

Issued: May 5, 2003

Problem Set 11

Due: May 9, 2003

Problem 1: Cheap Heat

The heat engine described in the Chapter 13 of the notes catches your fancy because it seems to be versatile. You don't need a heat engine but you do want to heat your house more efficiently. You figure that you can make the device work as a heat pump, transferring energy from the lower temperature outdoors to your higher temperature living room. The cycle of the heat engine is pictured here (the cycle of your heat pump may be different but has the same rectangular shape).

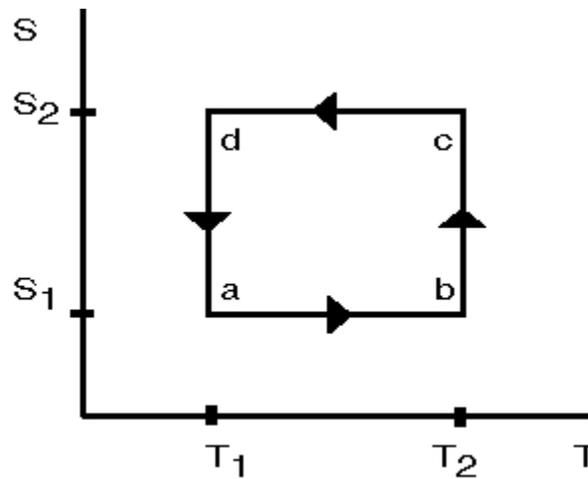


Figure 11-1: Heat Engine Cycle

You decide to simplify the design by using only one dipole, so there are only two states, called up and down, with energies

| State | Energy |
|-------|----------|
| up | $-m_d H$ |
| down | $m_d H$ |

Table 11-1: System Parameters

where $m_d = 1.165 \times 10^{-29}$ Joule-meters/ampere.

In a heat pump the idea is to take heat from the low-temperature (T_1) environment (in this case the outdoors) into the working material (in this case the magnetic dipole) in order to heat the material inside the high-temperature environment. You decide to run the system using a rectangular path similar to the one pictured above.

- a. To use the device as a heat pump, you can either run it as shown above or in the reverse direction. Which should you choose? Hint: a reversible heat engine, when run backwards, can serve as either a refrigerator or a heat pump.

Your living room is at room temperature (17 degrees Celsius). You want your device to work when the temperature outdoors is freezing (0 degrees Celsius).

- b. Express the indoor and outdoor temperature in Kelvin.

Continuing your design, you want to calculate how much energy can be delivered to the living room. You figure that an inexpensive permanent magnet could be moved toward and away from the dipole, and its field at the dipole could be made as large as 3000 amperes/meter. You decide to run the cycle as follows:

| | | | | |
|----------------|----------|----------|----------|----------|
| Point | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
| <i>H</i> (A/m) | ? | 3000 | 1000 | ? |

Table 11-2: More Parameters

Now you want to calculate the work and heat into the system during each of the four legs of the cycle. On each leg, the change in energy is related to the work and heat as $dE = dw + dq$. According to Chapter 13, the work and heat are related to changes dH and dS as $dw = (E/H)dH$ and $dq = TdS$. The way the cycle is defined it is relatively easy to find dq since on each leg either S or T is constant.

- c. In order to analyze this cycle you must first determine the magnetic field H_a and H_d at points a and d . Focus first on the leg c to d , where $dS = 0$ and $dT = T_2 - T_1$. This equation from Chapter 13 is helpful:

$$TdS = \left(\sum_i p_i (E_i(H) - E)^2 \right) \frac{1}{k_B T} \left(\frac{1}{T} dT - \frac{1}{H} dH \right) \tag{11-1}$$

Use this equation to determine a relationship among T_1 , T_2 , H_c , and H_d .

- d. From this relationship find H_d .

The “coefficient of performance” of a heat pump is defined as the ratio of heat released to the hot environment to the work done on the system, in our case by the magnetic field. The larger this coefficient, the less power is needed for the same amount of heating. Surprisingly, the coefficient of performance can be found without actually solving for the amount of energy converted. The coefficient is therefore universal, not depending on the details of the refrigerator but only on the two temperatures T_1 and T_2 .

- e. Find the coefficient of performance of this heat pump.
- f. Now focus on the leg b to a . Determine a relationship among T_1 , T_2 , H_a , and H_b .
- g. Calculate the magnetic field H_a .
- h. Next you want to know the work and heat into the system during the four legs of the cycle. First consider the top leg, d to c . What is the heat into the system dq on that leg?

