Recitation 8

What's game theory?

- Traditional economics
  - my decision affects my welfare but not other people's welfare
  - e.g.: I'm in a supermarket - whether I decide or not to buy a tomato does not affect another customer's welfare (it doesn't affect the price of tomatoes) and it does not affect the company's profits (markets clear - so if I don't buy this tomato, someone else will)
  - idea: when there is a market, a given customer or a given company are too small to affect other people's welfare in a significant way
- But that does not always hold. Examples?
  - market served only by 2 firms (duopole): if firm A decreases its price, it affects the share of consumers captured by firm B and the profits it makes
  - soccer, penalty kick: a goalie who was to decide whether to dive left or right; a striker who has to target left or right

What's game theory?

- Game theory was designed to model this kind of situations
  - small number of players
  - what each player does affects not only his welfare but also other players' welfare
  - players can choose simultaneously, or sequentially. We focus on the first case here (and in the pset).

Payoff matrix

- Imagine a game with 2 players: Player 1, Player 2
- Strategies
  - Player 1 has 2 possible strategies: he can play "Top" or "Bottom"
  - Player 2 has 2 possible strategies: he can play "Left" or "Right"
  - So there are 4 possible cases: "Top" "Left" (1 plays "Top" and 2 plays "Left"), "Top Right", "Bottom Left" and "Bottom Right"

Payoff matrix

- Payoffs
  - payoffs obtained by Player 1 in all 4 cases

		Player 2: Left	Player 2: Right
Player 1	Top	5	3
	Bottom	6	8

  - payoffs obtained by Player 2 in all 4 cases

		Player 2: Left	Player 2: Right
Player 1	Top	-1	3
	Bottom	4	2

- Payoff matrix: let's put all the relevant information together

		Player 2: Left	Player 2: Right
Player 1	Top	5,-1	3,3
	Bottom	6,4	8,2

Dominant strategy

- In this game, for Player 1, the strategy “Bottom” **strictly dominates** the strategy “Top”. Indeed:
  - suppose Player 2 chooses “Left”. Then Player 1 is strictly better choosing “Bottom”: $6 > 5$
  - suppose Player 2 chooses “Right”. Then Player 1 is strictly better choosing “Bottom”: $8 > 3$
  - So, for Player 1, the payoff from “Bottom” is strictly greater than the payoff from “Top”, regardless what Player 2 does
  - Even without knowing what strategy Player 2 chooses, Player 1 knows that he should play “Bottom”

Prisoner's dilemma

- Setting:
  - 2 men are arrested
  - The police do not have anough information to convict them
  - They put the 2 men in separate rooms and offer both the same deal:
    - if one betrays the other, and the other remains silent, the betrayer goes free and the other goes to jail for 10 years
    - if both remain silent, both go to jail for 1 year
    - if both betray, they both go to jail for 4 years
- Payoff matrix?

		Prisoner 2	
		Betrays	Silent
Prisoner 1	Betrays	-4,-4	0,-10
	Silent	-10,0	-1,-1

Prisoner's dilemma

- Outcome of the game?
  - Betraying is a strictly dominant strategy for Player 1
  - Betraying is a strictly dominant strategy for Player 2
  - They both betray and get -4,-4 - when both remaining silent would have been better for both
  - Why? They would have liked to coordinate, but could not
- But, at least, we can solve the game, looking at strictly dominant strategies. Is it always the case?

Iterative deletion of strictly dominated strategies.

- Let's go back to our initial game:

		Player 2: Left	Player 2: Right
Player 1	Top	5,-1	3,3
	Bottom	6,4	8,2

- Remember: Player 1 has a strictly dominant strategy: "Bottom"
- Does Player 2 have a strictly dominant strategy?
- Can we say more?
  - Player 2 knows that Player 1 will play "Bottom": he can rule out the possibility that 1 plays "Top", he can delete this strictly dominated strategy
  - Thus, he decides to play "Left": $4 > 2$
  - Although Player 2 did not have a strictly dominant strategy, we have solved the game.

