

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Electrical Engineering and Computer Science,  
Department of Mechanical Engineering,  
Division of Bioengineering and Environmental Health,  
Harvard-MIT Division of Health Sciences and Technology

Quantitative Physiology: Cells and Tissues  
2.791J/2.794J/6.021J/6.521J/BE.370J/BE.470J/HST.541J

**Homework Assignment #6**

Issued: October 21, 2004

Due: October 28, 2004

**Reading**

Lecture 20 — Volume 2: 4.1-4.1.2.3 4.2-4.2.2.2

Lecture 21 — Volume 2: 4.2.3-4.2.3.1

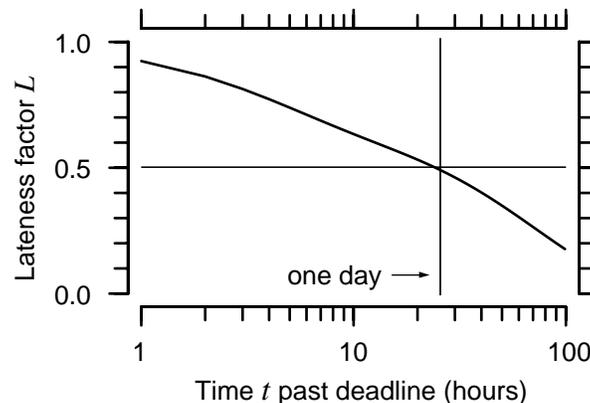
Lecture 22 — Volume 2: 4.2.3-4.2.3.2

**Announcements**

Laboratory reports are **DUE Monday October 25 at noon**. You should submit one copy of the final draft of the report. You should include the following items in an appendix: final proposal, copy of your critique of a peer's first draft, peer critique of your first draft, critique of your first draft by writing program, critique of your first draft by technical staff, photocopies of notes taken during original lab session. Note that there is a **SEVERE LATENESS PENALTY**. The grade for a late report will be multiplied by a lateness factor

$$L = 0.3e^{-t/4} + 0.7e^{-t/72}$$

where  $t$  is the number of hours late. The lateness factor is plotted below. Notice that the maximum grade for a report that is more than ONE DAY LATE is less than 50%.



This lateness factor is applied regardless of the reason for the lateness, except for health related problems or personal problems certified by the Dean's office. Specifically, lateness due to computer and/or printer problems is not exempt.

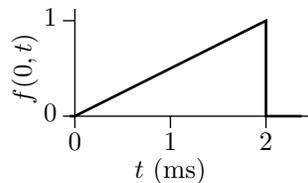
**Exam 2 will be held on Thursday, November 4 from 7:30 PM to 9:30 PM**

The exam is closed-book: notes on both sides of two  $8\frac{1}{2} \times 11$  sheets of paper may be used for reference. Calculators may be used, but computers and wireless devices may not be used. There will be no recitations on the day of the exam.

**Exercise 1.** Give a *physical* explanation for why the conduction velocity is larger in larger diameter fibers, if all other factors are the same.

**Exercise 2.** Give a *physical* explanation of the meaning of Equation 2.18 (in volume 2 of the text) without the use of equations.

**Exercise 3.** Let the function  $f(z, t)$  represent a solution to the wave equation. This solution is shown in the following figure as a function of time  $t$  at the position  $z = 0$ .



Notice that  $f(0, t)$  is non-zero for  $0 < t < 2$  ms and zero elsewhere.

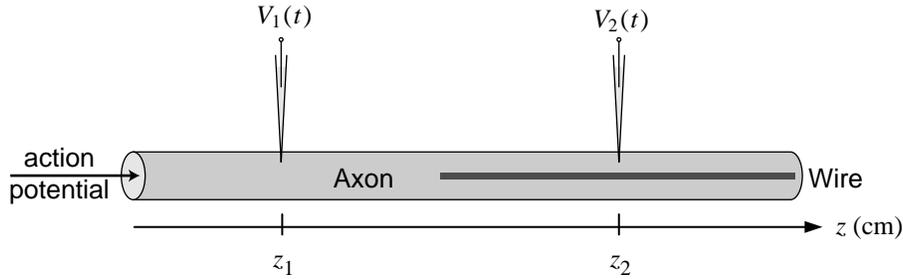
- Suppose that the wave is propagating in the  $+z$ -direction with a propagation velocity of 100 mm/ms. Plot  $f(z, t)$  versus  $z$  at time  $t = 2$  ms.
- Suppose that the wave is propagating in the  $-z$ -direction at a propagation velocity of 100 mm/ms. Plot  $f(z, t)$  versus  $z$  at time  $t = 2$  ms.

