

14.581 International Trade
— Lecture 14: Firm Heterogeneity Theory (I) —
Melitz (2003)

Firm-Level Heterogeneity and Trade

What's wrong with previous theories?

- Nineties have seen a boom in the availability of micro-level data
- **Problem:** previous theories are at odds with (or cannot account for) many micro-level facts:
 - ① Within a given industry, there is *firm-level heterogeneity*
 - ② *Fixed costs matter in export* related decisions
 - ③ Within a given industry, more productive firms are more likely to export
 - ④ Trade liberalization leads to intra-industry reallocation across firms
 - ⑤ These reallocations are correlated with productivity and export status

Firm-Level Heterogeneity and Trade

What does Melitz (2003) do about it?

- Melitz (2003) will develop a model featuring facts 1 and 2 that can explain facts 3, 4, and 5
- This is by far the most influential trade paper in the last 10 years
- **Two building blocks:**
 - ① Krugman (1980): CES, IRS technology, monopolistic competition
 - ② Hopenhayn (1992): equilibrium model of entry and exit
- From a normative point of view, Melitz (2003) may also provide “new” source of gains from trade if trade induces reallocation of labor from least to most productive firms (more on that later)

- 1 A Refresher on Monopolistic Competition
 - 1 Krugman (1979)
 - 2 An Important Special Case: CES Utility
- 2 Melitz (2003)
 - 1 Krugman (1980) meets Hopenhayn (1992)
 - 2 Selection into Exports and the Impact of Trade

Monopolistic Competition

Basic idea

- **Monopoly pricing:**

Each firm faces a downward-sloping demand curve

- **No strategic interaction:**

Each demand curve depends on the prices charged by other firms

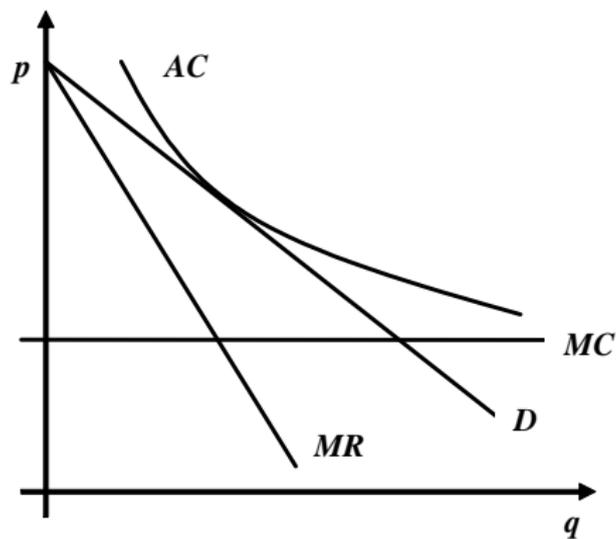
- but since the number of firms is large, each firm ignores its impact on the demand faced by other firms

- **Free entry:**

Firms enter the industry until profits are driven to zero for all firms

Monopolistic Competition

Graphical analysis



Krugman (1979)

Endowments, preferences, and technology

- **Endowments:** All agents are endowed with 1 unit of labor
- **Preferences:** All agents have the same utility function given by

$$U = \int_0^n u(c_i) di$$

where:

- $u(0) = 0$, $u' > 0$, and $u'' < 0$ (love of variety)
- $\sigma(c) \equiv -\frac{u'}{cu''} > 0$ is such that $\sigma' \leq 0$ (why?)
- **IRS Technology:** Labor used in the production of each “variety” is

$$l_i = f + q_i / \varphi$$

where $\varphi \equiv$ common productivity parameter

Krugman (1979)

Equilibrium conditions

1 Consumer maximization:

$$p_i = \lambda^{-1} u'(c_i)$$

2 Profit maximization:

$$p_i = \left[\frac{\sigma(c_i)}{\sigma(c_i) - 1} \right] \cdot \left(\frac{w}{\varphi} \right)$$

3 Free entry:

$$\left(p_i - \frac{w}{\varphi} \right) q_i = wf$$

4 Good and labor market clearing:

$$q_i = Lc_i$$

$$L = nf + \int_0^n \frac{q_i}{\varphi} di$$

Krugman (1979)

Equilibrium conditions rearranged

- Symmetry $\Rightarrow p_i = p$, $q_i = q$, and $c_i = c$ for all $i \in [0, n]$
- c and p/w are simultaneously characterized by

$$\text{(PP):} \quad \frac{p}{w} = \left[\frac{\sigma(c)}{\sigma(c) - 1} \right] \frac{1}{\varphi}$$

