

# Discrimination: Theory

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## Outline

- (1) Preliminaries
- (2) Taste-based discrimination: [Becker \(1957\)](#)
- (3) Statistical discrimination: [Phelps \(1972\)](#), [Arrow \(1973\)](#), [Aigner and Cain \(1977\)](#)

## 1 Preliminaries

We observe systematic differences in labor market outcomes across men and women, and across racial groups. While a variety of models we have covered so far in 14.661 and 14.662 could in theory fully explain these differences, economists have long focused attention on the idea that gender- or race-based discrimination may be another empirically relevant explanation. While attention often focuses on gaps in employment rates and wages, differences are also often observed in other measures as well such as occupation, non-wage compensation, job characteristics, and job mobility.

Two excellent review articles on the economics of discrimination are the [Altonji and Blank \(1999\)](#) *Handbook of Labor Economics* chapter and - more recently - a *Journal of Economic Literature* review by [Lang and Lehmann \(2012\)](#). I will draw on both in structuring today's lecture.

### 1.1 Empirical regularities in group differences

Although models of discrimination can be applied in a variety of contexts, in the introduction of *The Economics of Discrimination* [Becker \(1957\)](#) motivates his focus on racial discrimination by saying: “*One might venture the generalization that no single domestic issue has occupied more space in our newspapers in the postwar period than discrimination against minorities...*” [Lang and Lehmann \(2012\)](#) begin by documenting a set of empirical regularities about black-white labor market differences that economic theories should try to explain. Note that their goal in presenting these descriptive facts is not to address whether observed group differences are explained by discrimination - that is a task we will move to in the next lecture when we cover the empirics of discrimination. Rather, the goal here is just to give you a sense of what facts theories of discrimination should try to explain.

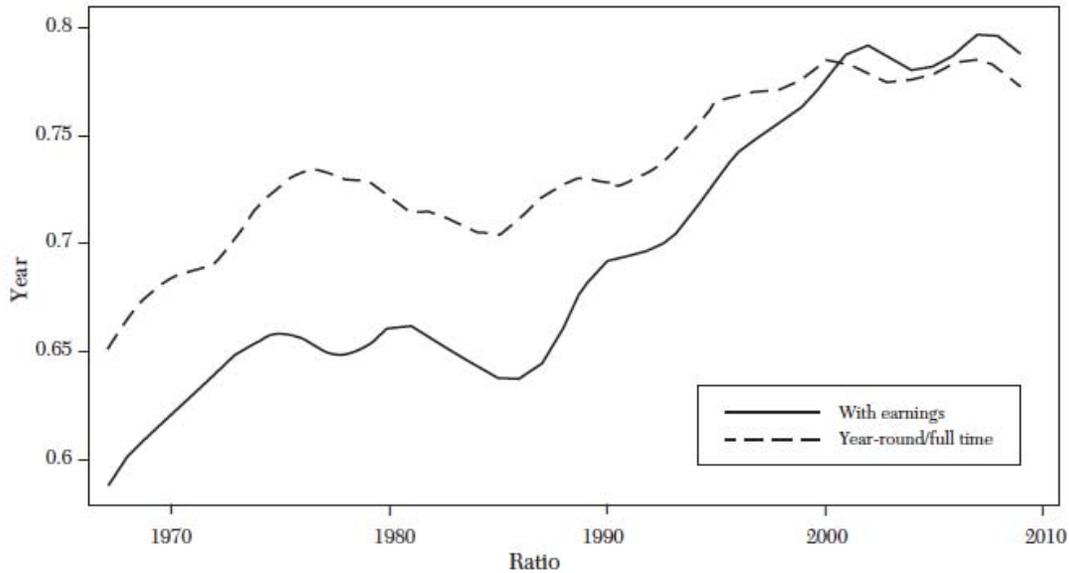
They focus almost exclusively on documenting differential labor market experiences of black and white men, because they argue that differences in the patterns of participation between

black and white women make analysis difficult. Nonparticipation among prime-age males is concentrated among low-skilled workers regardless of race. In contrast, the same is not true for women.

There is a large raw wage differential between black and white men. As we will discuss, much of this differential can be explained by differences in the skills these men bring to the labor market. By adolescence, on tests of cognitive ability, the differential between blacks and whites is typically reported as being on the order of one standard deviation. Potential influences on that test score gap discussed by Lang and Lehmann include residential segregation and school quality.

Lang and Lehmann argue we know much less about wage differentials between black and white women. Raw wage differentials between black and white women have historically been considerably lower than between black and white men, and have at times been reversed. However, this finding is thought to at least partially reflect the differential selection of black and white women into the labor force: white women with wages are noticeable less positively selected than are black women, which results in a significant underestimate of the black-white wage gap among women.

**Lang and Lehmann (2012) Figure 1.** This shows the (smoothed) ratio of black to white median annual earnings among men at or older than age 20 and (separately) those working year-round/full-time jobs. Although the magnitudes differ, the broad patterns are similar for the two series: the relative earnings of black men in these groups rose sharply from the late 1960s until the mid-to-late 1970s, and then fell somewhat until the mid-1980s, after which they rose again until roughly 2000; they have since remained flat. Lang and Lehmann stress that these patterns should not be ascribed solely to changes in labor market discrimination, as much of the improvement in the early period is undoubtedly due to the declining labor force participation of black men. In addition, early improvements can also be credited to both the rise in the relative level of educational attainment and the relative quality of the schools attended by blacks. However, they argue that it is difficult to come up with plausible estimates of the effects of human capital that would fully explain the wage convergence in the 1960s and 1970s. On the other hand, they make the absence of further convergence in the late 1970s and much of the 1980s even more surprising. They note that the very large gains made by black men after the mid-to-late 1980s cannot be accounted for by non earners in the CPS since there was little change during this period (the proportion of black men in prison or jail grew, but is insufficient to explain the observed degree of convergence). The main take-away is that there is a large “raw” wage gap between black and white men.



