Now, conditional probability will let us explain a lot of the confused arguments that people brought up about Monty Hall. And we'll see that it is a little bit confusing and where there is some correct sounding arguments that give you the wrong answer.

So let's go back and look at our Monty Hall tree that allowed us to derive the sample space and probability space for the whole process of the prize being placed and the contest picking a door and Carol opening a door. Now, this tree was way more complicated than we needed if all we were trying to do was figure out the probability of winning if you switch. But having the tree will allow us to discuss a whole bunch of other events in their probabilities that will get us a grip on some of the arguments that gave the wrong answer.

So let's look at the event, first of all that the goat is at 2. Now, this is the branch where the prize is at 2. And so in all the other branches the goat is at 2, I which means that we have these eight of the 12 outcomes in the event-- goat is at 2.

Now, let's also look at the event that the prize is at 1. That's just this branch of the tree, OK? So one of the arguments is that when the contestant is at the point where they've seen that the open door and they're trying to decide whether to stick or switch, they know that the goat is at the door 2. Say without loss of generality that that was the door that they got to look at behind, that Carol opened.

And so we want to ask the probability, given that he picked 1, what's the probability that the prize is at 1 given that the goat is at 2? That means that if you're at door 1 then you should stick if that probability is high and otherwise you shouldn't stick.

So we can look at this event, the prize at 1 given the goat at 2, and what we can see is that it's taking up exactly half of the outcomes for goat at 2 and the same kind of outcomes-- red ones and green ones. The red ones are worth an 1/18 and the green ones are worth a 1/9 in probability, and that implies that the probability that the prize is at 1 given that the goat is at 2 is 1/2. It really is.

And that's the argument that people were saying. They said, look, when the contestant sees that the goat is at door 2, and they're trying to decide whether the goat-- the prize is at the door-- is it door 1 or at the other door, and it's equally likely. And so it doesn't matter whether they stick or switch. That's a correct argument but it's not calculating the probability of the stick strategy winning.

Why? Well, because there's more information that's available than goat is at 2. The contestant not only knows that the goat is at 2 and trying to figure out the probability that the prize is at 1, but the contestant knows what door he picked.

So let's suppose that the contestant did pick door 1 and learned that the goat was at door 2, that's a different event. If the blue one is marked off at the places where the contestant picks one, this is where the door is picked-- is 1 and here's 1 and here's 1. This 1 splits into one event, this 1 splits into one event, but this choice of 1 splits into two outcomes.

And so when we look at the event that both the goat is at 2 and the contestant picked 1, which is what the contest really knows when they get to see that there's a goat at door 2, we wind up with the overlap of just three outcomes. Two outcomes that have probability 1/8 and one outcome that has probability a 1/9. It's just those three.

And the result is that the probability that the prize is at 1 given that you picked 1 and the goat is at 2-- so this is the event-- goat at 2 and picked 1, these three outcomes. The prize is at 1 is these two outcomes, which are each worth an 1/18 and this is an outcome that's worth a 1/9. So the prize at 1 outcomes amount to 1/2 of the total probability of this event, goat at 2 picked at 1. So, again, the probability that the prize is at 1 given that the contestant picked 1 and saw the goat at 2 is a 1/2 also. That's confusing.

So it seems as though the contestant may as well stick because at the point that he has to decide whether to stick or switch, and he knows where-- he sees where the goat is and he knows what door he's picked, it's 50-50 whether he should stick or switch. The probability that the prize is at door 1 that he picked is a 1/2, so it really doesn't matter if he stays there or if he decides to switch to the unopened door.

But wait a minute, that's not right because the contestant not only knows what door he picked, not only knows that there's a goat behind a given door that Carol has opened, but he knows that Carol has opened that door. That's how he got to know that the goat was there.

So let's go back and look at the tree. What basically the previous two arguments are conditioning on the wrong events. It's a typical mistake and one that you really have to watch out for.

So if you use the correct event, what we're looking at is the contestant knows that they've picked door 1. That's the outcomes of picked door 1 are marked here. In addition, the contestant will get to know, for example, in a play of the game that Carol has opened door 2. Carol opening door 2 is quite a different event from the goat being at 2.

This is a picture of the outcomes in Carol opening door 2, and we're interested in the intersection of them. That is, just this guy that's in both and this guy that's in both. There they are.

And so what we can do is identify that the event that you picked 1 and that Carol opened door 2 consists simply of two outcomes-- one worth an 1/18 and one worth a 1/9. Now, of these two outcomes, which one has the prize at 1? Well, only that one. Remember the first component here is where the prize is.

And so the prize at 1 event among the given picked 1 and opened 2 is just this red outcome. Now, the red outcome has probability 1/18 and the green outcome has probability that's twice as much. So that means that relative to this event, the probability that the prize is at 1 given that you picked 1 and opened 2 is actually 1/18 over 1/18 plus 1/9, or 1/3.

So given that you picked 1 and you get to see what Carrol did, the probability the prize is at the door you picked is only 1/3, which means that if you stick you only have a 1/3 chance of winning. You should switch. And if you do, you'll have a 2/3 probability of winning.

So when we finally condition on everything that we know, which is the contestant knows what door he picked and what door Carol opened, then we discover that it correctly-- as we deduced previously-- that the probability of switching wins is 2/3. So we're not trying to rederive the fact that the probability of switching wins is 2/3. We're trying to illustrate a very basic blunder that you have to watch out for, which is when you're trying to reason about some situation and you condition on some event that you think summarizes what's going on, if you don't get the conditioning event right, you're going to get the wrong answer.

So it's easy to see how many people got confused, and, in fact, finding the right event can be tricky. When in doubt, the 4 step method with constructing the tree where you're not even thinking about conditional probabilities but you're just examining the individual outcomes is a good fall back to avoid these kinds of confusing situations.