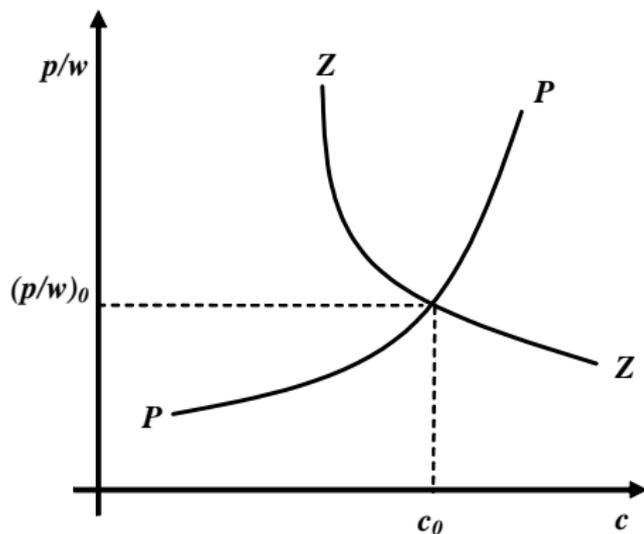
$$\text{(ZP):} \quad \frac{p}{w} = \frac{f}{q} + \frac{1}{\varphi} = \frac{f}{Lc} + \frac{1}{\varphi}$$

- n can then be computed using market clearing conditions

$$n = \frac{1}{f/L + c/\varphi}$$

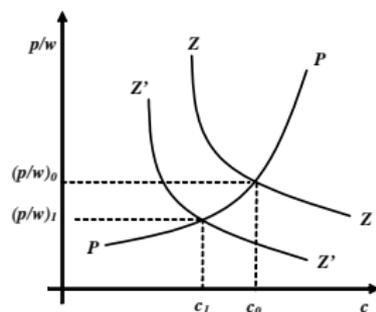
Krugman (1979)

Graphical analysis



Krugman (1979)

Gains from trade revisited



- Suppose that two identical countries open up to trade
 - This is equivalent to a doubling of country size (which would have no effect in a neoclassical trade model)
- Because of IRS, opening up to trade now leads to:
 - **Increased product variety:** $c_1 < c_0 \Rightarrow \frac{1}{f/2L+c_1/\varphi} > \frac{1}{f/L+c_0/\varphi}$
 - **Pro-competitive/efficiency effects:** $(p/w)_1 < (p/w)_0 \Rightarrow q_1 > q_0$

CES Utility

Trade economists' most preferred demand system

- Constant Elasticity of Substitution (CES) utility corresponds to the case where:

$$U = \int_0^n (c_i)^{\frac{\sigma-1}{\sigma}} di,$$

where $\sigma > 1$ is the elasticity of substitution between pair of varieties

- This is the case considered in Krugman (1980)
- What is it to like about CES utility?
 - Homotheticity ($u(c) \equiv (c)^{\frac{\sigma-1}{\sigma}}$ is actually the *only* functional form such that U is homothetic)
 - Can be derived from discrete choice model with i.i.d extreme value shocks (See Feenstra Appendix B)
- Is it empirically reasonable?

CES Utility

Special properties of the equilibrium

- Because of monopoly pricing, CES \Rightarrow constant markups:

$$\frac{p}{w} = \left[\frac{\sigma}{\sigma - 1} \right] \frac{1}{\varphi}$$

- Because of zero profit, constant markups \Rightarrow constant output per firm:

$$\frac{p}{w} = \frac{f}{q} + \frac{1}{\varphi}$$

- Because of market clearing, constant output per firm \Rightarrow constant number of varieties per country:

$$n = \frac{L}{f + q/\varphi}$$

- So, gains from trade **only** come from access to Foreign varieties
 - IRS provide an intuitive reason why Foreign varieties are different
 - But consequences of trade would now be the same if we had maintained CRS with different countries producing different goods

CES Utility

Special properties of the equilibrium

- Decentralized equilibrium is efficient
- Decentralized equilibrium solves:

$$\begin{aligned} & \max_{q_i, n} \int_0^n p_i(q_i) q_i di \\ \text{subject to} & : nf + \int_0^n \frac{q_i}{\varphi} di \leq L. \end{aligned}$$

- A central planner would solve:

$$\begin{aligned} & \max_{q_i, n} \int_0^n (q_i)^{\frac{\sigma-1}{\sigma}} di \\ \text{subject to:} & \quad nf + \int_0^n \frac{q_i}{\varphi} di \leq L. \end{aligned}$$

- Under CES, $p_i(q_i) q_i \propto q_i^{1-\frac{1}{\sigma}} \Rightarrow$ Two solutions coincide
 - This is unique to CES (in general, entry is distorted)
 - This implies that many properties of perfectly competitive models will carry over to this environment

- Like in Krugman (1980), representative agent has CES preferences:

$$U = \left[\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution

- Consumption and expenditures for each variety are given by

$$q(\omega) = Q \left[\frac{p(\omega)}{P} \right]^{-\sigma} \quad (1)$$

$$r(\omega) = R \left[\frac{p(\omega)}{P} \right]^{1-\sigma} \quad (2)$$

where:

$$P \equiv \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}, \quad R \equiv \int_{\omega \in \Omega} r(\omega), \quad \text{and} \quad Q \equiv R/P$$