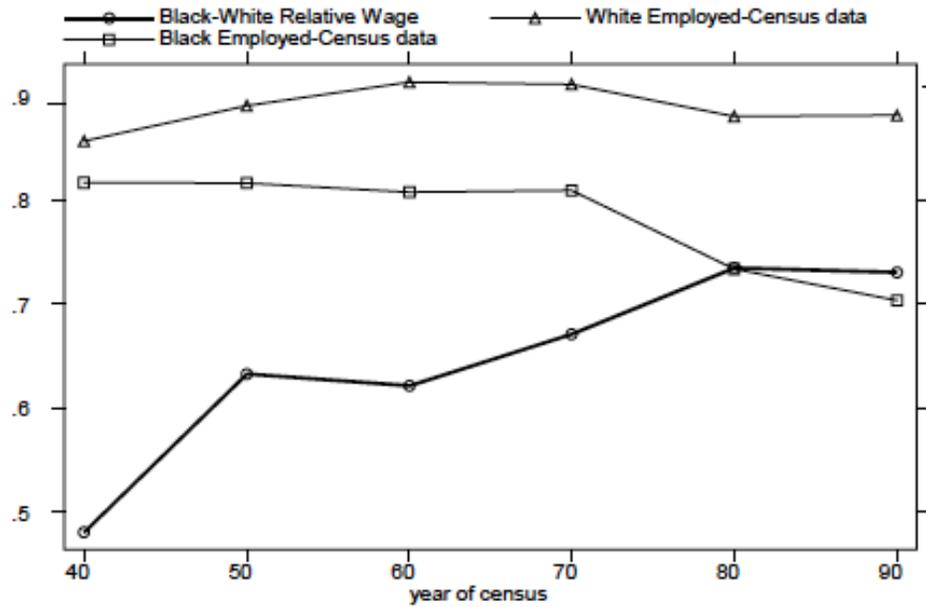
*Figure 1. Ratio of Median Earnings: Black Men/White Men, 1967–2009*

Courtesy of Kevin Lang, Jee-Yeon K. Lehmann, and the American Economic Association. Used with permission.

**Chandra (2003) Figure 1.** Lang and Lehmann argue that much less attention has been paid to racial employment and unemployment differentials than to wage differentials, although the former are in many ways more dramatic. Unlike the black-white wage gap, very little of the unemployment differential can be accounted for by education or other characteristics. Figure 1 from Chandra (2003) is reproduced below. The solid black line plots the black-white relative wage time series, similar (although over a shorter time period) to what we saw in Lang and Lehmann (2012) Figure 1. Chandra overlays on the employment-to-population ratios for blacks and (separately) whites. Over time, the employment to population ratio for blacks has fallen much faster than for whites - implying that interpreting wage gaps on their own is problematic given changing selection into the labor force. Of course, as we discussed with the Mulligan-Rubinstein paper, selection into the labor market could be changing even over time periods when the relative employment to population ratio is steady. Lang and Lehmann (2012) discuss some papers analyzing these trends: while non-participation (due to e.g. incarceration) is important, blacks also experience longer unemployment durations.

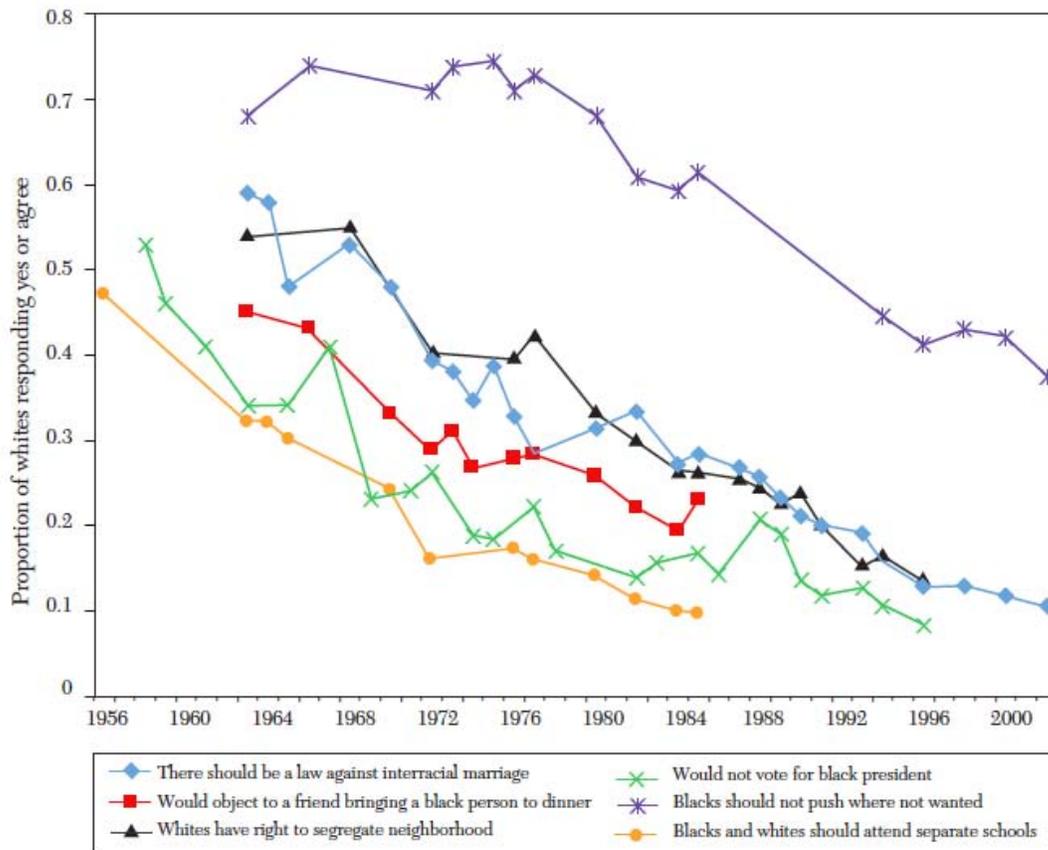
**Figure 1: Black-White Relative Wages and Employment Population Ratios, for Men aged 25-55**

