

Problem Set 5: Energy Method and Waves

Due Friday, March 18, 2011

The reading for this problem set is chapters 3, 10, 14, and 16 from Shen and Kong.

Problem 5.1 — Electrostatic Voltmeter

The picture below shows a example of a electrostatic voltmeter which uses pair of pie shaped plates.



Figure 5.2.1: Example of electrostatic voltmeter.

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Electrostatic voltmeters are often constructed as shown in Figure 5.3.2.  $N$  pairs of pie shaped plates form the stator and rotor of a series of capacitors (the figure shows six pairs of rotor plates and six pairs of stator plates). The rotor plates are attached to a conducting shaft that is free to rotate through an angle  $\theta$ . In the electrostatic voltmeter a pointer is attached to this shaft so that the deflection  $\theta$  is indicated on a calibrated scale (not shown).

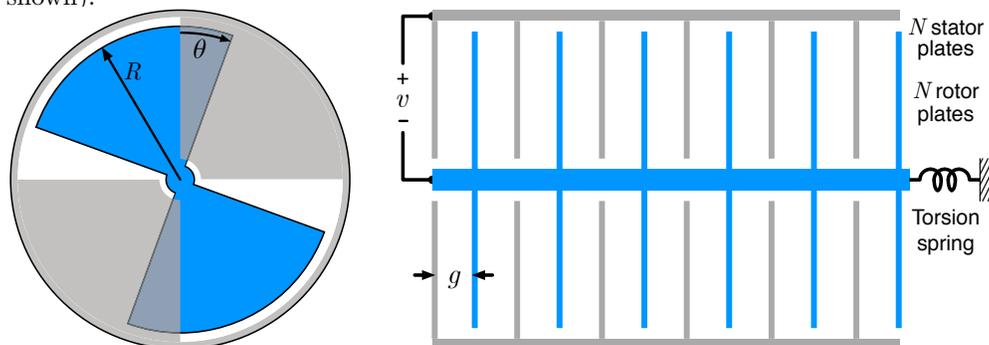


Figure 1: Figure 5.2.2 Capacitive plates of an electrostatic voltmeter (top and side views). The rotor is blue (darker), and the stator is grey (lighter).

- (a) Determine  $q(v, \theta)$ , where  $q$  is the charge on the stator and  $v$  is the voltage applied between the rotor and stator. The device is constructed so that fringing fields can be ignored and that the area of the plates is large compared with the cross section of the shaft. In addition, it is operated in the region of  $0 < \theta < \frac{\pi}{2}$  such that the plates overlap but not completely.
- (b) Use the energy method relation  $\tau_e = -dw_e/d\theta$  to find the torque of electric origin on the rotor.
- (c) The shaft is attached to a torsion spring which has a torque  $\tau_s$  when subjected to the deflection  $\theta$ , where  $\theta$  and  $\tau_s$  are related by  $\tau_s = k(\theta - \alpha)$ . Here  $k$  is the spring constant, and a “zero adjust” knob on the electrostatic voltmeter is used to set  $\alpha$  in such a way that a pointer attached to the shaft indicates 0 when  $\theta = \alpha$ . A constant voltage  $v$  is applied to the terminals. Balance the torques to find static deflection of the pointer ( $\theta - \alpha$ ) as a function of  $v$ .
- (d) In lecture, it was stated that maximum electric field sustainable in air is  $10^7$  V/m over small distances. For a MEMS device,  $g$  will be on the order of a micron. What is the maximum voltage that you could measure with this device without ionizing the air?
- (e) Now assume that  $N = 5$  plates,  $g = 3 \mu\text{m}$ , and  $R = 50 \mu\text{m}$ .  $\epsilon_0 = 8.85 \cdot 10^{-12}$  F/m.
- (i) What is the maximum and minimum voltage that the device can measure?
  - (ii) Choose  $K$  and  $\alpha$  such that the instrument gives the best possible resolution over this range. Decorate your answers with the appropriate units.
- (f) Now imagine the space between the plates is filled with a dielectric material with  $\epsilon_r = 50$ . Has the sensitivity of the device increased or decreased?

## Problem 5.2 - Quasi-static Limit

Recall that the limit for using the quasi-static (QS) approximation was given in lecture as  $\omega L \ll c$ , where  $L$  is the characteristic length scale and  $c$  is the speed of light.