- Like in Krugman (1980), labor is the only factor of production
 - $L \equiv$ total endowment, $w = 1 \equiv$ wage
- Like in Krugman (1980), there are IRS in production

$$l = f + q/\varphi \quad (3)$$

- Like in Krugman (1980), monopolistic competition implies

$$p(\varphi) = \frac{1}{\rho\varphi} \quad (4)$$

- CES preferences with monopoly pricing, (2) and (4), imply

$$r(\varphi) = R(P\rho\varphi)^{\sigma-1} \quad (5)$$

- These two assumptions, (3) and (4), further imply

$$\pi(\varphi) \equiv r(\varphi) - l(\varphi) = \frac{r(\varphi)}{\sigma} - f$$

- **Comments:**

- ① Higher productivity φ in the model implies higher *measured* productivity

$$\frac{r(\varphi)}{l(\varphi)} = \frac{1}{\rho} \left[1 - \frac{f}{l(\varphi)} \right]$$

- ② More productive firms produce more and earn higher revenues

$$\frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2} \right)^\sigma \quad \text{and} \quad \frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2} \right)^{\sigma-1}$$

- ③ φ can also be interpreted in terms of quality. This is isomorphic to a change in units of account, which would affect prices, but nothing else

- By definition, the CES price index is given by

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

- Since all firms with productivity φ charge the same price $p(\varphi)$, we can rearrange CES price index as

$$P = \left[\int_0^{+\infty} p(\varphi)^{1-\sigma} M\mu(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}}$$

where:

- $M \equiv$ mass of (surviving) firms in equilibrium
- $\mu(\varphi) \equiv$ (conditional) pdf of firm-productivity levels in equilibrium

- Combining the previous expression with monopoly pricing (4), we get

$$P = M^{\frac{1}{1-\sigma}} / \rho \tilde{\varphi}$$

where

$$\tilde{\varphi} \equiv \left[\int_0^{+\infty} \varphi^{\sigma-1} \mu(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}$$

- One can do the same for all aggregate variables

$$R = Mr(\tilde{\varphi}), \Pi = M\pi(\tilde{\varphi}), Q = M^{\frac{\sigma}{\sigma-1}} q(\tilde{\varphi})$$

- Comments:**

- These are the same aggregate variables we would get in a Krugman (1980) model with a mass M of identical firms with productivity $\tilde{\varphi}$
- But productivity $\tilde{\varphi}$ now is an *endogenous* variable which may respond to changes in trade cost, leading to *aggregate* productivity changes

- In order to determine how $\mu(\varphi)$ and $\tilde{\varphi}$ get determined in equilibrium, one needs to specify the entry and exit of firms
- Timing is similar to Hopenhayn (1992):
 - 1 There is a large pool of identical potential entrants deciding whether to become active or not
 - 2 Firms deciding to become active pay a fixed cost of entry $f_e > 0$ and get a productivity draw φ from a cdf G
 - 3 After observing their productivity draws, firms decide whether to remain active or not
 - 4 Firms deciding to remain active exit with a constant probability δ

Melitz (2003)

Aside: Pareto distributions

- In variations and extensions of Melitz (2003), most common assumption on the productivity distribution G is Pareto:

$$G(\varphi) \equiv 1 - \left(\frac{\underline{\varphi}}{\varphi}\right)^\theta \text{ for } \varphi \geq \underline{\varphi}$$

$$g(\varphi) \equiv \theta \underline{\varphi}^\theta \varphi^{-\theta-1} \text{ for } \varphi \geq \underline{\varphi}$$

- Pareto distributions have two advantages:
 - ① Combined with CES, it delivers closed form solutions
 - ② Distribution of firm sizes remains Pareto, which is not a bad approximation empirically (at least for the upper tail)
- But like CES, Pareto distributions will have very strong implications for equilibrium properties (more on this later)

- In a stationary equilibrium, a firm either exits immediately or produces and earns the same profits $\pi(\varphi)$ in each period
- In the absence of time discounting, expected value of a firm with productivity φ is

$$v(\varphi) = \max \left\{ 0, \sum_{t=0}^{+\infty} (1 - \delta)^t \pi(\varphi) \right\} = \max \left\{ 0, \frac{\pi(\varphi)}{\delta} \right\}$$

- There exists a unique productivity level $\varphi^* \equiv \inf \left\{ \varphi \geq 0 : \frac{\pi(\varphi)}{\delta} > 0 \right\}$
- Productivity cutoff φ^* can also be written as:

$$\pi(\varphi^*) = 0$$

- Once we know φ^* , we can compute the pdf of firm-productivity levels

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)} & \text{if } \varphi \geq \varphi^* \\ 0 & \text{if } \varphi < \varphi^* \end{cases}$$