**Panel A: All Schooling Groups**



Courtesy of Amitabh Chandra. Used with permission.

**Lang and Lehmann (2012) Figure 3.** This documents various measures of prejudice (primarily drawn from the General Social Society Surveys), plotting the proportion of whites responding yes or agree. There have been large declines over time in these expressions of prejudiced views on issues such as school segregation, social interaction, and blacks in politics. Lang and Lehmann are careful to caveat that whites may be more cautious in expressing what are now socially unacceptable views, but note that behavioral evidence supports some degree of real change. For example, the share of Americans reporting disapproval of marriage between a white and a black declined from 94% in 1958 to 17% in 2007; the frequency of black-white marriages increased over eight-fold over the same time period, albeit from a very low level. Whether more subtle or subconscious forms of prejudice have also declined is an ongoing topic of current research.



*Figure 3. Trends in Prejudice Measures, 1956–2003.*

Courtesy of Kevin Lang, Jee-Yeon K. Lehmann, and the American Economic Association. Used with permission.

Lang and Lehmann summarize these facts by arguing that a theory of discrimination should explain the following regularities while relying on either strong prejudice in only a small portion of the population, or widespread mild prejudice:

1. There is a notable wage gap between blacks and whites. This gap is smaller or nonexistent for very high-skill workers and possibly for very low-skill workers.
2. There is a notable employment gap between blacks and whites that is somewhat smaller

among high-skill than among low-skill workers. Black have both longer unemployment duration and a higher rate of entry into unemployment.

3. The black-white earnings gap has fallen, albeit sporadically, over the last five decades, but the unemployment gap has remained constant and may even have risen after adjusting for the increased human capital of black men in the labor force.

The authors note that statistical discrimination models generally do not address employment while taste-based search models typically do not allow for within-race heterogeneity and therefore cannot address wage differentials at different skill levels.

## 1.2 Sources of group differences

Altonji and Blank (1999) begin their discussion of theories of group differences by distinguishing between two sources of group differences:

1. Group differences in preferences, comparative advantage, and skill
2. Labor market discrimination

Our focus, and Altonji and Blank's focus, is on models of labor market discrimination, but their chapter briefly discusses the first set of hypotheses. On preferences, Altonji and Blank note that the role of group differences in preferences is emphasized primarily in discussion of gender differences rather than racial differences; a major issue with this hypothesis is the source of gender differences in preferences, and the question of how and why preferences may evolve over time. On comparative advantage, Altonji and Blank note that, historically, differences in physical strength may have given men an advantage in certain labor market tasks; however, they argue that the labor market consequences of any biologically based comparative advantage should have declined over time. On differences in skill, we will discuss several papers analyzing why pre-labor market discrimination (or expectations of future labor market discrimination) may generate group differences in human capital investments (Lundberg and Startz, 1983; Coate and Loury, 1993). Altonji and Blank (1999) conclude that differences in preferences, comparative advantage, and pre-market human capital accumulation may complement theories of discrimination.

## 1.3 Defining discrimination

It is necessary to define what we mean by discrimination. Arrow (1973) motivates his definition of discrimination as follows: *“The fact that different groups of workers, be they skilled or unskilled, black or white, male or female, receive different wages, invites the explanation that the different groups must differ according to some characteristic valued on the market. In standard economic theory, we think first of differences in productivity. The notion of discrimination involves the additional concept that personal characteristics of the worker unrelated to productivity are also valued on the market.”* Altonji and Blank (1999) adopt a similar definition. Let an individual's log wage  $Y_i$  be a function of an exogenous vector of characteristics  $X_i$  that determine worker

productivity, and an indicator  $Z_i$  for whether the individual is a member of a minority group. Then we can say that the group is discriminated against if  $\alpha < 0$  in the following equation:

$$Y_i = \beta X_i + \alpha Z_i + e_i \tag{1}$$

Altonji and Blank highlight three problems that arise with this approach. First, defining what “equal productivity” means is not straightforward. In Chapter 1 of *The Economics of Discrimination*, Becker illustrates this difficulty with an example: “...discrimination and prejudice are not usually said to occur when someone prefers looking at a glamorous Hollywood actress rather than at some other woman; yet they are said to occur when he prefers living next to whites rather than next to Negroes.” Second, the technology determining  $\beta$  may not be exogenous. Altonji and Blank cite as an example that changes in technology in fire-fighting and the military have altered the influence of physical strength on productivity, and increased the average productivity of women relative to men. Third, as discussed above, investments in human capital that determine the  $X_i$ ’s may be a function of either pre-labor market discrimination or expectations of future labor market discrimination.