- (a) How fast would a 1 cm motor have to spin to break the QS limit (state your answer in RPMs).
- (b) Clearly we are not going to break the approximation with the 1 cm motor. Let’s go the other way. How big does a 3 KHz motor need to be to break the QS approximation?
- (c) It is possible to visualize the QS limit as a result of the finite speed of light. The condition states that *a change in the system must be known by all other parts before another change is made*. Let’s re-derive the QS limit using this idea.
- (i) How long does it take for light to cross a distance  $L$ ? (Call this time  $t_p$ .)
  - (ii) A change is made at  $t = 0$ , and time  $\Delta t$  is waited before making another change. What is the constraint between  $t_p$  and  $\Delta t$  such that the QS limit holds?
  - (iii) Relate the period  $\Delta t$  to the frequency  $\omega$  to re-derive the QS condition.

**Problem 5.3 – Short Problems on Waves**

- (a) The magnetic field of a plane wave is given by:

$$\vec{\mathbf{H}} = \hat{y}H_0 \cos(kz - \omega t)$$

This plane wave is propagating in free space.

- (i) Write an expression for the *electric* field of this plane wave.
  - (ii) What is the direction of propagation for this wave (assume  $\omega$  and  $k$  are positive)?
  - (iii) If we assume that the power carried by the wave is 1,366 watts per square meter (roughly the power carried by the sunlight hitting the top of the atmosphere) what are the electric field amplitude (call it  $E_0$ ) and the magnetic field amplitude  $H_0$ ?
- (b) Consider a wave with the following electric field:

$$\vec{\mathbf{E}} = \hat{x}E_0 \cos(2\pi \times 10^7 t + 5\pi y)$$

This wave is propagating in some material that has the same permeability as free space: that is  $\mu = \mu_0$ .

- (i) In what direction is this wave propagating?
- (ii) What is the frequency of this wave (in Hz)?
- (iii) What would be the wavelength of an electromagnetic wave of this frequency in free space?
- (iv) What is the speed of light in the material?
- (v) What is the relative permittivity  $\epsilon_r$  in this material?
- (vi) Find the expression for the magnetic field associated with this electric field.
- (vii) If the electric field amplitude is 5V/m, what are the magnetic field amplitude and the Poynting vector for this wave?

## Problem 5.4 – EM Waves Passing Through a Color Filter

*This problem examines the use of the Lorentz oscillator model for describing the response of an absorbing media.*

Red, green, and blue color filters are an integral part of a pixelated full-color liquid crystal display (LCD). The white color of the LCD backlight is passed through the LCD color filters to generate colored light. The absorption spectrum of the color filters determines the color of light that will pass through.

In this problem we will analyze a particular LCD color filter, by modelling the optical properties of the filter with the Lorentz oscillator model, and using the following parameters:

Plasma Frequency,  $\frac{\omega_p}{2\pi} = f_p = 2415$  THz

Optical Transition Frequency,  $\frac{\omega_0}{2\pi} = f_0 = 724$  THz

Damping Factor,  $\frac{\gamma}{2\pi} = 555$  THz

Recall that the dielectric constant given by the Lorentz oscillator model is:

$$\epsilon(\omega) = \epsilon_0 \left( 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 + j\omega\gamma} \right)$$

All of the wavelengths given below correspond to **free space**. Therefore, to switch between  $f$  and  $\lambda$ , we use the relation  $c = f\lambda$ .

- (a) Give expressions for the real and imaginary parts of the dielectric constant for the color filter described above.
- (b) Using MATLAB, plot the real and imaginary parts **versus**  $\omega$  of the dielectric constant for the color filter in the wavelength range  $\lambda = 400$  nm to  $\lambda = 700$  nm.
- (c) An electromagnetic wave that entered the color filter (starting at  $z = 0$ , traveling in the  $+\hat{z}$  direction) has the form  $E_y(z) = E_0 \cos(\omega t - kz)$ . Since the wave is now in an absorbing medium, it will decay in amplitude.
  - (i) Give an expression for  $E_y(z)$  in phasor notation.
  - (ii) Since we are now in an absorbing medium, what is the complex wave vector,  $\tilde{k}$ , in terms of  $\sqrt{\epsilon_{real} - j\epsilon_{imag}}$ ? Assume that permeability of the color filter is  $\mu_0$ .
  - (iii) Using MATLAB and the expression in (ii) for the wave vector in the color filter, plot  $|E_y(z)|$  vs.  $z$  for  $\lambda = 400$  nm,  $\lambda = 550$  nm and  $\lambda = 700$  nm. At what value of  $z$  for each wavelength has  $|E_y(z)|$  reduced by a factor of 10? Assume  $E_0 = 1$ .
  - (iv) Based on the results of (iii), if white light is incident on this color filter, what color of visible light would pass through with highest intensity?

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6.007 Electromagnetic Energy: From Motors to Lasers  
Spring 2011

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