

6.007 – Electromagnetic Energy: From Motors to Lasers
Spring 2011

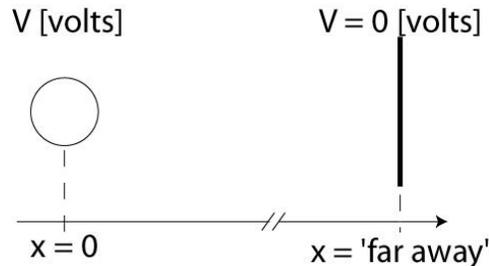
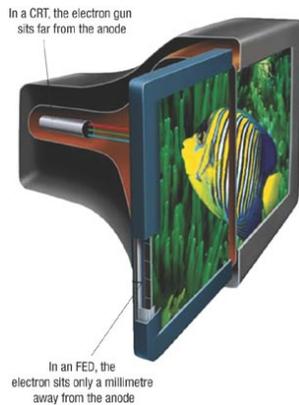
Problem Set 2: Energy and Static Fields

Due Wednesday, February 16, 2011

The reading for this problem set is chapters 9 and 13 from Shen and Kong.

Problem 2.1 – Field Emission Displays

An interesting alternative to current display technologies (Cathode Ray Tube, LCD, Plasma) is the Field Emission Display. You can think of the Field Emission Display as an array of tiny single color Cathode Ray Tubes (CRTs), with each CRT corresponding to one subpixel in the total display (with a red, a blue, and two green subpixels forming one pixel). At each of the subpixels there is a source of electrons at some electric potential and a grounded phosphor screen. To turn a subpixel on we need to raise the electric field at the source until we begin emitting electrons. This happens when the electric field reaches roughly 10^9 [V/m]. The emitted electrons then accelerate toward the screen, causing the screen to glow when they hit.



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Dean, Kenneth. "A New Era: Nanotube Displays."
Nature Photonics 1: 273-275, copyright 2007.

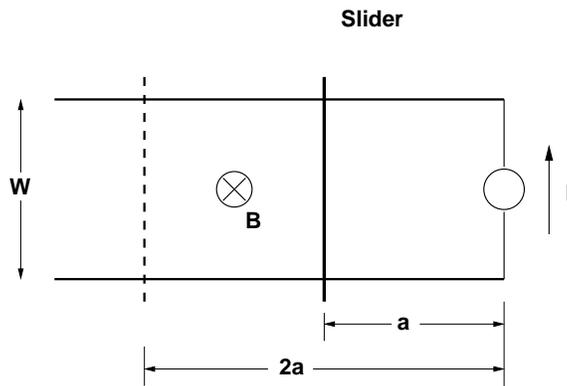
Figure 1: Left: Image from 'A new era: Nanotube displays' (Nature Photonics), Right: Our approximation of a single subpixel

We can model the electron source of a single pixel in the Field Emission Display as a charged conducting sphere and a ground plane at infinity.

- For a spherical conductor with total charge Q what is the surface electric field magnitude as a function of the sphere's radius R ?
- What is the total charge Q needed on a conducting sphere with radius R for the sphere to have potential V relative to a ground plane at infinity?
- At what radius is the surface electric field magnitude equal to 10^8 [V/m] for a charged spherical conductor at potential $V = -6$ [volts]?
- For the conducting sphere in part (c) what are the total charge Q and the surface charge density q_s ?

Problem 2.2 – Short Problem on Magnetostatics

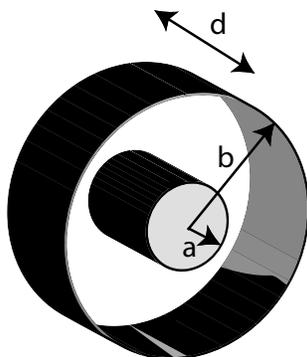
Below is a picture of a circuit consisting of a current source I connected to two conducting rails, separated by the dimension W . A conducting bar makes good electrical contact with those rails and can slide in the horizontal direction. The system is immersed in a magnetic field that goes into the paper as shown. The magnetic field made by the current I is small and may be ignored.



- What is the magnitude and direction of the magnetic force on the bar?
- The bar is made to move from position a to position $2a$ by some external mechanism not shown. How much work is done by that mechanism?

Problem 2.3 – Coaxial Cable

Consider the coaxial, air-filled cable shown in the figure below. The inner and outer strands are perfectly conducting.



