6.453 Quantum Optical Communication
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6.453 *Quantum Optical Communication* Lecture 12

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6.453 *Quantum Optical Communication* - Lecture 12

- Announcements
  - Turn in problem set 6
  - Pick up problem set 6 solutions, problem set 7, lecture notes, slides
- Single-Mode and Two-Mode Linear Systems
  - Attenuators
  - Phase-Insensitive Amplifiers
  - Phase-Sensitive Amplifiers
  - Entanglement

Single-Mode Linear Systems: Quantum Case

- Attenuation

$$\hat{a}_{\text{in}} \rightarrow \boxed{0 < L < 1} \rightarrow \hat{a}_{\text{out}} = \sqrt{L}\,\hat{a}_{\text{in}} + \sqrt{1-L}\,\hat{a}_L$$

minimum-noise case: $\hat{a}_L$ in its vacuum state

- Phase-Insensitive Amplification

$$\hat{a}_{\text{in}} \rightarrow \boxed{G > 1} \rightarrow \hat{a}_{\text{out}} = \sqrt{G}\,\hat{a}_{\text{in}} + \sqrt{G-1}\,\hat{a}_G^\dagger$$

minimum-noise case: $\hat{a}_G$ in its vacuum state

Output State of the Attenuator

- Quantum Characteristic-Function Analysis:

$$\chi_A^{\rho_{\text{out}}}(\zeta^*, \zeta) = \chi_A^{\rho_{\text{in}}}(\sqrt{L}\zeta^*, \sqrt{L}\zeta)\chi_A^{\rho_L}(\sqrt{1-L}\zeta^*, \sqrt{1-L}\zeta)$$

$$= \chi_A^{\rho_{\text{in}}}(\sqrt{L}\zeta^*, \sqrt{L}\zeta)e^{-(1-L)|\zeta|^2}, \text{ for vacuum-state } \hat{a}_L$$

$$= e^{-\zeta^*\sqrt{L}\alpha_{\text{in}} + \zeta\sqrt{L}\alpha_{\text{in}}^* - |\zeta|^2}, \text{ for coherent-state } \hat{a}_{\text{in}}$$

$$= \langle\sqrt{L}\alpha_{\text{in}}|e^{-\zeta^*\hat{a}_{\text{out}}}e^{\zeta\hat{a}_{\text{out}}^\dagger}|\sqrt{L}\alpha_{\text{in}}\rangle$$

- Attenuation Preserves State Classicality:

$$P_{\text{in}}(\alpha, \alpha^*) \longrightarrow \frac{1}{L}P_{\text{in}}\left(\frac{\alpha}{\sqrt{L}}, \frac{\alpha^*}{\sqrt{L}}\right)$$

Output State of the Phase-Insensitive Amplifier

- Quantum Characteristic-Function Analysis:

$$\chi_A^{\rho_{\rm out}}(\zeta^*,\zeta) = \chi_A^{\rho_{\rm in}}(\sqrt{G}\zeta^*,\sqrt{G}\zeta)\chi_N^{\rho_G}(-\sqrt{G-1}\zeta,-\sqrt{G-1}\zeta^*)$$

$$= \chi_A^{\rho_{\rm in}}(\sqrt{G}\zeta^*,\sqrt{G}\zeta),\ \text{for vacuum-state } \hat{a}_G$$

$$= e^{-\zeta^*\sqrt{G}\alpha_{\rm in}+\zeta\sqrt{G}\alpha_{\rm in}^*-G|\zeta|^2},\ \text{for coherent-state } \hat{a}_{\rm in}$$

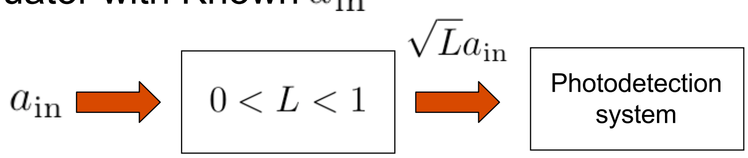
$$= \langle\sqrt{G}\alpha_{\rm in}|e^{-\zeta^*\hat{a}_{\rm out}}e^{\zeta\hat{a}_{\rm out}^\dagger}|\sqrt{G}\alpha_{\rm in}\rangle e^{-(G-1)|\zeta|^2}$$

