

6.007 – Applied Electromagnetics
 Spring 2011

Problem Set 10

Due: Friday, May 6, 2011

The reading for this problem set is chapter 8 from French and Taylor.

Problem 10.1 – Homojunction p-n LED

- (a) Draw the energy level diagram of a homojunction p-n Light Emitting Diode in the zero bias condition and the forward bias conditions. Label which side of the junction is p type and n type. Indicate the relative positions of the Fermi energy levels and the direction in which voltage is applied.
- (b) What is the size of the bandgap needed for the LED to emit red light at 650 nm? Green light at 510 nm?
- (c) You are the CTO of a company which makes white light LEDs with outstanding efficiency of 100 lm/W. You have a customer who puts the diode in a circuit with a variable power supply and a 400 Ω resistor and measured 0.3 V of voltage across the LED. How many lumens should he expect to get from the LED? Assume the LED is operated at 300°K and has a saturation current I_s of 0.5 μ A
- (c) Your customer decides he wants more lumens of light for his experiment and has planned to use the power supply at its maximum capacity of 30 V without changing the rest of the setup. However, you know that the LED will burn out for currents above 65 mA. Should you tell your technical support staff to warn the customer? Assume the voltage drop across the LED is small compared with the voltage drop across the resistor.

Problem 10.2 – Energy Bands and Optoelectronic Devices

Energy band diagrams of two different semiconductors are drawn below, each as undoped, p-type doped, or n-type doped semiconductor.

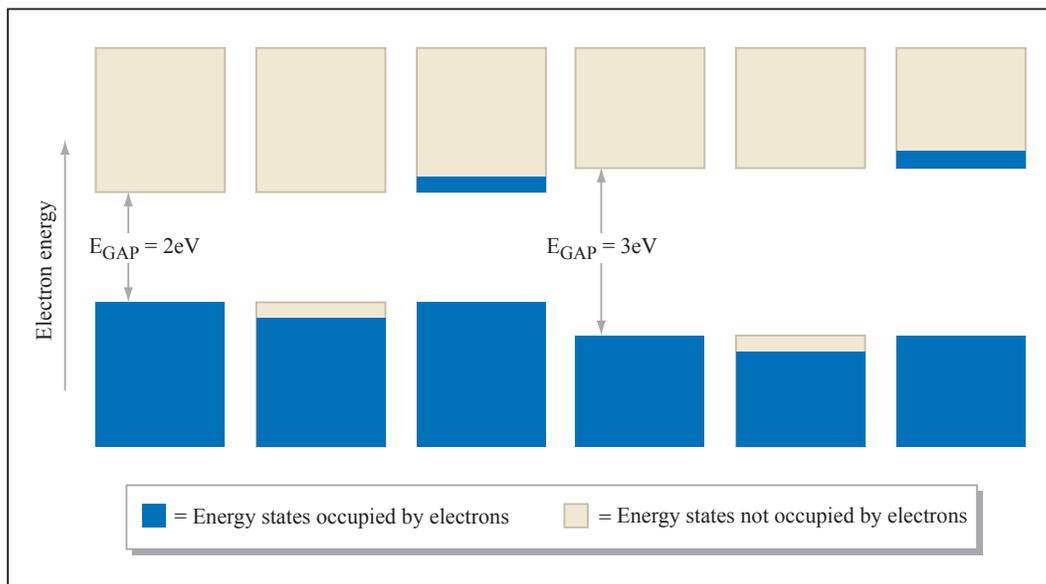
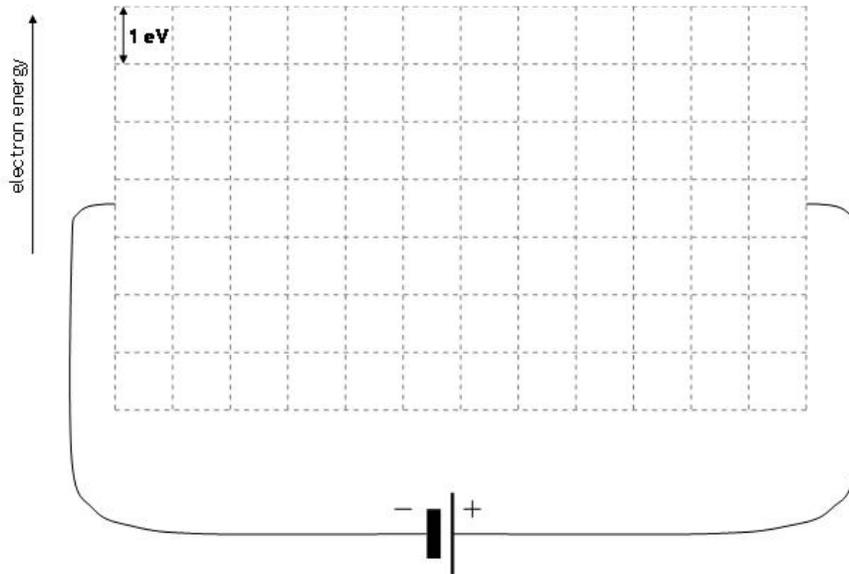
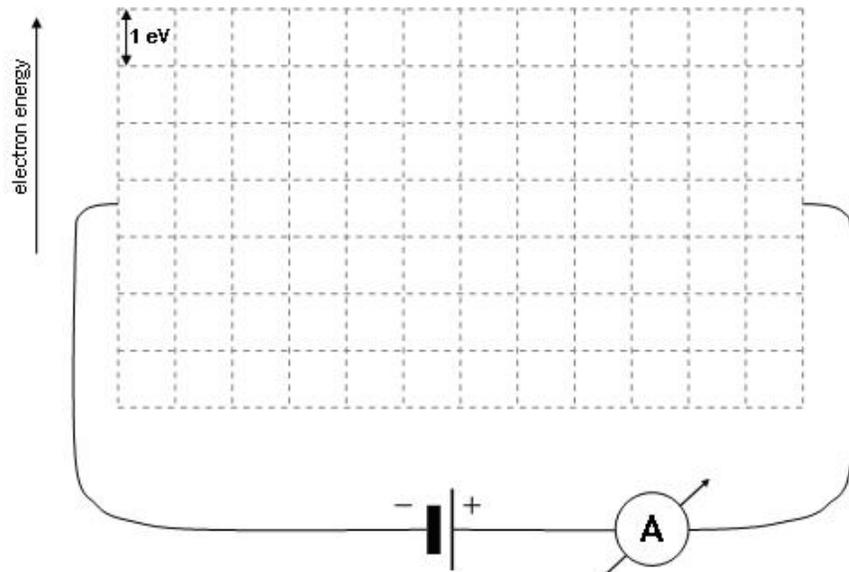