To go further you have to calculate the probabilities, since you need them to find the energy E at each of the four corners. You already know the temperature and magnetic field at each corner, so it is straightforward to find α and then the probabilities using these equations from Chapter 13:

$$p_i = e^{-\alpha} e^{-E_i/k_B T} \tag{11-2}$$

$$\alpha = \ln \left(\sum_i e^{-E_i/k_B T} \right) \tag{11-3}$$

Because the magnetic energy is so small compared with thermal energy $k_B T$, the probabilities are all very close to 0.5. You may find it necessary to retain a lot of significant figures, or else use a suitable approximation.

- i. Find the two probabilities p_{up} and p_{down} at each of the four corners of the cycle. Note that for adiabatic legs, the probabilities do not change so for example they are the same at points a and b . Hint: you can check your results with these approximate answers – at point a , $p_{up} = 0.5 + 4.3664 \times 10^{-6}$ and at point c , $p_{down} = 0.5 - 1.455 \times 10^{-6}$.
- j. Find the energy $E = \sum_i E_i p_i$ for each of the four corners.
- k. Find the difference between the two entropies, $S_2 - S_1$. Since S_1 and S_2 are very nearly equal, you will have to keep many significant figures before taking the difference between them.
- l. Find the heat dq for each of the four legs (you have already found one of these). Hint: the heat is zero for two of the legs.
- m. Find the work dw for each of the four legs by calculating the change in energy and subtracting the heat.
- n. Find the net work into the system in one cycle, starting at point a and ending up at the same point. Hint: it is positive, meaning that energy actually is put in as work.
- o. Calculate the ratio of heat out at the higher temperature T_2 to work in during one cycle. Compare this to the coefficient of performance calculated earlier.
- p. How many cycles would this machine require to heat one gram of air one degree Celsius? (Assume that the specific heat of air is approximately 0.715 Joules per degree Celsius per gram.)
- q. You want to speed up the machine by using more dipoles. You have heard that Avogadro's number 6.02×10^{23} is the number of molecules in a reasonable size chunk of material, so you decide to use that many dipoles. Assuming you can get it all working, how many cycles would your new machine need to heat the same mass the same amount?

Problem 2: Information is Cool

An exciting application of thermodynamics is in the study of biological systems. The energy cycles that we have seen in class are the starting point for modelling processes that occur throughout nature, whether it be the dynamics of a muscle cell or an animal's thermal regulation facilities. For this problem we will examine forms of energy loss and storage in the human body. Please note that the unit Calorie, used to indicate the energy content of foods you consume, is actually a kilocalorie which is equal to 4.1868×10^3 Joules. The approximate basal metabolic rate of an average 70 kg adult male is 75 Calories/hour (that is, the body dissipates 75 Calories per hour to maintain its normal body temperature while performing normal bodily tasks). What a person doesn't take in through diet will ultimately be burned from the body's store of fat, and when a person takes in more Calories than is needed the excess energy is normally stored as fat. One kilogram of fat corresponds to 33.1×10^6 Joules of metabolized energy.

Have fun with these simple exercises in energy conversion.

- a. Express the basal metabolic rate, 75 Calories/hour, in Watts, or Joules per second. Optional question: You know what a 100-Watt light bulb looks like when turned on. Why don't people look like that?
- b. How many Calories will a person who does no exercise (75 Calories/hr) need to eat in one day to maintain normal body weight?

- c. A person who jogs normally burns 550, rather than 75, Calories/hour. Consider two people who follow exactly the same routine and eat the same food for one month, except that one jogs for thirty minutes each day. If the jogger neither gains nor loses weight during this month, how much weight does the non-jogger gain?
- d. Suppose Room xx-xxx is losing heat at a rate of 9000×10^3 Joules/hour to the outdoors, and that the 6.050/2.110 lecture has 24 students in the room, each dissipating heat at a rate of 100 Watts when awake, or 70 Watts when sleeping. The professor, who is dissipating 200 Watts, can either be interesting, thereby keeping the students awake, or boring, putting them to sleep. How many students should be kept awake in order for the temperature of the room to remain constant?
- e. Paul drinks his root beer cold (0 degrees Celsius), arguing that a significant portion of the Calories consumed are used to raise the temperature of the beverage to body temperature (98.6 degrees Fahrenheit), rather than going into fat. Are you convinced? Consider that a 355 ml can of A&W root beer has 170 Calories, and that about one Calorie is needed to raise one liter of water one degree Celsius.
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Turning in Your Solutions

You may turn in this problem set by e-mailing your written solutions, M-files, and diary.

Do this either by attaching them to the e-mail as *text* files, or by pasting their content directly into the body of the e-mail (if you do the latter, please indicate clearly where each file begins and ends). If you have figures or diagrams you may include them as graphics files (GIF, JPG or PDF preferred) attached to your email. The deadline for submission is the same no matter which option you choose.

Your solutions are due 5:00 PM on Friday, May 9, 2003. Later that day, solutions will be posted on the course website.

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