## 2 Taste-based discrimination

### 2.1 Overview

Theories of discrimination can be divided into two general classes: taste-based and statistical. Gary Becker’s seminal 1957 book *The Economics of Discrimination* (Becker, 1957) developed a model of taste-based discrimination, while the key references developing models of statistical discrimination are Phelps (1972), Arrow (1973), and Aigner and Cain (1977).

Becker’s model of taste-based discrimination is based on the following idea: if an individual has a “taste for discrimination,” she must act *as if* she is willing to pay something to be associated with some persons instead of others. For example, employers who are prejudiced against black workers act as if black workers are more expensive to hire than they truly are. Becker acknowledged a strange feature of his model, famously articulated by Kenneth Arrow (Arrow, 1972a,b, 1973): because prejudiced employers sacrifice profits by discriminating, they will ultimately be driven out of the market in a long run competitive setting. Arrow famously remarked that Becker’s employer discrimination model “predicts the absence of the phenomenon it was designed to explain” (Arrow, 1972a). Subsequent research proposed several modifications to the Becker framework that can generate equilibrium wage gaps, of which we will briefly discuss three: nepotism (Goldberg, 1982), search and adjustment costs (Black, 1995; Lang, Manove and Dickens, 2005), and employer-employee transitions of prejudiced individuals (Charles and Guryan, 2008).

## 2.2 Becker (1957): Model

Becker's 1957 book is worth reading, but covers more ground than we have time to discuss in this class. We'll follow the version of Becker's employer discrimination model laid out by [Altonji and Blank \(1999\)](#).

There are two groups:  $a$  is the majority group, and  $b$  is the minority group. Assume that members of the two groups are perfect substitutes in production. Define employer discrimination as a situation in which some employers are prejudiced against members of group  $b$ . Employers maximize a utility function that is the sum of profits plus the monetary value of utility from employing members of particular groups. Let  $d$  be Becker's "coefficient of discrimination" - the taste parameter of the firm. Then we can write the firm maximization problem as:

$$U = pF(N_b + N_a) - w_a N_a - w_b N_b - d N_b \quad (2)$$

where  $p$  is the price,  $F$  is the production function,  $N_g$  is the employment of members of group  $g$  ( $g \in a, b$ ), and  $w_g$  is the wage paid to members of group  $g$ .

Employers who are prejudiced have  $d > 0$ , and act as if the price of hiring a  $b$  worker is  $w_b + d$ . The firm will hire workers from group  $b$  only if  $w_a - w_b \geq d$ . Let  $G(d)$  denote the CDF of the prejudice parameter  $d$  in the population of firms. Firms will choose  $N_a$  and  $N_b$  according to:

$$\frac{dU}{dN_a} = 0 \quad (3)$$

$$\Rightarrow pF'(N_b + N_a) = w_a \quad (4)$$

for firms that hire  $a$  workers, and:

$$\frac{dU}{dN_b} = 0 \quad (5)$$

$$\Rightarrow pF'(N_b + N_a) = w_b + d \quad (6)$$

for firms that hire  $b$  workers. The number of workers hired is decreasing in  $w_a$  for firms employing  $a$  workers, and decreasing in  $w_b + d$  for firms employing  $b$  workers. Treating the price  $p$  as fixed and aggregating across firms in the economy leads to the market demand function  $N_b^d(w_a, w_b; G(d))$  for  $b$  workers and  $N_a^d(w_a, w_b; G(d))$  for  $a$  workers. Wages for the two groups are determined by the equalization of supply and demand:

$$N_a^d(w_a, w_b; G(d)) = N_a^s(w_a) \quad (7)$$

$$N_b^d(w_a, w_b; G(d)) = N_b^s(w_b) \quad (8)$$

where  $N_a^s(w_a)$  and  $N_b^s(w_b)$  are market supply functions.

A wage differential ( $w_b < w_a$ ) will arise if and only if the share of employers who are

prejudiced against members of group  $b$  is sufficiently high that the demand for  $b$  workers when  $w_b = w_a$  is less than the supply. That is, even if some employers discriminate, if they are a small share of the market minority workers will simply work at non-discriminatory employers, and there will be no wage differential observed in equilibrium.

On the other hand, if prejudiced firms are a sufficiently large share of the market, then some  $b$  workers will work at employers with  $d > 0$ , in which case there will be a wage differential ( $w_b < w_a$ ). Just as in the Rosen-style compensating differentials model, we have:

1. Sorting:  $b$  workers are employed by the least prejudiced firms. Only marginal firms will hire both groups of workers.
2. What matters is preference of the the marginal firm: the price on the attribute  $d$  is determined by the preference of the least prejudiced employer who hires  $b$  workers, *not* by the average prejudice among firms: what matters is the  $d$  for the marginal firm that employs  $b$  workers.