Image by MIT OpenCourseWare.

- (a) Label each semiconductor as undoped, p-type doped, or n-type doped.
- (b) Using the semiconductors from part (a) you are asked to design a light emitting device that emits red color. Please draw on the energy diagram below the series of semiconducting layers that would form the device. Label the band gap and type of each of the materials you use. Pay attention to the polarity of the battery attached below.



- (c) Using the semiconductors from part (a) you are asked to design a photodetector that is sensitive to blue color photons and other higher energy photons. Please draw on the energy diagram below the series of semiconducting layers that would form the device. Label the band gap and type of each of the materials you use. Pay attention to the polarity of the battery attached below.



Problem 10.3 –Flash Memory

Flash memory stores information by changing the charge on a floating gate (a piece of metal that is not contacted to other conductors). We interpret a floating gate with a positive charge as a 1 and a floating gate with a negative charge as a 0. The details by which the flash memory is read (e.g. how we detect the presence of a positive or negative charge) is a topic best left to 6.012. A one sentence explanation is: when there is a positive charge on the floating gate, current can flow from the source to the drain and we read a 1; when there is negative charge on the gate, current cannot flow and we read a 0.

The figure below shows the floating gate flash memory structure. The dielectric constant of SiO_2 is $3\epsilon_0$. The thickness of the oxide between the substrate and the floating gate is 10 nm and the thickness of the oxide between the control gate and floating gate is 45 nm. The floating gate is 10 nm thick.

