MASSACHUSETTS INSTITUTE of TECHNOLOGY
Department of Electrical Engineering and Computer Science

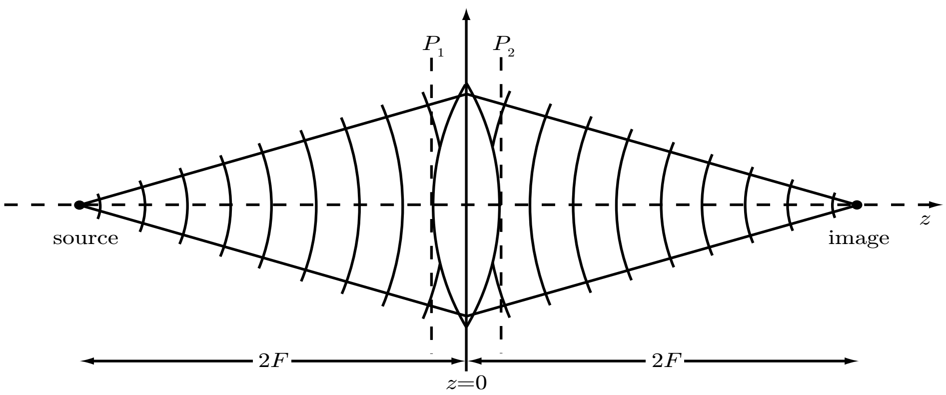
6.161 Modern Optics Project laboratory

Problem Set No. 9	**Fourier Optics**	Issued Thrs. 11/17/2005
Spring Term, 2005	-	Due Tues. 11/29/2005

Reading recommendation: Class Notes, Chapter 8. Be neat in your work!

Problem 9.1

It is well known that a lens will image a point-source at a distance $2F$ in front of the lens to a point at a distance $2F$ in back of the lens as shown.



(a) Write an expresion for the wavefront incident on the lens $\underline{U}_1(\bar{\rho}_1)$ at the plane, P_1, located at $z = 0_-$ (lens is thin).

(b) Using the paraxial approximation, simplify the expression obtained in part (a).

(c) Similarly, write an expression for the wavefront exiting the lens $\underline{U}_2(\bar{\rho}_2)$ at the plane, P_2, located at $z = 0_+$ (lens is thin).

(d) Using the paraxial approximation, simplify the expression obtained in part (c).

(e) What then is the expression for the thin lens transformation $\underline{t}_l(\bar{\rho})$?

Problem 9.2

In a classical two-lens coherent optical processor with lenses of focal length F, two signal transparencies of transmittance $g_a(x_1, y_1)$ and $g_b(x_1, y_1)$ are inserted in the $\bar{\rho}_1$ (input) plane with centers at $(a, 0, 0)$ and $(-a, 0, 0)$ respectively. The Fourier-plane filter $\underline{t}_h(\bar{\rho})$ is a sinusoidal grating with amplitude transmittance

$$\underline{t}_h(\bar{\rho}) = \underline{t}_h(F\lambda\,\bar{f}) = \frac{1}{2} + \frac{1}{2}\sin(2\pi\,f_g x + \phi)$$

where f_g and ϕ represent the grating frequency and position respectively.

(a) Calculate the amplitude distribution at the output plane of the processor.

(b) For the special case where f_g= a/λF and $\phi = 0$, comment on the significance of the output. Repeat the process for $\phi = 90^\circ$.

Problem 9.3 - Color Image Processing

In the polychromatic optical processor shown below, S is a white-light point source, and L_0, L_1 and L_2 are achromatic lenses. Two three-color (each monochromatic) signal transparencies $g_a(x, y, \lambda)$ and $g_b(x, y, \lambda)$ are placed in the input plane P_1 at the points (0, d) and (0, -d) respectively and in contact with a high-efficiency diffraction grating $G(x, y)$ with transmission function

$$t_G(x, y) = \frac{1}{2}[1 + cos(2\pi f_g x)]$$

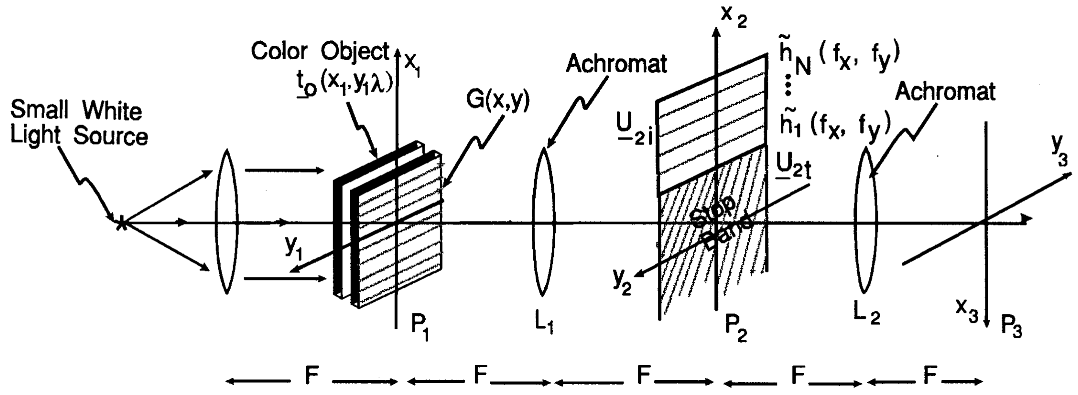


Figure 1: Typical white light optical processing system for a color object

(a) Write an expression for $U_{2i}(f_x, f_y, \lambda_n)$ where n is the color subscript. Here [n =1(red), 2(green), 3 (blue)].

(b) Suppose the Fourier-plane filter in the P_2 plane is given by $h(f_x, f_y) = \Sigma_n(1/2)[1 + sin(2\pi d f_{yn})]$ for $x_2 > 0$, and is opaque for $x_2 < 0$. Sketch the three subfilters, corresponding to the three values of n, in physical space.

(c) Write an expression for the output $U_3(f_x, f_y, \lambda)$ in the P_3 plane for one of the colors using its corresponding Fourier-plane subfilter.

(d) To perform full color subtraction, we must synthesize a composite filter. Make a sketch of the desired composite filter. On your diagram clearly specify and give expressions for: (1) its periodicity, Λ_{gy}, (2) the centroids and (3) widths of the filter segments (if any), and (4) the location of the desired subtraction image in the output plane.

(e) Assuming the two input signal color transparencies, $g_a(x, y, \lambda)$ and $g_b(x, y, \lambda)$, each have a spatial frequency bandwidth of B_x x B_y, and that each have a spatial extent that falls within a rectangle of dimension D_x x D_y. What are the conditions on λ_n, f_g, D_x, D_y and d so that the desired output image is clearly separated from other images in the output plane?

(f) Comment on the practical limitations of this system.