Becker and others (famously, Arrow) noted that if there is free entry or constant returns to scale, then prejudicial employers should be competed out of the market. In a competitive labor market, workers are paid their marginal product. Because they have lower costs, non-prejudiced employers will expand to the point where it is no longer necessary for  $b$  workers to work for prejudiced firms (implying that the wage gap will be eliminated). That is, market forces should eliminate discrimination unless it cannot provide sufficient segregation. Since not all firms are completely segregated, this prediction stands in sharp contrast to observed wage differentials.

### 2.3 Becker (1957): Implications and revisions

Subsequent research proposed several modifications to the Becker framework that can generate equilibrium wage gaps, of which we will discuss three: nepotism (Goldberg, 1982), search and adjustment costs (Black, 1995; Lang, Manove and Dickens, 2005), and employer-employee transitions of prejudiced individuals (Charles and Guryan, 2008).

Becker (1957) also discusses the consequences of employee discrimination and customer discrimination, which we will not cover here.

### 2.4 Modification #1: Nepotism

Goldberg (1982) argues that the objections raised against Becker's model are "essentially misdirected," arguing that once one reformulates the model in terms of "nepotism" towards whites instead of "discrimination" against blacks that Becker's theory is consistent with the existence and persistence of wage differentials.

Goldberg clarifies that the Becker framework views firms as maximizing utility rather than profits, the difference being the utility earned by the firm from choosing employees according to its taste parameter. The key difference between Goldberg's model and Becker's model is that Goldberg re-writes  $d$  as a nepotism coefficient rather than as a discrimination coefficient:

the firm acts as if the white wage was lower than it actually is, because the firm earns some non-monetary utility from hiring white workers.

Importantly, in Goldberg's framework the sellout price of a firm is not equal to its money profit level, but rather its utility level. Whereas in the original Becker model discriminating employers should be willing to sell their firm to non-discriminators (who can earn higher profits), in the nepotism case nepotistic employers earn a non-pecuniary return from staying in the market. Goldberg's model can thus generate long-run wage differentials (unlike the Becker model).

## 2.5 Modification #2: Search and adjustment costs

Two general types of search models have been explicated: random search models and directed search models. [Black \(1995\)](#) presents a random search model of employer discrimination with search costs in which he assumes that information about employment opportunities is costly to attain, and that workers randomly encounter firms in the market. Search costs imply that workers will sequentially search across employers to look for a good employment match. In this type of model, workers accept a job/wage if the expected value of the offer is greater than or equal to the expected value of additional search (taking into account the costs that would be incurred by additional search). The equilibrium is determined by the workers' reservation wage. With prejudiced employers in the market, minorities face a lower probability of finding a position that dominates their current offer, lowering their reservation wage. Because of this lower reservation wage, minorities are willing to accept an offer with a lower wage, which provides all employers (not just prejudiced employers) an incentive to offer minorities lower wages. In equilibrium, minority workers are employed only at unprejudiced firms, but they earn lower wages than comparable nonminority workers whenever any prejudiced firms are in the market. That is, unlike the Becker model, in Black's model the whole distribution of prejudicial tastes matter, not just the prejudice of the marginal firm.

[Lang, Manove and Dickens \(2005\)](#) offer a directed search model of employer discrimination in labor markets with posted wage offers: employers attach wage offers to announced job openings, a commonly observed labor market phenomenon, and workers subsequently decide where to apply (hence "directed" search). Employers in their model find black workers slightly less desirable than white workers; these differences are small, but are sufficient to ensure that employers will choose a white worker if both a white and black worker apply for the same job. As a result, black workers want to avoid the cost of applying to firms that are likely to receive applications from whites, and thus apply to firms with wage offers that are low enough to discourage white applicants. In equilibrium, blacks and whites are employed by different firms, and blacks receive lower wages.

[Borjas and Bronars \(1989\)](#) present a model of customer discrimination in a search model, where customers search for low prices but have a distaste for buying from minority entrepreneurs. [Saski \(1999\)](#) presents a model of employee discrimination that incorporates search costs.

## 2.6 Modification #3: Employer-employee transitions

In the NBER working paper version of [Charles and Guryan \(2008\)](#), the authors illustrate a third modification to the Becker model that can generate long-run wage differentials. They argue that in the long run prejudiced employers have two options: they can be unprofitable, or can shut down and transition to instead be a worker at another firm. If prejudiced employers consider the outside option of the co-worker interactions they will have if they shut down the firm, it does not necessarily follow that prejudiced employers shut down in the long run. Charles and Guryan walk through a version of this model in the theory appendix of their NBER working paper (#13661).

## 3 Statistical discrimination

In contrast to models of taste-based discrimination, models of statistical discrimination apply even if employers have no distaste for minority workers. In addition, wage differentials can persist in equilibrium in models of statistical discrimination, as opposed to being “competed away” in the original Becker framework.

There have been two main strands of literature on statistical discrimination. The first strand of literature has investigated the consequences of group differences in the precision of information that employers have about individual productivity; [Aigner and Cain \(1977\)](#) is the classic reference. This imperfect information problem gives firms an incentive to rely on observable characteristics (such as race or gender) to the extent they are correlated with productivity.

The second strand of literature has investigated how prior beliefs (stereotypes) about the productivity of group members can influence hiring and pay decisions. An important issue in this second literature is whether biased employer beliefs may be self confirming when the payoff for difficult-to-observe worker investments depends on employer beliefs. The key references on this topic are [Phelps \(1972\)](#), [Arrow \(1973\)](#), and - more recently - [Coate and Loury \(1993\)](#). For this second strand of literature, we will here focus on discussing the [Phelps \(1972\)](#) model; in a later lecture we will cover the [Coate and Loury \(1993\)](#) model in detail.