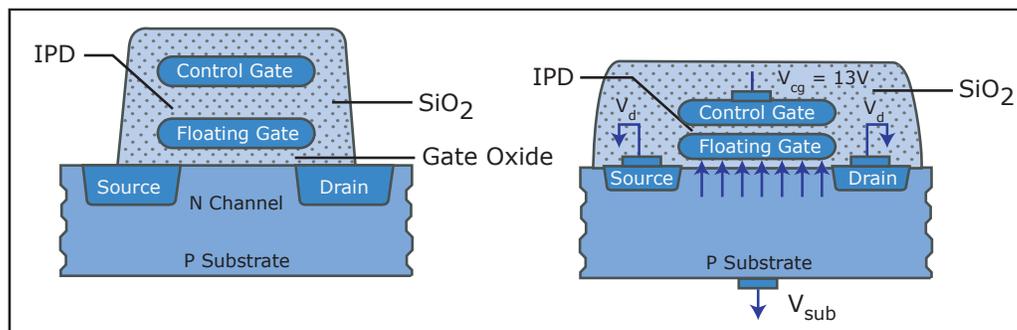
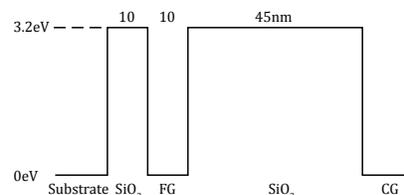


Image by MIT OpenCourseWare.

The interesting part (that you won't learn in 6.012) is how we *change* the charge on the floating gate. To write a bit, we must somehow put electrons on or take electrons off of the isolated bit of metal. The write operation is accomplished through electron tunneling and is explored below.

To begin, let's assume that we have already written information to our memory and charge is stored on the floating gate. Let's further assume that we have an excess of electrons on the floating gate (thus the bit represented is 0). We are interested in how long the bit will remain a 0. We know that the electrons will eventually tunnel through the oxide, depleting the charge on the gate. When the charge on the gate approaches zero (neutral), we will no longer be sure of the value of the bit.

If a single free electron is present in the floating gate, it will see the potential barrier shown below. The 3.2 eV potential barrier represents the energy difference between the conduction band of metal versus the conduction band SiO_2 (an insulator). The electron has an energy given by $E = k_b T/2$ where T is the temperature in Kelvin and k_b is the Boltzmann constant.



- What is the energy of the electron at room temperature, $T = 300$ K? What is its speed in the classical picture? The effective mass of the electron in the floating gate is $m_e^* = 0.26m_e$.
- Estimate the energy level n of the electron using the energy level spacings of the infinite potential well, $E_n = \hbar^2 \pi^2 n^2 / (2mL^2)$.

- (c) What is the transmission probability T that the electron will tunnel through thick dielectric? What is T for the thin dielectric? Recall from lecture that T is

$$T \approx \frac{4E_o(V - E_o)}{V^2} e^{-2\kappa L}$$

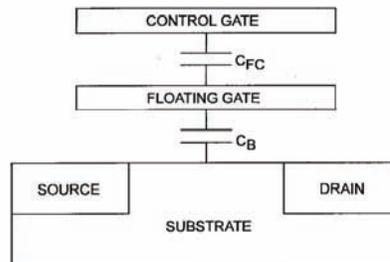
with

$$\kappa^2 = \frac{2m(V - E_o)}{\hbar^2}$$

where L is the width of the potential, V is the height of the potential above zero, and E_o is the energy of the particle above zero. Due to defects in the silicon dioxide layer between the floating gate and substrate, the effective width of the barrier is reduced to $L = 4.8$ nm.

- (d) Assume the classical picture of the electron rattling in the box. (This is especially valid if your n from (b) is much larger than 1.) In this view the electron moves with a classical speed, bouncing back and forth between the walls of the box. How frequently (in Hertz) does it “hit” the thin potential barrier?
- (e) Use your answers to (c) and (d) to find the half-life of the electron in the box. For how long will this memory cell store data?
- (f) To write a flash memory bit, we apply a 13 V potential across the floating gate as shown in the right figure. (The source, drain, and substrate electrodes are grounded, while the control gate is biased at 13 V.)