- Accordingly, the measure of aggregate productivity is given by

$$\tilde{\varphi}(\varphi^*) = \left[\frac{1}{1-G(\varphi^*)} \int_{\varphi^*}^{+\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}$$

- Let $\bar{\pi} \equiv \Pi/M$ denote average profits per period for surviving firms
- Free entry requires the total expected value of profits to be equal to the fixed cost of entry

$$0 \times G(\varphi^*) + \frac{\bar{\pi}}{\delta} \times [1 - G(\varphi^*)] = f_e$$

- **Free Entry Condition (FE):**

$$\bar{\pi} = \frac{\delta f_e}{1 - G(\varphi^*)} \quad (6)$$

- Holding constant the fixed costs of entry, if firms are less likely to survive, they need to be compensated by higher average profits

- Definition of φ^* can be rearranged to obtain a second relationship between φ^* and $\bar{\pi}$
- By definition of $\bar{\pi}$, we know that

$$\bar{\pi} = \Pi/M = \pi [\tilde{\varphi}(\varphi^*)] \Leftrightarrow \bar{\pi} = f \left[\frac{r[\tilde{\varphi}(\varphi^*)]}{\sigma f} - 1 \right]$$

- By definition of φ^* , we know that

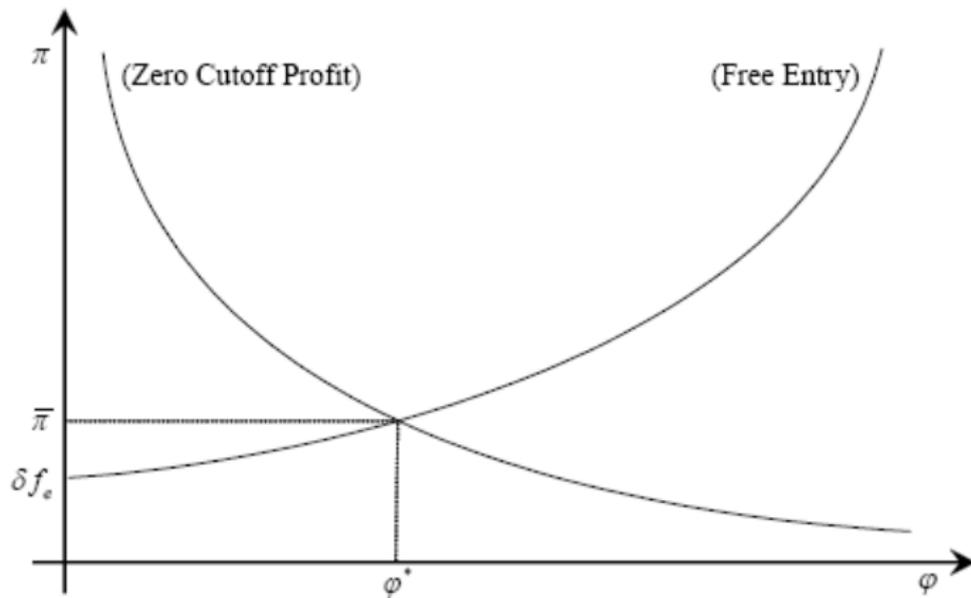
$$\pi(\varphi^*) = 0 \Leftrightarrow r(\varphi^*) = \sigma f$$

- Two previous expressions imply **ZCP condition**:

$$\bar{\pi} = f \left[\frac{r[\tilde{\varphi}(\varphi^*)]}{r(\varphi^*)} - 1 \right] = f \left[\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*} \right)^{\sigma-1} - 1 \right] \quad (7)$$

Melitz (2003)

Closed economy equilibrium



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Melitz (2003)

Aside: the shape of the ZCP schedule

- FE and ZCP, (6) and (7), determine a unique $(\bar{\pi}, \varphi^*)$, and therefore $\tilde{\varphi}$, independently of country size L
 - the only variable left to compute is M , which can be done using free entry and labor market clearing as in Krugman (1980)
- However, ZCP is not necessarily downward sloping:
 - it depends on whether $\tilde{\varphi}$ or φ^* increases relatively faster
 - ZCP is downward sloping for most common distributions
- In the Pareto case, it is easy to check that $\tilde{\varphi}/\varphi^*$ is constant:
 - So ZCP is flat and average profits are independent of φ^*

Melitz (2003)

Number of varieties and welfare

- Free entry and labor market clearing imply

$$L = R = \bar{r}M$$

- We can rearrange the previous expression

$$M = \frac{L}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f)}$$

- Like in Krugman (1980), welfare of a representative worker is given by

$$U = 1/P = M^{\frac{1}{\sigma-1}} \rho \tilde{\varphi}$$

- Since $\tilde{\varphi}$ and $\bar{\pi}$ are independent of L , growth in country size and costless trade will also have the same impact as in Krugman (1980):
 - welfare \nearrow because of \nearrow in total number of varieties in each country