- We will first consider the case of a capacitive setup for the coaxial cable. That is, the inner and outer strands of the cable are isolated. Assume that charge densities $+\rho \frac{C}{m}$ and $-\rho \frac{C}{m}$ are present on the inner and outer strands, respectively. What is the capacitance per unit length, C , of the cable in terms of the given parameters? The SI unit for capacitance is the farad, F . Show that your units are correct.
- Next, consider the case of an inductive setup for the coaxial cable. That is, the inner and outer strands of the cable are shunted at each end of the cable. Assume that current densities of $+I$ A and $-I$ A are present on the inner and outer strands, respectively (with the current

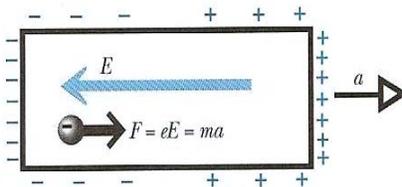
going into the paper). What is the inductance per unit length, L , of the cable in terms of the given parameters? The SI unit for inductance is the henry, H . Show that your units are correct.

- (c) Compute the value of $\frac{1}{\sqrt{LC}}$. What are the units? Where have you seen this value previously?

Problem 2.4 – Electrons in Metal¹

In 1917 Tolman and Stewart published a paper titled "The electromotive force produced by the acceleration of metals" [Physics Review 8, pages 97-116] in which they demonstrated that mobile charge carriers in metals are electrons, which was not known for certain before their experiment. Below we will consider a simple version of their experiment.

Suppose we accelerate a metal bar, which means that we accelerate the atomic cores, which are bound to each other. On average, the mobile electrons do not feel any forces, as the mutual repulsion by the mobile electrons is effectively canceled by the attraction to the atomic cores. As the bar and the atomic cores are accelerated, the mobile electrons are left behind, As a result, they pile up at the trailing end of the bar (they cannot easily leave the bar). This polarizes the bar, with negative charge on the trailing surface and positive charge (deficiency of electrons) on the leading surface. The surface charges produce an electric field \mathbf{E} in the opposite direction of acceleration.



Now let us take a metal bar of length $L = 0.1$ m and cross-sectional area $A = 0.0001$ m² and accelerate it horizontally with acceleration $a = 10$ m/s², as in the picture.

- (1) We know that the mass of the electron is 9×10^{-31} kg and the electron charge is 1.6×10^{-19} C. Assume a steady state condition is achieved very quickly. What is the voltage measured by a voltmeter attached to the end of the metal bar (assume the voltmeter does not interfere with the experiment, therefore it has negligible weight and infinite resistance)? Note: In steady state, the force due to acceleration has to match the electrostatic force.
- (2) If instead of the voltmeter, a lightbulb of 100Ω internal resistance is attached across the ends of the bar, what is the current that would be measured through the lightbulb?

¹This problem was adopted from Chabay & Sherwood, Matter and Interactions II

Problem 2.5 – Have You Ever Looked Inside a 9 V Battery?

If you took the casing off a 9 V battery (manufacturers strongly caution against disassembly of the battery, and 6.007 staff strongly agrees) this is what you would see:



- (1) Make a guess as to what the cylinders in the picture on the right represent, how many of them there are and how do they contribute to the 9 V operation of the battery?
- (2) The battery capacity of the 9 V is rated at 600 mA – hr. How many moles of charge are stored in the battery?
- (3) Dimensions of the 9 V battery is 45 mm x 25 mm x 15 mm. You are asked to make a capacitor that occupies the equivalent volume, with plates of the capacitor separated by an air gap of 0.1 μm , and folded to occupy the entire volume.
 - What is the capacitance of such a capacitor?
 - How many moles of charge would you be able to store in such a capacitor when it is charged to 9 V? Compare this stored charge to the value to the stored charge you found in question 2 above.
 - What is the electric field between the capacitor plates when it is fully charged? If the electric field is larger than the air breakdown field, explain what will happen when the capacitor is charged. Will it undergo an electrical breakdown?
- (4) Now let's do some cost analysis:
 - Given that you would spend \$ 3 to buy the 9 V battery, what is the cost per kW-hr of electrical energy produced by the battery. [Note: 1 kW-hr of electrical energy from the electrical grid costs \$ 0.15].
 - A gallon of gasoline stores 40 kW-hr of energy and cost \$ 3 per gallon. What is the cost of gasoline per kW-hr?

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