### 3.1 [Aigner and Cain \(1977\)](#): Model

Employers base hiring decisions on an indicator of skill  $y$  (say, a test) that measures a worker’s true skill level  $q$ . The measurement equation is:

$$y = q + u \tag{9}$$

where  $u \sim N(0, \sigma_u^2)$ ,  $u$  is independent of  $q$ , and  $q \sim N(\alpha, \sigma_q^2)$ . Employers observe  $y$  but not  $q$ , and use  $y$  to extract information about  $q$ . We are interested in deriving an expression for  $\hat{q}$ , an employer’s predicted value of true skill  $q$  given the observed indicator of skill  $y$ :  $\hat{q} = E[q|y]$ .

Because  $u$  is independent of  $q$ , we know that  $cov(u, q) = 0$  and that  $q$  and  $u$  are joint normally distributed:  $\begin{pmatrix} q \\ u \end{pmatrix} \sim N\left(\begin{pmatrix} \alpha \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_q^2 & 0 \\ 0 & \sigma_u^2 \end{pmatrix}\right)$ . Because  $y$  is a linear combination of two

normally distributed random variables, we know that  $y \sim N(\alpha, \sigma_q^2 + \sigma_u^2)$ .

Recall from our Roy model lecture notes that a property of the bivariate normal distribution is that if  $X$  and  $Y$  are jointly normally distributed with means  $\mu_x$  and  $\mu_y$ , variances  $\sigma_x^2$  and  $\sigma_y^2$ , and correlation  $\rho_{X,Y}$ , then the conditional distribution of  $Y$  given  $X = x$  is normally distributed  $\sim N\left(\mu_y + \rho_{X,Y}\left(\frac{\sigma_y}{\sigma_x}\right)(x - \mu_x), \sigma_y^2(1 - \rho_{X,Y}^2)\right)$  where  $\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X\sigma_Y}$ .<sup>1</sup> Thus, because  $q$  and  $y$  are jointly normally distributed then we can derive an expression for  $\hat{q}$  as:

$$\hat{q} = E(q|y) \tag{10}$$

$$= \mu_q + \rho_{y,q} \frac{\sigma_q}{\sigma_y} (y - \mu_y) \tag{11}$$

$$= \mu_q + \frac{\text{cov}(y,q)}{\sigma_y\sigma_q} \frac{\sigma_q}{\sigma_y} (y - \mu_y) \tag{12}$$

$$= \mu_q + \frac{\text{cov}(y,q)}{\sigma_y^2} (y - \mu_y) \tag{13}$$

Substituting  $\mu_q = \mu_y = \alpha$ , we have:

$$\hat{q} = \alpha + \frac{\text{cov}(y,q)}{\sigma_y^2} (y - \alpha) \tag{14}$$

$$= \alpha - \alpha \frac{\text{cov}(y,q)}{\sigma_y^2} + y \frac{\text{cov}(y,q)}{\sigma_y^2} \tag{15}$$

$$= \alpha \left(1 - \frac{\text{cov}(y,q)}{\sigma_y^2}\right) + y \frac{\text{cov}(y,q)}{\sigma_y^2} \tag{16}$$

Letting  $\gamma$  denote  $\frac{\text{cov}(y,q)}{\sigma_y^2}$ , we thus have:

$$\hat{q} = (1 - \gamma)\alpha + \gamma y \tag{17}$$

This is a signal extraction problem: under the assumptions laid out above, the expectation of a worker's productivity is a weighted average of her test score  $y$  and the group average  $\alpha$ , where the weights are determined by  $\gamma$ . Note that  $\gamma$  can be re-written as:

$$\gamma = \frac{\text{cov}(y,q)}{\sigma_y^2} \tag{18}$$

$$= \frac{\text{cov}(q+u,q)}{\text{var}(q+u)} \tag{19}$$

$$= \frac{\text{cov}(q,q) + \text{cov}(u,q)}{\text{var}(q) + \text{var}(u) + 2\text{cov}(q,u)} \tag{20}$$

$$= \frac{\text{var}(q) + 0}{\text{var}(q) + \text{var}(u) + 2 \cdot 0} \tag{21}$$

$$= \frac{\text{var}(q)}{\text{var}(q) + \text{var}(u)} \tag{22}$$

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<sup>1</sup>Details: see, e.g., page 175-177 of [Casella and Berger \(2001\)](#).

Writing  $\gamma = \frac{\text{var}(q)}{\text{var}(q) + \text{var}(u)}$  clarifies that if the test is less informative (higher  $\text{var}(u)$ ) then gamma will be smaller, and employers will put more weight on the group average  $\alpha$  in their estimate  $\hat{q}$ .

Now consider two groups of workers: whites and blacks. The two groups have possibly different means ( $\alpha_w$  and  $\alpha_b$ ) and possibly different variances of  $q$  and  $u$ . Employers are assumed to pay workers based on the information available information for each group:

$$\hat{q}^w = (1 - \gamma^w)\alpha^w + \gamma^w y \quad (23)$$

$$\hat{q}^b = (1 - \gamma^b)\alpha^b + \gamma^b y \quad (24)$$

The  $\gamma$  term will generally differ across groups ( $\gamma^w \neq \gamma^b$ ) if the variances of  $q$  and  $u$  differ. For example, if the test is more informative for whites ( $\text{var}(u_b) > \text{var}(u_w)$ ) then  $\gamma_w$  will be larger than  $\gamma_b$ , and employers will place more weight on individual test scores for whites than blacks.