- Phase-Insensitive Amplification Preserves Classicality:

$$P_{\rm in}(\alpha,\alpha^*) \longrightarrow \frac{1}{G}P_{\rm in}\left(\frac{\alpha}{\sqrt{G}},\frac{\alpha^*}{\sqrt{G}}\right)\star\frac{e^{-|\alpha|^2/(G-1)}}{\pi(G-1)}$$

Semiclassical Photodetection Results

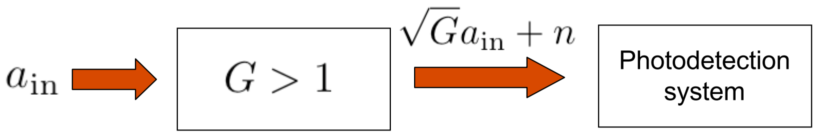
- Attenuator with Known $a_{\rm in}$



  - Direct detection: N Poisson with mean $L|a_{\rm in}|^2$
  - Homodyne detection: $\alpha_\theta \sim N(\sqrt{L}a_{{\rm in}_\theta}, 1/4)$
  - Heterodyne detection: α_1,α_2 SI , $\alpha_i \sim N(\sqrt{L}a_{{\rm in}_i}, 1/2)$

Semiclassical Photodetection Results

- Phase-Insensitive Amplifier with Known a_{in}



$$n = n_1 + jn_2, \text{ with } n_1, n_2 \text{ SI }, n_i \sim N(0, (G-1)/2)$$

  - Homodyne detection: $\alpha_\theta \sim N(\sqrt{G}a_{\text{in}_\theta}, (2G-1)/4)$
  - Heterodyne detection: $\alpha_1, \alpha_2 \text{ SI }, \alpha_i \sim N(\sqrt{G}a_{\text{in}_i}, G/2)$

Semiclassical Direct Detection Results

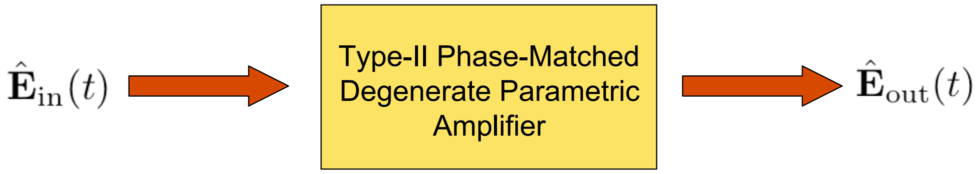
- Phase-Insensitive Amplifier with Known a_{in}

$$\langle N \rangle = \underbrace{G|a_{\text{in}}|^2}_{\text{amplified signal}} + \underbrace{(G-1)}_{\text{amplifier noise}}$$

$$\langle \Delta N^2 \rangle = \underbrace{\langle \mathcal{N} \rangle}_{\text{shot noise}} + \underbrace{\langle \Delta \mathcal{N}^2 \rangle}_{\text{excess noise}}, \text{ where } \mathcal{N} \equiv |\sqrt{G}a_{\text{in}} + n|^2$$

$$= \underbrace{[G|a_{\text{in}}|^2 + (G-1)]}_{\text{shot noise}} + \underbrace{[2G(G-1)|a_{\text{in}}|^2 + (G-1)^2]}_{\text{excess noise}}$$

Two-Mode Linear System: Parametric Amplifier



- Input and Output Field Operators:

$$\hat{\mathbf{E}}_{\text{in}}(t) = \frac{\hat{a}_{\text{in}_x} e^{-j\omega t}}{\sqrt{T}} \mathbf{i}_x + \frac{\hat{a}_{\text{in}_y} e^{-j\omega t}}{\sqrt{T}} \mathbf{i}_y + \text{ other terms}$$

$$\hat{\mathbf{E}}_{\text{out}}(t) = \frac{\hat{a}_{\text{out}_x} e^{-j\omega t}}{\sqrt{T}} \mathbf{i}_x + \frac{\hat{a}_{\text{out}_y} e^{-j\omega t}}{\sqrt{T}} \mathbf{i}_y + \text{ other terms}$$