Melitz (2003)

Open economy model

- In the absence of trade costs, we have seen trade integration does not lead to any intra-industry reallocation ($\tilde{\varphi}$ is fixed)
- In order to move away from such (counterfactual) predictions, Melitz (2003) introduces two types of trade costs:
 - ① **Iceberg trade costs:** in order to sell 1 unit abroad, firms need to ship $\tau \geq 1$ units
 - ② **Fixed exporting costs:** in order to export abroad, firms must incur an additional fixed cost f_{ex} (information, distribution, or regulation costs) *after learning their productivity φ*
- In addition, Melitz (2003) assumes that $c = 1, \dots, n$ countries are symmetric so that $w_c = 1$ in all countries

- Monopoly pricing now implies

$$p_d(\varphi) = \frac{1}{\rho\varphi}, p_x(\varphi) = \frac{\tau}{\rho\varphi}$$

- Revenues in the domestic and export markets are

$$r_d(\varphi) = R_d [P_d \rho \varphi]^{\sigma-1}, r_x(\varphi) = \tau^{1-\sigma} R_x [P_x \rho \varphi]^{\sigma-1}$$

- Note that by symmetry, we must have

$$P_d = P_x = P \text{ and } R_d = R_x = R$$

- Let $f_x \equiv \delta f_{ex}$. Profits in the domestic and export markets are

$$\pi_d(\varphi) = \frac{r_d(\varphi)}{\sigma} - f, \pi_x(\varphi) = \frac{r_x(\varphi)}{\sigma} - f_x$$

- Expected value of a firm with productivity φ is

$$v(\varphi) = \max \left\{ 0, \sum_{t=0}^{+\infty} (1 - \delta)^t \pi(\varphi) \right\} = \max \left\{ 0, \frac{\pi(\varphi)}{\delta} \right\}$$

- But total profits of are now given by

$$\pi(\varphi) = \pi_d(\varphi) + \max \{ 0, \pi_x(\varphi) \}$$

- Like in the closed economy, we let $\varphi^* \equiv \inf \left\{ \varphi \geq 0 : \frac{\pi(\varphi)}{\delta} > 0 \right\}$
- In addition, we let $\varphi_x^* \equiv \inf \left\{ \varphi \geq \varphi^* : \frac{\pi_x(\varphi)}{\delta} > 0 \right\}$ be the export cutoff
- In order to have both exporters and non-exporters in equilibrium, $\varphi_x^* > \varphi^*$, we assume that:

$$\tau^{\sigma-1} f_x > f$$

Melitz (2003)

Selection into exports

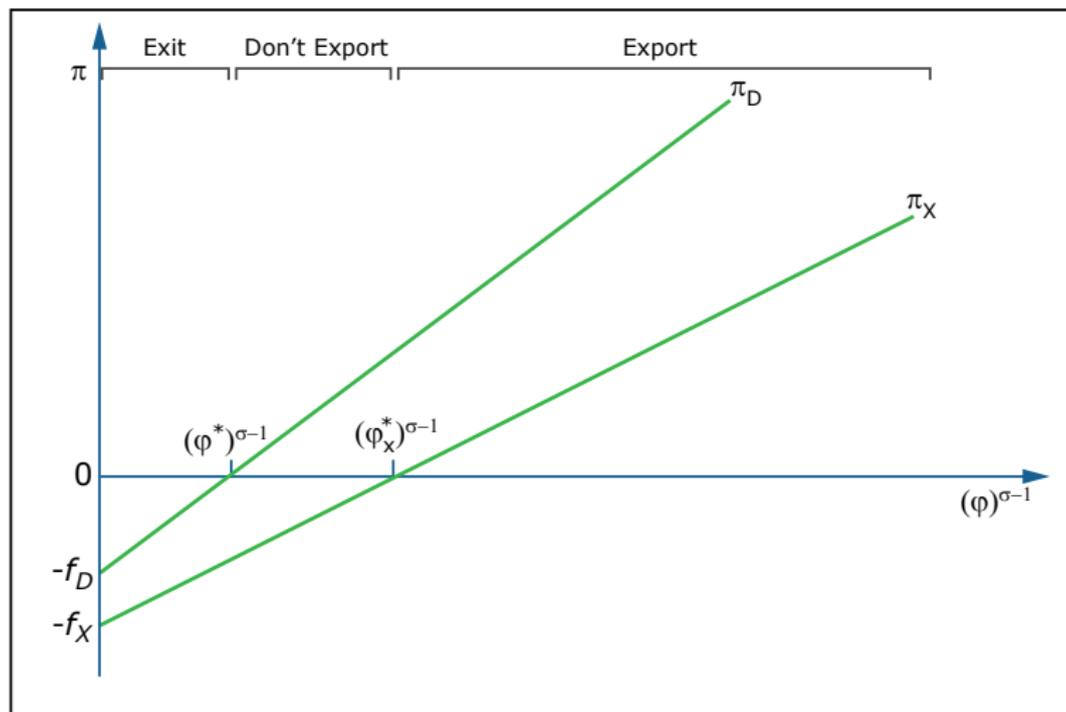


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Melitz (2003)

Are exporters more productive than non-exporters?