**Problem 1.** The following two experiments are performed on a squid giant axon:

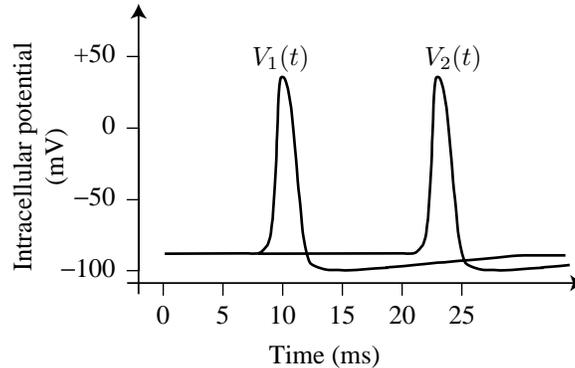
- Experiment #1:* The axon is placed in a large volume of sea water, and the size of the *transmembrane* action potential is measured by means of an intracellular micropipet and is found to have a peak-to-peak value of 100 mV. The conduction velocity is 36 m/s.
- Experiment #2:* The axon is placed in oil and the trans-membrane potential is still found to be 100 mV peak-to-peak. The peak-to-peak size of the *extracellular* action potential is 75 mV.

Estimate the expected conduction velocity in Experiment #2. State your assumptions.

**Problem 2.** A fine platinum wire with a resistance per unit length of  $130 \Omega/\text{cm}$  is inserted inside a portion of a squid axon as illustrated below.



The wire is so thin that its volume can be ignored. The axon ( $500 \mu\text{m}$  diameter) is electrically stimulated to produce a propagated action potential traveling in the  $+z$  direction. The action potential is recorded at two intracellular sites:  $V_1(t)$  is recorded at  $z = z_1$  and  $V_2(t)$  is recorded at  $z = z_2$ . The distance between the stimulus electrode (not shown) and  $z_1$  is 2 cm. Results are shown in the following figure.



The resistivity of the axoplasm of this axon is  $23 \Omega\text{-cm}$ . The resistance per unit length of the external solution is  $1.2 \Omega/\text{cm}$ . The wire begins at some location between  $z_1$  and  $z_2$ , but the exact position of the beginning is not known and should not be used in any of your calculations.

- Determine the instantaneous speed of the action potential as its peak passes the point  $z = z_1$ .
- Determine the instantaneous speed of the action potential as its peak passes the point  $z = z_2$ .
- Sketch the extracellular potential as a function of space ( $z$ ) that results at the time that the peak of the action potential passes the point  $z = z_1$ . Include distances  $z_1 - 4 < z < z_1 + 4$  cm. Indicate the scale for the y axis. Describe the important features of this plot.
- Sketch the extracellular potential as a function of space ( $z$ ) that results at the time that the peak of the action potential passes the point  $z = z_2$ . Include distances  $z_2 - 4 < z < z_2 + 4$  cm. Indicate the scale for the y axis. Describe the important features of this plot.

**Problem 3.** In measurement 1, a cylindrical unmyelinated axon of radius  $a$  is placed in a large volume of isotonic sea water and the conduction velocity of the action potential is measured to be  $\nu_1$ . The osmotic pressure of the sea water is then doubled by increasing the salt concentration, and the axon is allowed to come to osmotic equilibrium. Then, in measurement 2, the conduction velocity is measured to be  $\nu_2$ . Assume that during these experiments

- the number of ions transported through the axon membrane is negligible compared to the number of ions inside the axon,
- the volume of the axon is negligible compared to the volume of the bath,
- the *specific* electric properties of the **membrane** (the capacitance per unit area and the conductance per unit area) are the same in both measurements,
- the length of the axon remains unchanged, and
- the external resistance per unit length of the axon,  $r_o$ , is negligible compared to the internal resistance per unit length of the axon.

**Part a.** Let  $r_{i1}$  and  $r_{i2}$  represent the internal resistance per unit length of the axon in measurements 1 and 2, respectively. Determine the numerical value (or a numerical expression) for the ratio of these resistances:  $r_{i2}/r_{i1}$ . Briefly explain your reasoning.

**Part b.** Determine the numerical value (or a numerical expression) for the ratio of speeds:  $\nu_2/\nu_1$ . Briefly explain your reasoning.