

6.007 – Applied Electromagnetics
Spring 2011

Problem Set 9: Introduction to Quantum Mechanics

Due Wednesday, April 27, 2011

The reading for this problem set are chapters 3 and 4 from French and Taylor.

Problem 9.1 Short Problems on the Particle Nature of light

- (a) Earlier in the semester we learned that the power delivered to the surface of the earth by sunlight is roughly $1000 \frac{W}{m^2}$. If we assume that all of this power is carried by light at $\lambda = 500 \text{ nm}$, the approximate peak emission wavelength of the sun, how many photons hit one square meter of the earth's surface every second. Assume it's daytime over the square meter.
- (b) One proposed spaceship propulsion system is called the 'solar sail'. These operate using momentum transfer, by either reflecting or absorbing photons from an external source. Let's assume the Earth is one big solar sail.
1. Assuming elastic collisions (complete reflection) how much energy is transferred by one photon at wavelength $\lambda = 500 \text{ nm}$?
 2. Repeat for inelastic collisions (complete absorption).
 3. Assuming the radius of the earth is $R_{earth} = 6371 \text{ km}$, and that about 30% of the incident light from the sun is reflected and the rest is absorbed, how hard is the sun trying to push the earth away with light?

Problem 9.2 – Photons and radio waves

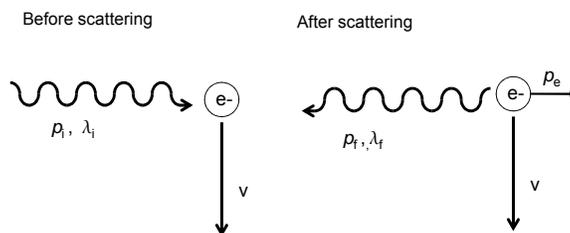
A radio station broadcasts at a frequency of 1.3 MHz with a total radiated power of 3 kW.

- (a) What is the wavelength of this radiation?
- (b) What is the energy (in electron-volts) of the individual quanta that compose the radiation? How many photons are emitted per second? Per cycle of oscillation?
- (c) A certain radio receiver must have 4 microwatts of radiation power incident on its antenna in order to provide intelligible reception. How many 1.3 MHz photons does it require per second? Per cycle of oscillation? [Fun fact: In comparison, your eye has a sensitivity of 10 photons, depending on the wavelength.^a]
- (d) Do your answers in (b) and (c) indicate that the particle nature of electromagnetic radiation (quantization of light, ie photons) can be neglected in these circumstances?

^ahttp://math.ucr.edu/home/baez/physics/Quantum/see_a_photon.html

Problem 9.3 – Compton scattering

It is proposed to measure the velocity of an electron with an energy of 100 eV by taking pictures at two different times (and at two different places), using a very sensitive digital camera and a pulsed laser. This is called a "time of flight" measurement.



In the experiment, light from the laser is incident onto the electron. The laser light is focused onto a square region of 1 wavelength in length at each location where the electron is detected. The pulsed laser can provide pulses of light at 600 nm, each with a duration of 1 psec, every 10 nsec. At least 100 photons must scatter off the electron in order to detect the electron. Assume that all 100 photons are incident perpendicular to the initial trajectory of the electron and scatter back at 180° . [You might need the following formulas for error propagation: For $f=x+y$ or $f=x-y$, $\Delta f = \sqrt{(\Delta x)^2 + (\Delta y)^2}$. For $f=xy$ or $f=x/y$, $\frac{\Delta f}{f} = \sqrt{(\frac{\Delta x}{x})^2 + (\frac{\Delta y}{y})^2}$.]

- Calculate the velocity of the electron. How far would you expect the electron to travel between each pulse of light?
- Attempting to measure the velocity of the electron impacts the velocity directly: Due to conservation of momentum the electron's momentum will be changed when a photon scatters off of it. We can get an estimate of the momentum transferred to the electron. How much transverse velocity will the electron acquire, assuming that the photons are incident perpendicular to the initial trajectory, undergo Compton scattering, and scatter back?
- What is the uncertainty in the photon momentum due to the focusing of laser light? Assume that when each photon scatters off the electron it transfers all of its momentum uncertainty to the electron. Estimate the fractional quantum uncertainty in the velocity of the electron $\Delta v/v$ after 100 photons scatter off it.
- What is the spread in photon energy ΔE associated with a 1 psec pulse?

Problem 9.4 – Heisenberg uncertainty principle in the infinite potential well.

This problem shows that the Heisenberg uncertainty principle is embedded in solutions to the Schrödinger equation.

Consider an electron in an infinite potential well of width L :

- The quantum ground state is: $\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$
- The classical probability distribution is: $P(x) = 1/L$

Calculate the following quantities:

- $\langle x \rangle_{\text{classical}}$ and $\langle x \rangle_{\text{QM}}$
- $\langle \Delta x^2 \rangle_{\text{classical}}$ and $\langle \Delta x^2 \rangle_{\text{QM}}$
- $\langle p \rangle_{\text{QM}}$ where $p = -i\hbar \frac{d}{dx}$ is the quantum momentum operator.
- $\langle \Delta p^2 \rangle_{\text{QM}}$
- The uncertainty product $\sqrt{\langle \Delta x^2 \rangle \langle \Delta p^2 \rangle}$ for the quantum case in terms of \hbar .
- Repeat (e) for a well of width $10L$. In which case is the overall uncertainty less?

Problem 9.5 – Probability Distribution

In quantum mechanics, a particle is represented by a wave function. All the information of the particle like its position and momentum (or rather the expected value of the position and momentum) are "encoded" in the wavefunction. The square of this wavefunction is the probability distribution of the particle i.e. it tells us the probability of finding the particle at a particular point in space. The probability distribution of a particle (which is the square of wavefunction) is described as follows

$$\rho(x) = Ae^{-\lambda(x-a)^2} \quad (1)$$

- (a) Calculate the normalization constant A ?
- (b) Why does the probability distribution need to be normalized?
- (c) Find the expectation value, variance and standard deviation of the probability distribution.
- (d) Briefly describe the physical meaning of expectation value and standard deviation.

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