Best responses and Nash equilibrium

- Can we always solve games using this method (iterative deletion of strictly dominated strategies)? Unfortunately no.
- Consider a slightly different version of the game:

		Player 2: Left	Player 2: Right
Player 1	Top	5,-1	3,3
	Bottom	6,4	2,2

- Now, does any player have a strictly dominant strategy? What can we do?

Best responses and Nash equilibrium

1. The concept of Best response
   - Player 1: What is his best response to “Left”? and to “Right”?
   - Player 2: What is his best response to “Top”? and to “Bottom”?
2. Nash equilibrium
   - A combination of strategies that are best responses to each other
   - No player wants to deviate from the equilibrium: satisfying solution to the game, even if no dominant strategy
   - Show that “Top, Right” is a Nash equilibrium
   - Is there another Nash equilibrium?

Is there always a Nash equilibrium?

- Consider the following game:

		Player 2	
		Left	Right
Player 1	Top	5,5	8,2
	Bottom	9,1	2,8

- Is there a Nash equilibrium in this game?
- Lesson: there might be 0, 1, or more than 1 Nash equilibria

Mixed strategies

- Pure and mixed strategies
  - so far we only considered **pure strategies**: we were forcing both players to choose 1 and only 1 strategy, and play it with probability 1
  - mixed strategy: Player 1 plays "Top" with probability $x \in [0, 1]$ and "Bottom" with probability $1 - x$
- Expected payoffs of mixed strategies
  - if Player 2 plays "Left" and Player 1 chooses the strategy x, he gets $5x + 9\,(1 - x)$
  - if Player 2 plays "Right", Player 1 gets $8x + 2\,(1 - x)$

Mixed strategy Nash equilibrium

- Now suppose both players play a mixed strategy
  - Player 1 plays mixed strategy x ("Top" with probability $x \in [0, 1]$ and "Bottom" with probability $1 - x$)
  - Player 2 plays mixed strategy y ("Left" with probability $y \in [0, 1]$ and "Right" with probability $1 - y$)
- Mixed strategy x is a best response to y if each of the pure strategies played with non-0 probability in the mix ("Top" and "Bottom") are best responses to y - ie they must yield the exact same payoff
- **Mixed strategy Nash equilibrium**: both mixed strategies are best responses to each other

Mixed strategy Nash equilibrium

- Let's go back to our game, where we did not find any pure strategy NE

		Player 2	
		Left	Right
Player 1	Top	5,5	8,2
	Bottom	,1	2,8

- Is there a Mixed strategy NE in this game? How can we find it?
- Suppose there is, and let's call the 2 mixed strategies x and y.

Mixed strategy Nash equilibrium

- For x to be a best response to y we need that both "Top" and "Bottom" be best responses to y.
    - payoff to play "Top" $= 5y + 8\,(1-y)$
    - payoff to play "Bottom" $= 9y + 2\,(1-y)$
    - if "Top" and "Bottom" are both best responses, their payoff must be equal: $5y + 8\,(1-y) = 9y + 2\,(1-y)$ and $y = 0.6$
- Similarly: for y to be a best response to x we need that both "Left" and "Right" be best responses to x.
    - payoff to play "Left" $= 5x + 1\,(1-x)$
    - payoff to play "Right" $= 2x + 8\,(1-x)$
    - if "Left" and "Right" are both best responses, their payoff must be equal: $5x + 1\,(1-x) = 2x + 8\,(1-x)$ and $x = 0.7$
- This gives us a Mixed Strategy Nash Equilibrium: $[[0.7, 0.3]\,;[0.6, 0.4]]$

Final considerations

- There can be more than 2 players, and 2 strategies per player
- The pset mentions "symmetric" mixed strategy NE:
  - 2 players can choose between the same set of strategies
  - they choose fixed strategies that put same probabilities on the available pure strategies
- When you are asked to tell what is the NE:
  - define it by the (pure or mixed) strategies chosen by the players
  - NOT by the payoffs received under the equilibrium

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