Two-Mode Linear System: Parametric Amplifier

- Two-Mode Bogoliubov Transformation:

$$\begin{aligned} \hat{a}_{\text{out}_x} &= \mu \hat{a}_{\text{in}_x} + \nu \hat{a}^\dagger_{\text{in}_y} \\ \hat{a}_{\text{out}_y} &= \mu \hat{a}_{\text{in}_y} + \nu \hat{a}^\dagger_{\text{in}_x} \end{aligned}$$

where $|\mu|^2 - |\nu|^2 = 1$

- Phase-Insensitive Amplifier:

$$\hat{a}_{\text{in}_x} = \text{signal input}, \hat{a}_{\text{out}_x} = \text{signal output}$$

$$\hat{a}_{\text{in}_y} \text{ in vacuum state}$$

$$\mu = \sqrt{G} > 1, \nu = \sqrt{G-1} > 0$$

Two-Mode Linear System: Parametric Amplifier

- Bogoliubov Transformations for the Diagonal Basis:

$$\hat{a}_{\text{out}_{\pm 45}} = \mu \hat{a}_{\text{in}_{\pm 45}} \pm \nu \hat{a}^{\dagger}_{\text{in}_{\pm 45}}$$

- Phase-Sensitive Amplifier: $\mu = \sqrt{G} > 1, \nu = \sqrt{G-1} > 0$

$$\langle \hat{a}_{\text{out}_{45_1}} \rangle = \underbrace{(\sqrt{G} + \sqrt{G-1})}_{\text{amplification}} \langle \hat{a}_{\text{in}_{45_1}} \rangle$$

$$\langle \hat{a}_{\text{out}_{45_2}} \rangle = \underbrace{(\sqrt{G} - \sqrt{G-1})}_{\text{attenuation}} \langle \hat{a}_{\text{in}_{45_2}} \rangle$$

Output State of the Parametric Amplifier

- Quantum Characteristic-Function Analysis:

$$\begin{aligned} \chi_W^{\rho_{\text{out}}}(\zeta_x^*, \zeta_y^*, \zeta_x, \zeta_y) &\equiv \langle e^{-\zeta_x^* \hat{a}_{\text{out}_x} - \zeta_y^* \hat{a}_{\text{out}_y} + \zeta_x \hat{a}^{\dagger}_{\text{out}_x} + \zeta_y \hat{a}^{\dagger}_{\text{out}_y}} \rangle \\ &= \chi_W^{\rho_{\text{in}}}(\xi_x^*, \xi_y^*, \xi_x, \xi_y) \end{aligned}$$

$$\xi_x \equiv \sqrt{G}\zeta_x - \sqrt{G-1}\zeta_y^* \text{ and } \xi_y \equiv \sqrt{G}\zeta_y - \sqrt{G-1}\zeta_x^*$$

- Important Special Case: Vacuum-State Inputs

$$\begin{aligned} \chi_A^{\rho_{\text{out}}}(\zeta_x^*, \zeta_y^*, \zeta_x, \zeta_y) &= e^{-G(|\zeta_x|^2 + |\zeta_y|^2) + 2\sqrt{G(G-1)}\text{Re}(\zeta_x \zeta_y)} \\ &\neq \chi_A^{\rho_{\text{out}_x}}(\zeta_x^*, \zeta_x) \chi_A^{\rho_{\text{out}_y}}(\zeta_y^*, \zeta_y) \end{aligned}$$

output state is *entangled*

Coming Attractions: Lecture 13

- Lecture 13:
  Four-Mode Quantum Systems
  - Polarization entanglement
  - Qubit teleportation