} \equiv a_{k1}(\omega) y_1/k$ . Equation (8) implies

$$\hat{y}_1 \geq \hat{l}, \hat{k} \geq \hat{y}_2 \text{ or } \hat{y}_2 \geq \hat{l}, \hat{k} \geq \hat{y}_1$$

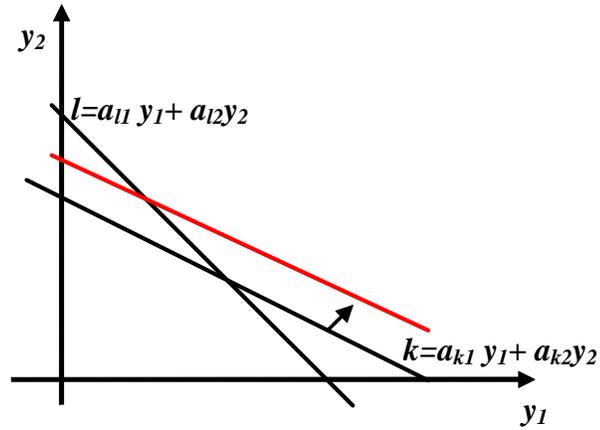
By (i),  $\lambda_{k1} < \lambda_{l1}$ . So (ii) requires  $\hat{y}_2 > \hat{y}_1$ . Combining the previous inequalities, we get

$$\hat{y}_2 > \hat{k} > \hat{l} > \hat{y}_1$$

- Like for FPI and FPE Theorems:
  - $(p_1, p_2)$  is exogenously given  $\Rightarrow$  factor prices and factor requirements are not affected by changes factor endowments
  - Empirically, Rybczynski Theorem suggests that impact of immigration may be very different in closed vs. open economy
- Like for SS Theorem, we have a “magnification effect”
- Like for FPI, FPE, and SS Theorems, sharpness of the result hinges on “dimensionality”

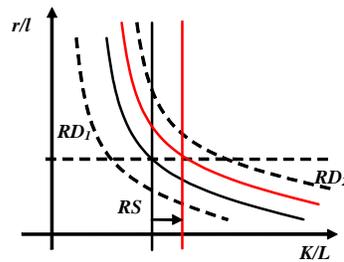
Rybczynski (1965) Theorem: graphical analysis (I)

- Since good prices are fixed, it is as if we were in Leontieff case



Rybczynski (1965) Theorem: graphical analysis (II)

- Rybczynski effect can also be illustrated using relative factor supply and relative factor demand:



- *Cross-sectoral reallocations* are at the core of HO predictions:
  - For relative factor prices to remain constant, *aggregate* relative demand must go up, which requires expansion capital intensive sector

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