- In the model, more productive firms (higher φ) select into exports
- Empirically, this directly implies larger firms (higher $r(\varphi)$)
- **Question:** Does that also mean that firms with higher *measured* productivity select into exports?
- **Answer:** Yes. For this to be true, we need

$$\frac{r_d(\varphi) + nr_x(\varphi)}{l_d(\varphi) + nl_x(\varphi)} > \frac{r_d(\varphi)}{l_d(\varphi)},$$

which always holds if $\tau^{\sigma-1} f_x > f$

- **Comment:** Like in the closed economy, this crucially relies on the fact that fixed labor costs enter the denominator

- In the open economy, aggregate productivity is now given by

$$\tilde{\varphi}_t = \left\{ \frac{1}{M_t} \left[M\tilde{\varphi}^{\sigma-1} + nM_x (\tilde{\varphi}_x/\tau)^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}}$$

where:

- $M_t \equiv M + nM_x$ is the total number of varieties
- $\tilde{\varphi} = \left[\frac{1}{1-G(\varphi^*)} \int_{\varphi^*}^{+\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}$ is the average productivity across all firms
- $\tilde{\varphi}_x = \left[\frac{1}{1-G(\varphi_x^*)} \int_{\varphi_x^*}^{+\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}$ is the average productivity across all exporters

- Once we know $\tilde{\varphi}_t$, we can still compute all aggregate variables as:

$$P = M_t^{\frac{1}{1-\sigma}} / \rho \tilde{\varphi}_t,$$

$$R = M_t r(\tilde{\varphi}_t),$$

$$\Pi = M_t \pi(\tilde{\varphi}_t),$$

$$Q = M_t^{\frac{\sigma}{\sigma-1}} q(\tilde{\varphi}_t)$$

- **Comment:**

- Like in the closed economy, there is a tight connection between welfare ($1/P$) and average productivity ($\tilde{\varphi}_t$)
- But in the open economy, this connection heavily relies on symmetry: welfare depends on the productivity of *foreign*, not *domestic* exporters

- The condition for free entry is unchanged
- **Free Entry Condition (FE):**

$$\bar{\pi} = \frac{\delta f_e}{1 - G(\varphi^*)} \quad (8)$$

- The only difference is that average profits now depend on export profits as well

$$\bar{\pi} = \pi_d(\tilde{\varphi}) + np_x \pi_x(\tilde{\varphi}_x)$$

where:

- $p_x = \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)}$ is probability of exporting conditional on successful entry

- By definition of the cut off productivity levels, we know that

$$\pi_d(\varphi^*) = 0 \Leftrightarrow r_d(\varphi^*) = \sigma f$$

$$\pi_x(\varphi_x^*) = 0 \Leftrightarrow r_x(\varphi_x^*) = \sigma f_x$$

- This implies

$$\frac{r_x(\varphi_x^*)}{r_d(\varphi^*)} = \frac{f_x}{f} \Leftrightarrow \varphi_x^* = \varphi^* \tau \left(\frac{f_x}{f} \right)^{\frac{1}{\sigma-1}}$$

- By rearranging $\bar{\pi}$ as a function of φ^* , we new **ZCP condition**:

$$\bar{\pi} = f \left[\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*} \right)^{\sigma-1} - 1 \right] + n p_x f_x \left[\left(\frac{\tilde{\varphi}_x(\varphi^*)}{\varphi_x^*(\varphi^*)} \right)^{\sigma-1} - 1 \right]$$

Melitz (2003)