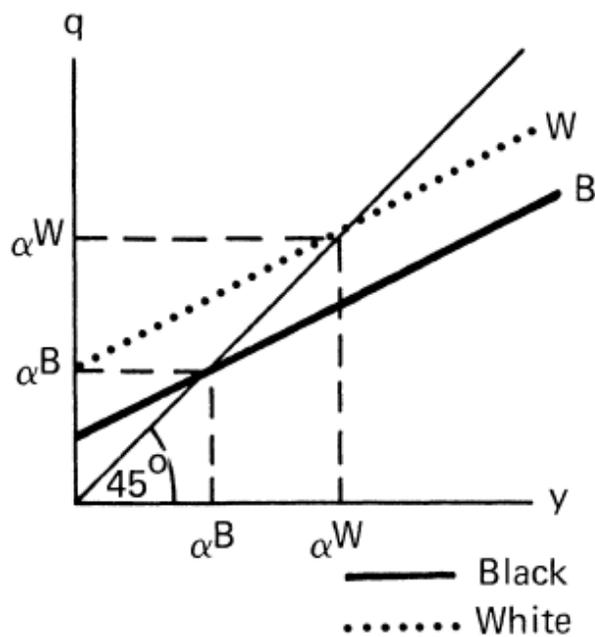
Two special cases of this model are frequently exposted:

1. Mean differences, equal variances:  $\alpha_b < \alpha_w$ ,  $\text{var}(u_b) = \text{var}(u_w)$ , and  $\text{var}(q_b) = \text{var}(q_w)$ .
2. Equal means, different variances:  $\alpha_b = \alpha_w$ ,  $\text{var}(u_b) > \text{var}(u_w)$ , and  $\text{var}(q_b) = \text{var}(q_w)$ .

### 3.1.1 Mean differences, equal variances

First, assume  $\alpha_b < \alpha_w$ ,  $\text{var}(u_b) = \text{var}(u_w)$ , and  $\text{var}(q_b) = \text{var}(q_w)$ . For example, Phelps (1972) discusses this case, where employers view blacks as having lower skill level on average but the test is equally informative for blacks and whites.

Recall our expression for  $\hat{q}^g$  for group  $g$ :  $\hat{q}^g = (1 - \gamma^g)\alpha^g + \gamma^g y$ . Aigner and Cain's Figure 5 illustrates this case graphically, plotting test score  $y$  on the  $x$ -axis and predicted skill  $\hat{q}$  on the  $y$ -axis.



**Figure 5.** Prediction of Productivity ( $q$ ), by Race and Test Score ( $y$ ), Assuming the Slopes Are Equal.

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Because of the differences in means ( $\alpha_b < \alpha_w$ ), the predicted  $\hat{q}$  will be lower for blacks relative to whites for a given test score  $y$ . That is, even though the test score is an unbiased signal for both workers, the expected productivity of blacks is lower than the expected productivity of whites. The lines for whites and blacks have equal slopes because of the assumption of equal variances for  $q$  and  $u$  (and hence for  $y$ ). Note that when  $y_i = \alpha^g$ , the expectation of  $q_i$  conditional on  $y_i$  will equal  $\alpha^g$ :

$$\hat{q}^g = (1 - \gamma^g)\alpha^g + \gamma^g y^g \quad (25)$$

$$= \alpha^g - \gamma^g \alpha^g + \gamma^g \alpha^g \quad (26)$$

$$= \alpha^g \quad (27)$$

This clarifies why the black line ( $B$ ) intersects the 45 degree line at  $\alpha_B$ , and why the white line ( $W$ ) intersects the 45 degree line at  $\alpha_W$ .

Note that because  $u$  is mean zero, the expectation of the test score  $y_i$  conditional on  $q_i$  is equal to  $q_i$  for each group:  $E(y_i|q_i, g) = E(q_i + u_i|q_i, g) = q_i + E(u_i|q_i, g) = q_i$  since  $E(u_i|q_i, g) = 0$  given that  $u$  is independent of  $q$ . That is, the expectation of the productivity signal ( $y_i$ ) is equal to true productivity ( $q_i$ ): the signal is unbiased. However, in general it will not be true that  $E(q_i|y_i, g) = q_i$ .<sup>2</sup> In words, the expectation of the productivity signal ( $y_i$ ) is equal to true productivity ( $q_i$ ); however, the expectation of productivity given the signal is - in general - not equal to actual productivity. There is not equal pay for equal productivity, but there is equal pay for equal expected productivity.

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<sup>2</sup>This will be true in some special cases, for example if there is no noise in the test score ( $\sigma_u^2 = 0$ ), in which case  $\gamma = 1$  and  $E(q_i|y_i, g) = y_i = q_i$ .

### 3.1.2 Equal means, different variances

Next, assume  $\alpha_b = \alpha_w$  and  $\text{var}(u_b) > \text{var}(u_w)$ ;  $\text{var}(q_b) = \text{var}(q_w)$ . Aigner and Cain (1977) discuss this case, where blacks and whites have the same skill level on average but the test is more informative for whites than blacks. For example, differences in cultural characteristics such as language may make it more difficult for employers to understand blacks than whites (Lang, 1986).