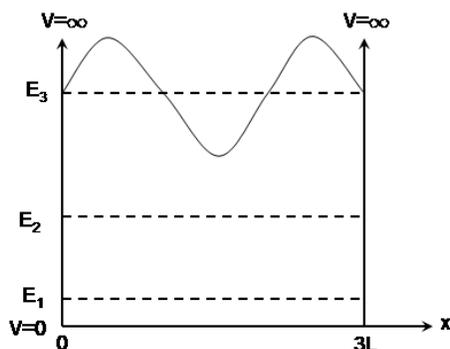
Sketch the magnitude of the electric field across the structure. Assume that the gates and substrate are metals (hence the electric field inside of them is zero). It may help to visualize the structure as capacitors in the following manner:



- (g) What is the potential drop (in volts) across the thick dielectric? What is the potential drop across the thin dielectric?
- (h) Use your answer to (g) to sketch the potential well that will be experienced by an electron incident from the substrate. [This will be a modified form of the original potential because now there is a voltage. It may help to look at the lab handout.] Sketch the ground state wave function that would be a solution to the Schrödinger equation for this potential for an electron incident from the substrate. Finally sketch the probability distribution associated with the wavefunction.

Problem 10.4 – Working with Wave Functions

- (a) The time-independent wavefunction of the infinite square-well potential's second excited state is shown in the figure below.



The time-independent wavefunction has the form:

$$\psi_3(x) = A \sin(k_3 x)$$

- (i) Find k_3 and A in terms of the other variables given.
- (ii) Show that for this wavefunction $\langle x \rangle = \frac{3}{2}L$.
- (iii) Find $\langle E \rangle$ for this wavefunction, which we'll refer to as E_3 . What is the full wavefunction, including time dependence, $\Psi_3(x, t)$? Is this a stationary state?
- (b) Out of the first three lowest energy states, which ones correspond to wavefunctions that are even, and which ones correspond to wavefunctions that are odd? Given that a system only absorbs light when a transition occurs between an even state and an odd state, at which two wavelengths would you expect to see absorption peaks if you were to shine light on this system? You may leave answers in terms of E_1 , E_2 and E_3 .

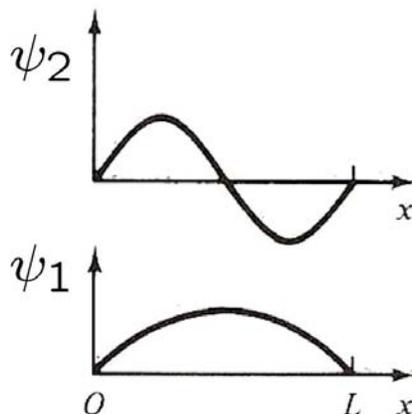
Problem 10.5 – Quantum superposition

So far, we have limited our quantum mechanical analysis to time-independent solutions of Schrodinger's Equation. However, in mixed states, or states which contain more than one eigenstate, the total solution may contain time-dependence which is physically meaningful. Let us examine this in the context of the infinite square well.

- (a) Consider the ground state and first excited state of the infinite square well of width L .

$$\psi_1(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}x\right)$$

$$\psi_2(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi}{L}x\right)$$



The complete solutions of these two eigenstates are

$$\Psi_1(x) = \frac{\sqrt{2}}{L} \sin\left(\frac{\pi}{L}x\right) e^{j(E_1/\hbar)t}$$

$$\Psi_2(x) = \frac{\sqrt{2}}{L} \sin\left(\frac{2\pi}{L}x\right) e^{j(E_2/\hbar)t}$$

What are the energies E_1 , E_2 and oscillation frequencies ω_1 , ω_2 of each state, respectively? Leave answers in terms of m , \hbar and L .

- (b) What are the probability distributions $|\Psi_1(x)|^2$ and $|\Psi_2(x)|^2$? Are these time-dependent?
- (c) What is the probability distribution of the mixed state $\Psi = \frac{1}{\sqrt{2}}\Psi_1(x) + \frac{1}{\sqrt{2}}\Psi_2(x)$? Is this time-dependent, and what is the period?
- (d) For $L=5$, sketch or plot the probability distribution in part (c) for three different times $t = \frac{\pi}{\omega_1 - \omega_2}$, $t = 0$, and $t = -\frac{\pi}{\omega_1 - \omega_2}$. You do not have to be exact but give a correct interpretation of how the wave packet evolves. In the particle picture, what is the semiclassical interpretation for how the particle is behaving?

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