The Impact of Trade

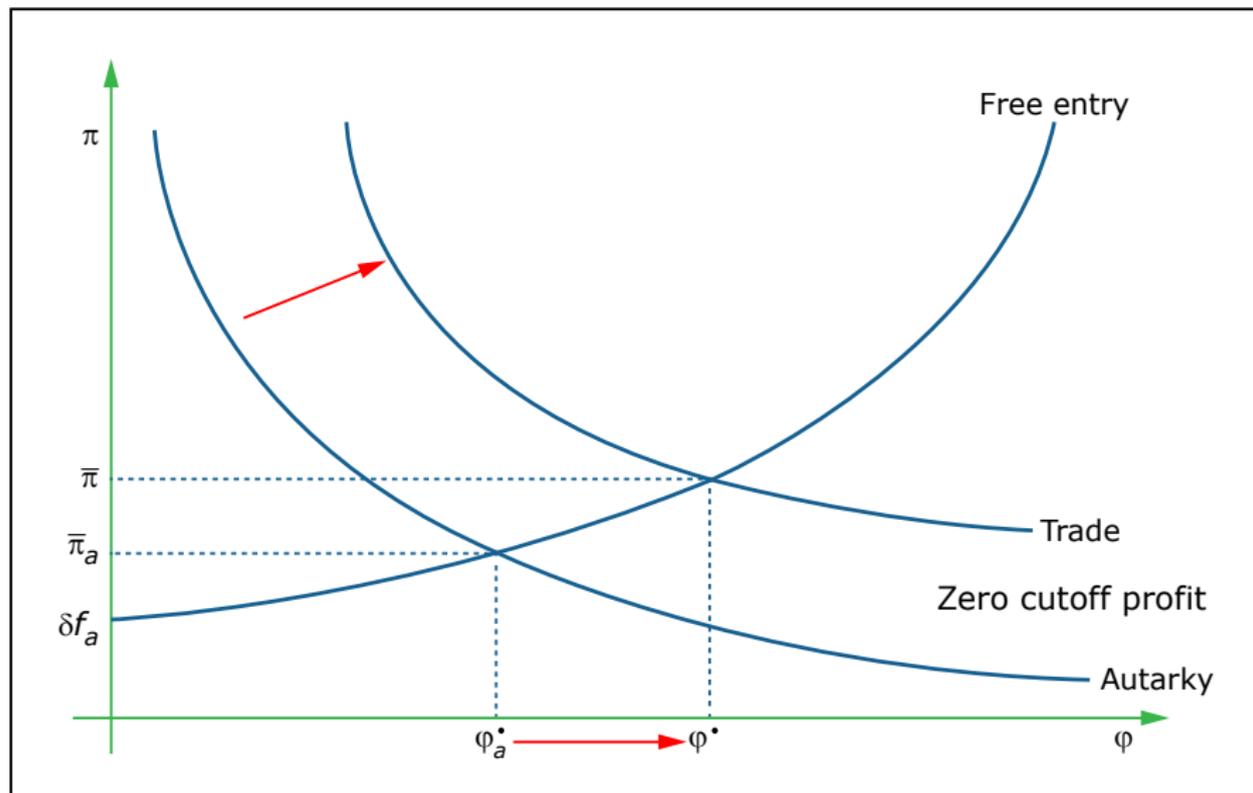


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- In line with empirical evidence, exposure to trade forces the least productive firms to exit: $\varphi^* > \varphi_a^*$
- **Intuition:**
 - *For exporters:* Profits \nearrow due to export opportunities, but \searrow due to the entry of foreign firms in the domestic market ($P \searrow$)
 - *For non-exporters:* only the negative second effect is active
- **Comments:**
 - The \nearrow in φ^* is not a new source of gains from trade. It's *because* there are gains from trade ($P \searrow$) that $\varphi^* \nearrow$ increases
 - Welfare unambiguously \nearrow though number of domestic varieties \searrow

$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f + p_x n f_x)} < M_a$$

Melitz (2003)