Aigner and Cain's Figure 1B illustrates this case graphically, plotting test score  $y$  on the  $x$ -axis and predicted skill  $\hat{q}$  on the  $y$ -axis. Each worker is paid according to her expected productivity, resulting in equal average wages for the two groups (given the assumption of equal mean skill levels across groups). The line for whites is steeper than the line for blacks: because  $\text{var}(u_b) > \text{var}(u_w)$ , the  $\gamma$  term is smaller for blacks than for whites, implying that for blacks more weight is placed on the group average relative to the individual test score; hence,  $\hat{q}$  is less sensitive to  $y$  for blacks relative to whites. This rotation implies that whites with scores  $y$  above the mean receive higher wages than blacks, and the reverse is true for  $y$  scores below the mean (blacks receive higher wages than whites). There is discrimination in the sense that there is different pay granted to individuals with the same test score.

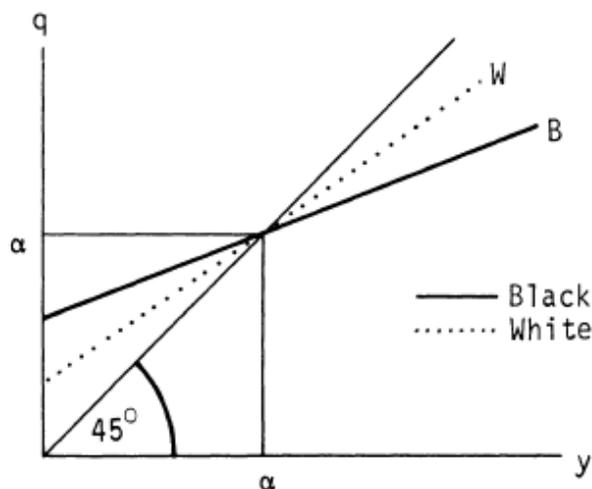


Figure 1B. Predictions of Productivity ( $q$ ) by Race and Test Score ( $y$ ), Assuming a Steeper Slope for Whites.

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### 3.1.3 Defining discrimination

If there is equal pay for equal expected productivity, does that constitute discrimination? Obviously different definitions of discrimination could lead to different conclusions. Lundberg and Startz (1983) propose an alternative definition of discrimination, which is when groups with equal average initial endowments of productive ability do not receive equal average compensation in equilibrium. Their goal in adopting that definition is to extend the Aigner-Cain framework to

accommodate endogenous human capital investments that may be made in the presence of labor market discrimination.

## References

- Aigner, Dennis and Glen Cain**, “Statistical theories of discrimination in labor markets,” *Industrial and Labor Relations Review*, 1977, 30 (2), 175–187.
- Altonji, Joseph and Rebecca Blank**, “Race and gender in the labor market,” in Orley Ashenfelter and David Card, eds., *Handbook of Labor Economics Volume 3C*, 1999, pp. 3143–3259.
- Arrow, Kenneth**, “Models of job discrimination,” in Anthony H. Pascal, ed., *Racial Discrimination in Economic Life*, 1972, pp. 187–204.
- , “Some mathematical models of race in the labor market,” in Anthony H. Pascal, ed., *Racial Discrimination in Economic Life*, 1972, pp. 83–102.
- , “The theory of discrimination,” in Orley Ashenfelter and Albert Rees, eds., *Discrimination in Labor Markets*, 1973, pp. 3–33.
- Becker, Gary**, *The Economics of Discrimination*, University of Chicago Press, 1957.
- Black, Dan**, “Discrimination in an equilibrium search model,” *Journal of Labor Economics*, 1995, 13 (2), 309–334.
- Borjas, George and Stephen Bronars**, “Consumer discrimination and self-employment,” *Journal of Political Economy*, 1989, 97 (3), 581–605.
- Casella, George and Roger Berger**, *Statistical Inference, Second Edition*, Duxbury Press, 2001.
- Chandra, Amitabh**, “Is the convergence in the racial wage gap illusory?,” 2003. NBER working paper #9476.
- Charles, Kerwin and Jonathan Guryan**, “Prejudice and wages: An empirical assessment of Becker’s The Economics of Discrimination,” *Journal of Political Economy*, 2008, 116 (5), 773–809.
- Coate, Stephen and Glenn Loury**, “Will affirmative action eliminate negative stereotypes?,” *American Economic Review*, 1993, 83 (5), 1220–1240.
- Goldberg, Matthew**, “Discrimination, nepotism, and long-run wage differentials,” *Quarterly Journal of Economics*, 1982, 97 (2), 307–319.
- Lang, Kevin**, “A language theory of discrimination,” *Quarterly Journal of Economics*, 1986, 101 (2), 363–382.
- **and Jee-Yeon Lehmann**, “Racial discrimination in the labor market: Theory and empirics,” *Journal of Economic Literature*, 2012.
- , **Michael Manove, and William Dickens**, “Racial discrimination in labor markets with posted wage offers,” *American Economic Review*, 2005, 95 (4), 1327–1340.
- Lundberg, Shelly and Richard Startz**, “Private discrimination and social intervention in competitive labor markets,” *American Economic Review*, 1983, 73 (3), 340–3347.
- Phelps, Edmund**, “The statistical theory of racism and sexism,” *American Economic Review*, 1972, 62 (4), 659–661.
- Saski, Masaru**, “An equilibrium search model with coworker discrimination,” *Journal of Labor Economics*, 1999, 17 (2), 377–407.

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