The Impact of Trade

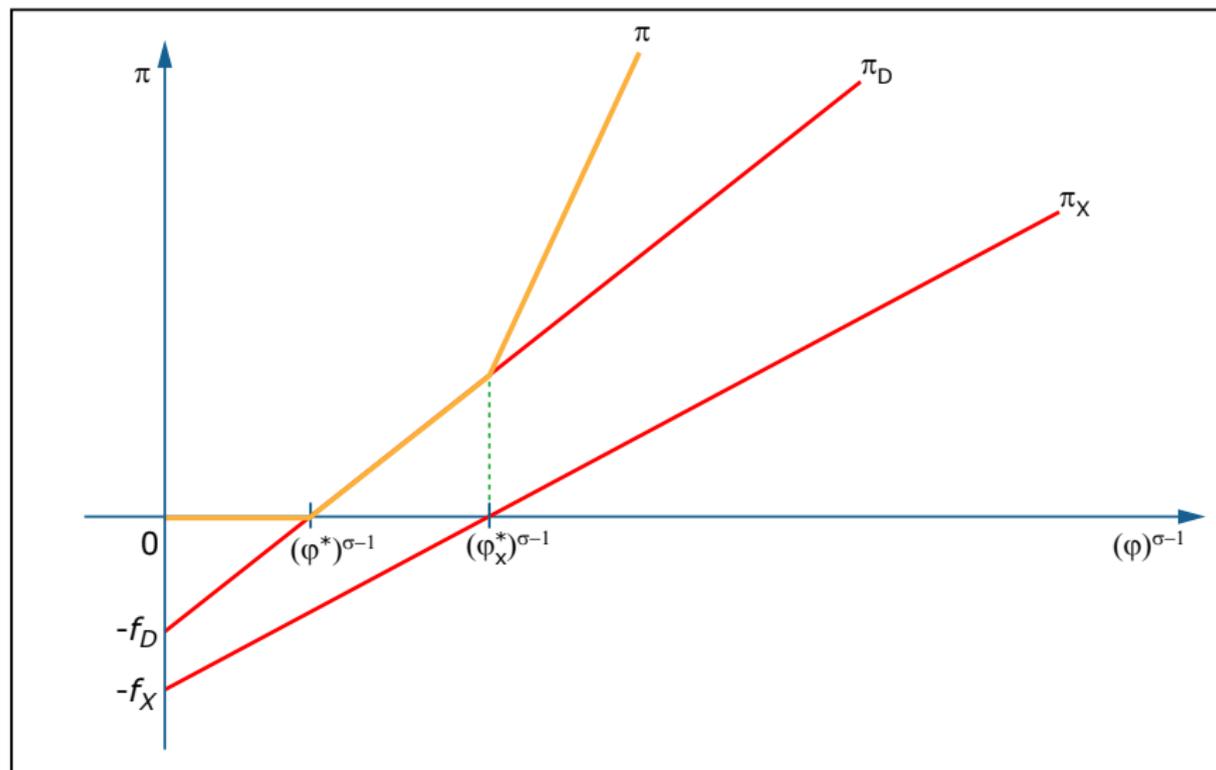


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The Impact of Trade

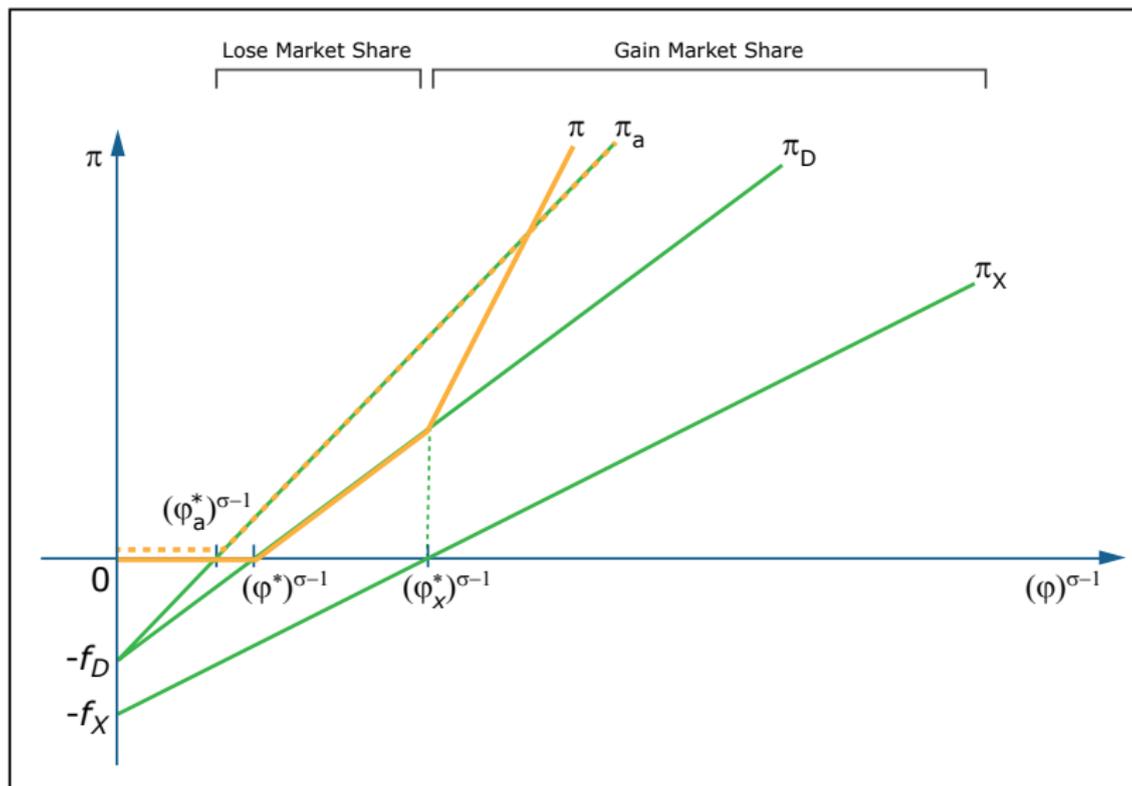


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Melitz (2003)

Other comparative static exercises

- Starting from autarky and moving to trade is theoretically standard, but not empirically appealing
- Melitz (2003) also considers:
 - ① Increase in the number of trading partners n
 - ② Decrease in iceberg trade costs τ
 - ③ Decrease in fixed exporting costs f_x
- Same qualitative insights in all scenarios:
 - Exit of least efficient firms
 - Reallocation of market shares from less to more productive firms
 - Welfare gains

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